

Computer algebra independent integration tests

4-Trig-functions/4.6-Cosecant/4.6.1.2-d-csc-ⁿ-a+b-csc-^m

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Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [59]. This is test number [129].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100.00 (59)	% 0.00 (0)
Mathematica	% 89.83 (53)	% 10.17 (6)
Maple	% 69.49 (41)	% 30.51 (18)
Maxima	% 42.37 (25)	% 57.63 (34)
Fricas	% 69.49 (41)	% 30.51 (18)
Sympy	% 5.08 (3)	% 94.92 (56)
Giac	% 64.41 (38)	% 35.59 (21)
Mupad	% 55.93 (33)	% 44.07 (26)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

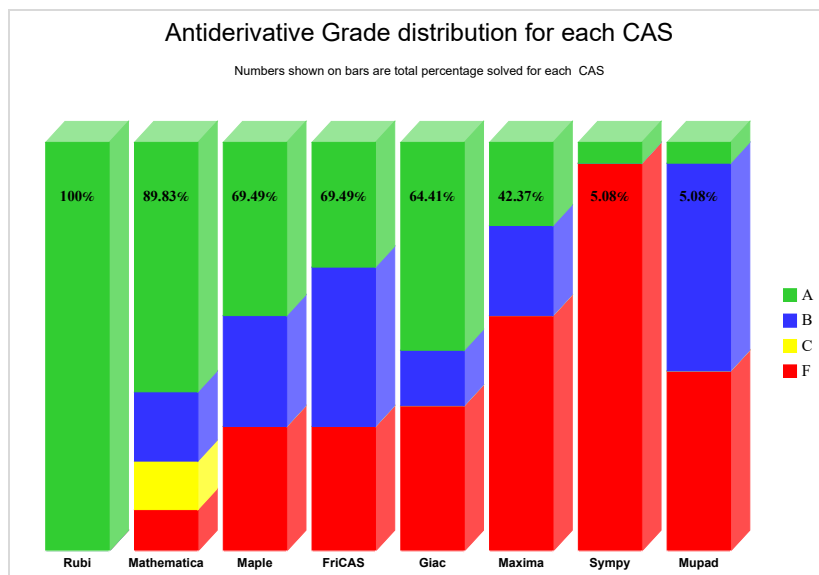
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

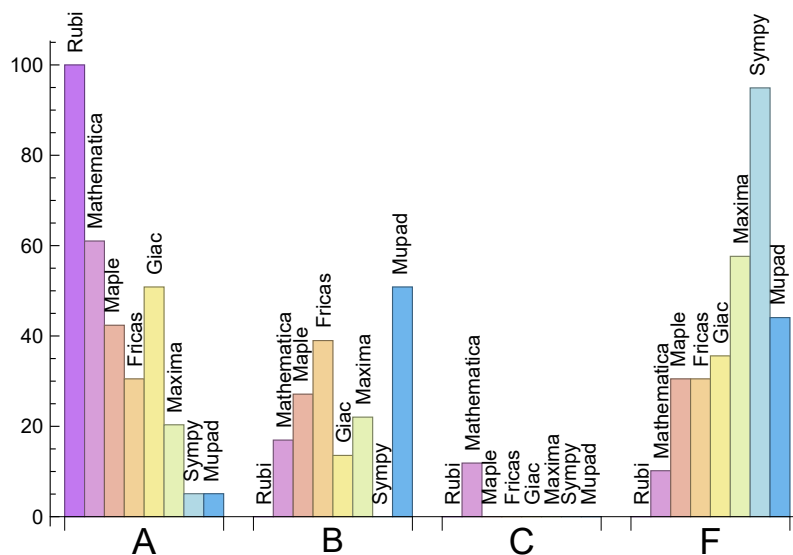
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	61.02	16.95	11.86	10.17
Maple	42.37	27.12	0.00	30.51
Maxima	20.34	22.03	0.00	57.63
Fricas	30.51	38.98	0.00	30.51
Sympy	5.08	0.00	0.00	94.92
Giac	50.85	13.56	0.00	35.59
Mupad	5.08	50.85	0.00	44.07

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This is the typical normal failure **F**.

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	6	100.00 %	0.00 %	0.00 %
Maple	18	100.00 %	0.00 %	0.00 %
Maxima	34	61.76 %	0.00 %	38.24 %
Fricas	18	100.00 %	0.00 %	0.00 %
Sympy	56	94.64 %	5.36 %	0.00 %
Giac	21	90.48 %	0.00 %	9.52 %
Mupad	26	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

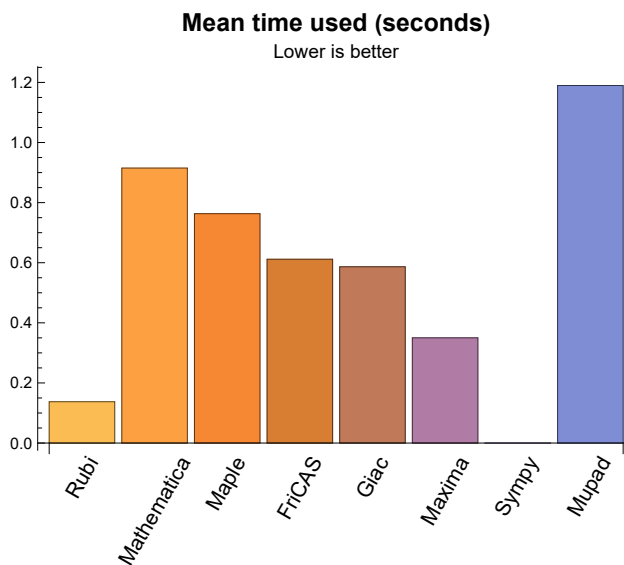
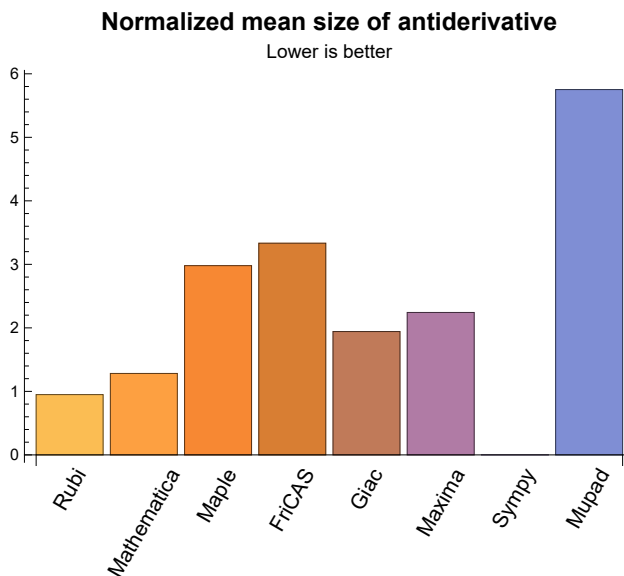
1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.14	113.10	0.95	69.00	1.00
Mathematica	0.92	95.26	1.28	76.00	1.16
Maple	0.76	248.12	2.98	100.00	1.53
Maxima	0.35	109.96	2.24	95.00	1.85
Fricas	0.61	252.10	3.33	181.00	3.05
Sympy	0.00	0.00	0.00	0.00	0.00
Giac	0.59	127.71	1.94	81.50	1.52
Mupad	1.19	742.73	5.75	89.00	1.81

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.



1.4 list of integrals that has no closed form antiderivative

{57,58,59}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima abs_integrate was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user slelievre at <https://>

ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount = 1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')
the_variable = evalin(symengine, 'x')
anti = int(integrand, the_variable)
```

Which gives $\sin(x)^2/2$

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
 2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
 3. integer. Leaf size of result.
 4. integer. Leaf size of the optimal antiderivative.
 5. number. CPU time used to solve this integral. 0 if failed.
 6. string. The integral in Latex format
 7. string. The input used in CAS own syntax.
 8. string. The result (antiderivative) produced by CAS in Latex format
 9. string. The optimal antiderivative in Latex format.
 10. integer. 0 or 1. Indicates if problem has known antiderivative or not
 11. String. The result (antiderivative) in CAS own syntax.
 12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
- The following field present only in Rubi and Mathematica Tables*
13. integer. 1 if result was verified or 0 if not verified.
- The following fields present only in Rubi Tables*
14. integer. Number of rules used.
 15. integer. Integrand leaf size.
 16. real number. Ratio of field 14 over field 15
 17. integer. 1 if result was verified or 0 if not verified.
 18. String of form "{n,n,...}" which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 2, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 28, 29, 30, 31, 32, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 53, 57, 58, 59 }

B grade: { 1, 3, 4, 5, 19, 20, 36, 37, 38, 52 }

C grade: { 21, 22, 23, 24, 25, 26, 27 }

F grade: { 33, 34, 35, 54, 55, 56 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 11, 12, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 52, 53, 57, 58, 59 }

B grade: { 8, 9, 10, 13, 14, 15, 16, 17, 18, 19, 20, 47, 48, 49, 50, 51 }

C grade: { }

F grade: { 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 54, 55, 56 }

2.1.4 Maxima

A grade: { 4, 5, 6, 16, 36, 37, 38, 52, 53, 57, 58, 59 }

B grade: { 1, 2, 3, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17 }

C grade: { }

F grade: { 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 54, 55, 56 }

2.1.5 FriCAS

A grade: { 5, 6, 7, 8, 9, 10, 16, 43, 44, 45, 46, 47, 48, 52, 53, 57, 58, 59 }

B grade: { 1, 2, 3, 4, 11, 12, 13, 14, 15, 17, 18, 19, 20, 36, 37, 38, 39, 40, 41, 42, 49, 50, 51 }

C grade: { }

F grade: { 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 54, 55, 56 }

2.1.6 Sympy

A grade: { 57, 58, 59 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 37, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 52, 53, 57, 58, 59 }

B grade: { 13, 14, 15, 17, 18, 36, 38, 51 }

C grade: { }

F grade: { 16, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 54, 55, 56 }

2.1.8 Mupad

A grade: { 57, 58, 59 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53 }

C grade: { }

F grade: { 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 54, 55, 56 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	113	89	120	168	0	96	89
normalized size	1	1.00	2.05	1.62	2.18	3.05	0.00	1.75	1.62
time (sec)	N/A	0.071	0.817	0.280	0.323	0.541	0.000	0.334	0.350
Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	83	67	97	134	0	73	69
normalized size	1	1.00	1.89	1.52	2.20	3.05	0.00	1.66	1.57
time (sec)	N/A	0.066	0.345	0.274	0.330	0.475	0.000	0.365	0.256
Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	63	45	68	91	0	53	49
normalized size	1	1.00	2.33	1.67	2.52	3.37	0.00	1.96	1.81
time (sec)	N/A	0.087	0.159	0.228	0.330	0.518	0.000	0.560	0.253

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	44	24	31	53	0	24	23
normalized size	1	1.00	2.20	1.20	1.55	2.65	0.00	1.20	1.15
time (sec)	N/A	0.056	0.054	0.248	0.323	0.489	0.000	0.597	0.183

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	26	14	16	22	0	13	13
normalized size	1	1.00	2.17	1.17	1.33	1.83	0.00	1.08	1.08
time (sec)	N/A	0.020	0.024	0.204	0.336	0.501	0.000	0.445	0.172

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	47	41	50	54	0	32	27
normalized size	1	1.00	1.68	1.46	1.79	1.93	0.00	1.14	0.96
time (sec)	N/A	0.013	0.093	0.525	0.416	0.556	0.000	0.422	0.216

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	32	40	78	35	0	44	46
normalized size	1	1.00	1.28	1.60	3.12	1.40	0.00	1.76	1.84
time (sec)	N/A	0.047	0.077	0.396	0.426	0.610	0.000	0.534	0.236

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	42	100	128	53	0	56	59
normalized size	1	1.00	1.05	2.50	3.20	1.32	0.00	1.40	1.48
time (sec)	N/A	0.061	0.131	0.522	0.429	0.519	0.000	0.685	0.273

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	49	121	180	70	0	67	78
normalized size	1	1.00	0.92	2.28	3.40	1.32	0.00	1.26	1.47
time (sec)	N/A	0.067	0.172	0.507	0.428	0.449	0.000	0.289	0.347

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	57	185	230	81	0	91	93
normalized size	1	1.00	0.86	2.80	3.48	1.23	0.00	1.38	1.41
time (sec)	N/A	0.073	0.251	0.434	0.425	0.587	0.000	0.344	0.402

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	108	83	142	124	0	60	52
normalized size	1	1.00	1.89	1.46	2.49	2.18	0.00	1.05	0.91
time (sec)	N/A	0.066	0.325	0.709	0.427	0.474	0.000	0.736	0.406

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	123	125	228	181	0	86	78
normalized size	1	1.00	1.40	1.42	2.59	2.06	0.00	0.98	0.89
time (sec)	N/A	0.110	1.038	0.794	0.436	0.520	0.000	0.449	2.178

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	80	535	417	318	0	250	-1
normalized size	1	1.00	1.23	8.23	6.42	4.89	0.00	3.85	-0.02
time (sec)	N/A	0.091	1.902	0.920	0.474	0.565	0.000	1.010	0.000

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	69	273	200	212	0	195	-1
normalized size	1	1.00	1.57	6.20	4.55	4.82	0.00	4.43	-0.02
time (sec)	N/A	0.030	0.089	0.631	0.446	0.456	0.000	1.162	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	32	199	148	120	0	353	-1
normalized size	1	1.00	1.23	7.65	5.69	4.62	0.00	13.58	-0.04
time (sec)	N/A	0.016	0.054	0.716	0.442	0.546	0.000	1.055	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	54	221	83	219	0	0	-1
normalized size	1	1.00	0.87	3.56	1.34	3.53	0.00	0.00	-0.02
time (sec)	N/A	0.058	0.136	0.745	0.452	0.537	0.000	0.000	0.000

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	129	1141	150	427	0	304	-1
normalized size	1	1.00	1.59	14.09	1.85	5.27	0.00	3.75	-0.01
time (sec)	N/A	0.102	0.428	0.811	0.452	0.605	0.000	1.930	0.000

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	139	1961	0	546	0	348	-1
normalized size	1	1.00	1.39	19.61	0.00	5.46	0.00	3.48	-0.01
time (sec)	N/A	0.148	0.507	0.614	0.000	0.483	0.000	1.615	0.000

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	108	113	0	283	0	0	-1
normalized size	1	1.00	2.92	3.05	0.00	7.65	0.00	0.00	-0.03
time (sec)	N/A	0.059	0.400	2.354	0.000	0.804	0.000	0.000	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	101	117	0	296	0	0	-1
normalized size	1	1.00	2.66	3.08	0.00	7.79	0.00	0.00	-0.03
time (sec)	N/A	0.071	0.794	2.108	0.000	1.084	0.000	0.000	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	102	0	0	0	0	0	-1
normalized size	1	1.00	0.40	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.281	0.383	2.641	0.000	0.952	0.000	0.000	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	46	0	0	0	0	0	-1
normalized size	1	1.00	0.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.128	0.230	4.174	0.000	0.906	0.000	0.000	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	110	0	0	0	0	0	-1
normalized size	1	1.00	0.43	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.145	0.460	2.982	0.000	0.701	0.000	0.000	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	514	514	120	0	0	0	0	0	-1
normalized size	1	1.00	0.23	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.299	1.178	2.413	0.000	0.891	0.000	0.000	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	470	470	109	0	0	0	0	0	-1
normalized size	1	1.00	0.23	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.253	0.968	2.178	0.000	1.454	0.000	0.000	0.000

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	508	508	46	0	0	0	0	0	-1
normalized size	1	1.00	0.09	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.275	0.756	2.682	0.000	1.653	0.000	0.000	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	552	552	72	0	0	0	0	0	-1
normalized size	1	1.00	0.13	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.307	1.218	1.804	0.000	1.888	0.000	0.000	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.058	0.170	1.950	0.000	0.944	0.000	0.000	0.000

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	73	0	0	0	0	0	-1
normalized size	1	1.00	1.06	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.070	0.990	2.448	0.000	0.646	0.000	0.000	0.000

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	178	0	0	0	0	0	-1
normalized size	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.190	4.613	3.273	0.000	0.528	0.000	0.000	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	126	0	0	0	0	0	-1
normalized size	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.103	1.348	1.997	0.000	0.582	0.000	0.000	0.000

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	60	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.056	0.205	1.544	0.000	0.536	0.000	0.000	0.000

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.615	1.204	0.000	0.833	0.000	0.000	0.000

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	3.771	2.262	0.000	0.532	0.000	0.000	0.000

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	7.711	4.686	0.000	0.606	0.000	0.000	0.000

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	568	139	125	217	0	205	314
normalized size	1	1.00	5.31	1.30	1.17	2.03	0.00	1.92	2.93
time (sec)	N/A	0.110	6.255	1.389	0.339	0.784	0.000	0.782	0.623

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	152	99	95	155	0	134	234
normalized size	1	1.00	2.08	1.36	1.30	2.12	0.00	1.84	3.21
time (sec)	N/A	0.047	0.628	1.368	0.336	0.551	0.000	0.474	0.417

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	76	52	43	77	0	74	105
normalized size	1	1.00	2.24	1.53	1.26	2.26	0.00	2.18	3.09
time (sec)	N/A	0.026	0.183	0.831	0.363	0.680	0.000	0.860	0.329

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	125	162	0	607	0	194	588
normalized size	1	1.00	1.12	1.45	0.00	5.42	0.00	1.73	5.25
time (sec)	N/A	0.407	1.727	0.272	0.000	0.736	0.000	0.657	0.789

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	144	112	0	524	0	141	515
normalized size	1	1.00	1.71	1.33	0.00	6.24	0.00	1.68	6.13
time (sec)	N/A	0.252	0.495	0.233	0.000	0.867	0.000	0.525	0.580

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	106	77	0	308	0	98	135
normalized size	1	1.00	1.71	1.24	0.00	4.97	0.00	1.58	2.18
time (sec)	N/A	0.155	0.205	0.214	0.000	0.672	0.000	0.346	0.485

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	62	53	0	245	0	63	129
normalized size	1	1.00	1.17	1.00	0.00	4.62	0.00	1.19	2.43
time (sec)	N/A	0.113	0.061	0.219	0.000	0.663	0.000	0.534	0.429

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	39	0	154	0	48	36
normalized size	1	1.00	1.00	0.98	0.00	3.85	0.00	1.20	0.90
time (sec)	N/A	0.070	0.024	0.180	0.000	0.529	0.000	0.451	0.287

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	59	70	0	238	0	77	184
normalized size	1	1.00	1.04	1.23	0.00	4.18	0.00	1.35	3.23
time (sec)	N/A	0.065	0.106	0.567	0.000	0.614	0.000	0.316	0.452

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	56	72	0	235	0	77	766
normalized size	1	1.00	0.92	1.18	0.00	3.85	0.00	1.26	12.56
time (sec)	N/A	0.107	0.095	0.457	0.000	0.492	0.000	0.520	0.652

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	78	142	0	285	0	112	1147
normalized size	1	1.00	0.95	1.73	0.00	3.48	0.00	1.37	13.99
time (sec)	N/A	0.261	0.113	0.488	0.000	0.676	0.000	0.637	0.840

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	98	213	0	329	0	149	1218
normalized size	1	1.00	0.89	1.94	0.00	2.99	0.00	1.35	11.07
time (sec)	N/A	0.398	0.232	0.436	0.000	0.617	0.000	0.609	0.945

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	129	405	0	410	0	252	1639
normalized size	1	1.00	0.90	2.81	0.00	2.85	0.00	1.75	11.38
time (sec)	N/A	0.588	0.308	0.378	0.000	1.131	0.000	0.662	1.258

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	139	247	0	493	0	158	2677
normalized size	1	1.00	1.29	2.29	0.00	4.56	0.00	1.46	24.79
time (sec)	N/A	0.171	0.445	0.691	0.000	0.612	0.000	0.530	4.179

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	216	796	0	933	0	297	5917
normalized size	1	1.00	1.27	4.68	0.00	5.49	0.00	1.75	34.81
time (sec)	N/A	0.319	1.079	0.775	0.000	0.754	0.000	0.381	8.803

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	279	1912	0	1554	0	535	8167
normalized size	1	1.00	1.17	8.00	0.00	6.50	0.00	2.24	34.17
time (sec)	N/A	0.504	2.025	0.823	0.000	0.754	0.000	0.378	12.384

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	66	36	49	33	0	49	39
normalized size	1	1.00	2.13	1.16	1.58	1.06	0.00	1.58	1.26
time (sec)	N/A	0.029	0.051	0.565	0.413	0.510	0.000	0.586	0.245

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	67	53	71	52	0	45	27
normalized size	1	1.00	0.99	0.78	1.04	0.76	0.00	0.66	0.40
time (sec)	N/A	0.039	0.045	0.635	0.410	0.538	0.000	0.509	0.291

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.343	4.514	2.102	0.000	0.623	0.000	0.000	0.000

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.226	2.770	1.316	0.000	0.532	0.000	0.000	0.000

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	1.972	1.721	0.000	0.508	0.000	0.000	0.000

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	15	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.07
time (sec)	N/A	0.010	1.504	1.238	0.000	0.568	0.000	0.000	0.000

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.032	6.362	1.067	0.000	0.504	0.000	0.000	0.000

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.040	6.238	4.447	0.000	0.505	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [39] had the largest ratio of [.6923]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	6	5	1.00	13	0.385
2	A	6	6	1.00	13	0.462
3	A	4	4	1.00	13	0.308
4	A	3	3	1.00	13	0.231
5	A	1	1	1.00	11	0.091
6	A	2	2	1.00	12	0.167
7	A	4	4	1.00	11	0.364
8	A	5	5	1.00	13	0.385
9	A	6	5	1.00	13	0.385
10	A	7	5	1.00	13	0.385

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
11	A	3	3	1.00	12	0.250
12	A	4	4	1.00	12	0.333
13	A	5	5	1.00	10	0.500
14	A	4	4	1.00	10	0.400
15	A	2	2	1.00	10	0.200
16	A	5	4	1.00	10	0.400
17	A	6	5	1.00	10	0.500
18	A	7	6	1.00	10	0.600
19	A	2	2	1.00	25	0.080
20	A	2	2	1.00	28	0.071
21	A	4	4	1.00	25	0.160
22	A	3	3	1.00	25	0.120
23	A	4	4	1.00	25	0.160
24	A	6	6	1.00	25	0.240
25	A	5	5	1.00	25	0.200
26	A	6	6	1.00	25	0.240
27	A	7	6	1.00	25	0.240
28	A	2	2	1.00	23	0.087
29	A	3	3	1.00	24	0.125
30	A	5	5	1.00	21	0.238
31	A	4	4	1.00	21	0.190
32	A	3	3	1.00	19	0.158
33	A	3	3	1.00	12	0.250
34	A	3	3	1.00	19	0.158
35	A	3	3	1.00	21	0.143
36	A	6	5	1.00	12	0.417

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
37	A	5	4	1.00	12	0.333
38	A	4	4	1.00	12	0.333
39	A	9	9	1.00	13	0.692
40	A	8	8	1.00	13	0.615
41	A	7	7	1.00	13	0.538
42	A	6	6	1.00	13	0.462
43	A	4	4	1.00	11	0.364
44	A	4	4	1.00	12	0.333
45	A	6	6	1.00	11	0.546
46	A	7	7	1.00	13	0.538
47	A	8	7	1.00	13	0.538
48	A	9	7	1.00	13	0.538
49	A	6	6	1.00	12	0.500
50	A	7	7	1.00	12	0.583
51	A	8	7	1.00	12	0.583
52	A	2	2	1.00	12	0.167
53	A	5	4	1.00	12	0.333
54	A	8	5	1.00	21	0.238
55	A	7	4	1.00	21	0.190
56	A	3	3	1.00	19	0.158
57	A	0	0	0.00	0	0.000
58	A	0	0	0.00	0	0.000
59	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

3.1 $\int \frac{\csc^5(x)}{a+a \csc(x)} dx$

Optimal. Leaf size=55

$$-\frac{4 \cot^3(x)}{3a} - \frac{4 \cot(x)}{a} + \frac{3 \tanh^{-1}(\cos(x))}{2a} + \frac{\cot(x) \csc^3(x)}{a \csc(x) + a} + \frac{3 \cot(x) \csc(x)}{2a}$$

[Out] $3/2*\operatorname{arctanh}(\cos(x))/a-4*\cot(x)/a-4/3*\cot(x)^3/a+3/2*\cot(x)*\csc(x)/a+\cot(x)*\csc(x)^3/(a+a*\csc(x))$

Rubi [A] time = 0.07, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3818, 3787, 3768, 3770, 3767}

$$-\frac{4 \cot^3(x)}{3a} - \frac{4 \cot(x)}{a} + \frac{3 \tanh^{-1}(\cos(x))}{2a} + \frac{\cot(x) \csc^3(x)}{a \csc(x) + a} + \frac{3 \cot(x) \csc(x)}{2a}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Csc}[x]^5/(a + a*\text{Csc}[x]), x]$

[Out] $(3*\text{ArcTanh}[\text{Cos}[x]])/(2*a) - (4*\text{Cot}[x])/a - (4*\text{Cot}[x]^3)/(3*a) + (3*\text{Cot}[x]*\text{Csc}[x])/(2*a) + (\text{Cot}[x]*\text{Csc}[x]^3)/(a + a*\text{Csc}[x])$

Rule 3767

$\text{Int}[\csc[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow -\text{Dist}[d^{(-1)}, \text{Subst}[\text{Int}[\text{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \text{Cot}[c + d*x]], x] /; \text{FreeQ}\{c, d\}, x \ \&\& \ \text{IGtQ}[n/2, 0]$

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Csc[c + d*x]
)*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
/; FreeQ[{c, d}, x]
```

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3818

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_.)), x_Symbol] := Simp[(d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/(f*(a +
b*Csc[e + f*x])), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^5(x)}{a + a \csc(x)} dx &= \frac{\cot(x) \csc^3(x)}{a + a \csc(x)} - \frac{\int \csc^3(x)(3a - 4a \csc(x)) dx}{a^2} \\
&= \frac{\cot(x) \csc^3(x)}{a + a \csc(x)} - \frac{3 \int \csc^3(x) dx}{a} + \frac{4 \int \csc^4(x) dx}{a} \\
&= \frac{3 \cot(x) \csc(x)}{2a} + \frac{\cot(x) \csc^3(x)}{a + a \csc(x)} - \frac{3 \int \csc(x) dx}{2a} - \frac{4 \text{Subst}\left(\int (1 + x^2) dx, x, \cot(x)\right)}{a} \\
&= \frac{3 \tanh^{-1}(\cos(x))}{2a} - \frac{4 \cot(x)}{a} - \frac{4 \cot^3(x)}{3a} + \frac{3 \cot(x) \csc(x)}{2a} + \frac{\cot(x) \csc^3(x)}{a + a \csc(x)}
\end{aligned}$$

Mathematica [B] time = 0.82, size = 113, normalized size = 2.05

$$20 \tan\left(\frac{x}{2}\right) - 20 \cot\left(\frac{x}{2}\right) + 3 \csc^2\left(\frac{x}{2}\right) - 3 \sec^2\left(\frac{x}{2}\right) - 36 \log\left(\sin\left(\frac{x}{2}\right)\right) + 36 \log\left(\cos\left(\frac{x}{2}\right)\right) + \frac{48 \sin\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)} - \frac{1}{2} \sin(x) \csc(x)$$

Antiderivative was successfully verified.

[In] Integrate[Csc[x]^5/(a + a*Csc[x]),x]

[Out] $(-20*\cot[x/2] + 3*\csc[x/2]^2 + 36*\log[\cos[x/2]] - 36*\log[\sin[x/2]] - 3*\sec[x/2]^2 + 8*\csc[x]^3*\sin[x/2]^4 + (48*\sin[x/2])/(\cos[x/2] + \sin[x/2]) - (\csc[x/2]^4*\sin[x])/2 + 20*\tan[x/2])/(24*a)$

fricas [B] time = 0.54, size = 168, normalized size = 3.05

$$\frac{32 \cos(x)^4 + 14 \cos(x)^3 - 48 \cos(x)^2 + 9 (\cos(x)^4 - 2 \cos(x)^2 - (\cos(x)^3 + \cos(x)^2 - \cos(x) - 1) \sin(x) + 1) \log\left(\frac{1}{2} \cos(x) + \frac{1}{2}\right) - 9 (\cos(x)^4 - 2 \cos(x)^2 - (\cos(x)^3 + \cos(x)^2 - \cos(x) - 1) \sin(x) + 1) \log\left(-\frac{1}{2} \cos(x) + \frac{1}{2}\right) + 2 * (16 \cos(x)^3 + 9 \cos(x)^2 - 15 \cos(x) - 6) \sin(x) - 18 \cos(x) + 12}{24 a^3} + \frac{2}{a \left(\tan\left(\frac{1}{2} x\right) + 1\right)} + \frac{66 \tan\left(\frac{1}{2} x\right)^3 - 21 \tan\left(\frac{1}{2} x\right)}{24 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^5/(a+a*csc(x)),x, algorithm="fricas")

[Out] $1/12*(32*\cos(x)^4 + 14*\cos(x)^3 - 48*\cos(x)^2 + 9*(\cos(x)^4 - 2*\cos(x)^2 - (\cos(x)^3 + \cos(x)^2 - \cos(x) - 1)*\sin(x) + 1)*\log(1/2*\cos(x) + 1/2) - 9*(\cos(x)^4 - 2*\cos(x)^2 - (\cos(x)^3 + \cos(x)^2 - \cos(x) - 1)*\sin(x) + 1)*\log(-1/2*\cos(x) + 1/2) + 2*(16*\cos(x)^3 + 9*\cos(x)^2 - 15*\cos(x) - 6)*\sin(x) - 18*\cos(x) + 12)/(a*\cos(x)^4 - 2*a*\cos(x)^2 - (a*\cos(x)^3 + a*\cos(x)^2 - a*\cos(x) - a)*\sin(x) + a)$

giac [A] time = 0.33, size = 96, normalized size = 1.75

$$\frac{3 \log\left(\left|\tan\left(\frac{1}{2} x\right)\right|\right)}{2 a} + \frac{a^2 \tan\left(\frac{1}{2} x\right)^3 - 3 a^2 \tan\left(\frac{1}{2} x\right)^2 + 21 a^2 \tan\left(\frac{1}{2} x\right)}{24 a^3} - \frac{2}{a \left(\tan\left(\frac{1}{2} x\right) + 1\right)} + \frac{66 \tan\left(\frac{1}{2} x\right)^3 - 21 \tan\left(\frac{1}{2} x\right)}{24 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^5/(a+a*csc(x)),x, algorithm="giac")

[Out] $-3/2*\log(\text{abs}(\tan(1/2*x)))/a + 1/24*(a^2*\tan(1/2*x)^3 - 3*a^2*\tan(1/2*x)^2 + 21*a^2*\tan(1/2*x))/a^3 - 2/(a*(\tan(1/2*x) + 1)) + 1/24*(66*\tan(1/2*x)^3 - 21*\tan(1/2*x)^2 + 3*\tan(1/2*x) - 1)/(a*\tan(1/2*x)^3)$

maple [A] time = 0.28, size = 89, normalized size = 1.62

$$\frac{\tan^3\left(\frac{x}{2}\right)}{24a} - \frac{\tan^2\left(\frac{x}{2}\right)}{8a} + \frac{7 \tan\left(\frac{x}{2}\right)}{8a} - \frac{1}{24a \tan\left(\frac{x}{2}\right)^3} + \frac{1}{8a \tan\left(\frac{x}{2}\right)^2} - \frac{7}{8a \tan\left(\frac{x}{2}\right)} - \frac{3 \ln\left(\tan\left(\frac{x}{2}\right)\right)}{2a} - \frac{2}{a \left(\tan\left(\frac{x}{2}\right) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csc(x)^5/(a+a*csc(x)),x)`

[Out] $1/24/a*\tan(1/2*x)^3-1/8/a*\tan(1/2*x)^2+7/8/a*\tan(1/2*x)-1/24/a/\tan(1/2*x)^3+1/8/a/\tan(1/2*x)^2-7/8/a/\tan(1/2*x)-3/2/a*\ln(\tan(1/2*x))-2/a/(\tan(1/2*x)+1)$

maxima [B] time = 0.32, size = 120, normalized size = 2.18

$$\frac{\frac{21 \sin(x)}{\cos(x)+1} - \frac{3 \sin(x)^2}{(\cos(x)+1)^2} + \frac{\sin(x)^3}{(\cos(x)+1)^3}}{24 a} + \frac{\frac{2 \sin(x)}{\cos(x)+1} - \frac{18 \sin(x)^2}{(\cos(x)+1)^2} - \frac{69 \sin(x)^3}{(\cos(x)+1)^3} - 1}{24 \left(\frac{a \sin(x)^3}{(\cos(x)+1)^3} + \frac{a \sin(x)^4}{(\cos(x)+1)^4} \right)} - \frac{3 \log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{2 a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(x)^5/(a+a*csc(x)),x, algorithm="maxima")`

[Out] $1/24*(21*\sin(x)/(\cos(x) + 1) - 3*\sin(x)^2/(\cos(x) + 1)^2 + \sin(x)^3/(\cos(x) + 1)^3)/a + 1/24*(2*\sin(x)/(\cos(x) + 1) - 18*\sin(x)^2/(\cos(x) + 1)^2 - 69*\sin(x)^3/(\cos(x) + 1)^3 - 1)/(a*\sin(x)^3/(\cos(x) + 1)^3 + a*\sin(x)^4/(\cos(x) + 1)^4) - 3/2*\log(\sin(x)/(\cos(x) + 1))/a$

mupad [B] time = 0.35, size = 89, normalized size = 1.62

$$\frac{7 \tan\left(\frac{x}{2}\right)}{8 a} - \frac{23 \tan\left(\frac{x}{2}\right)^3 + 6 \tan\left(\frac{x}{2}\right)^2 - \frac{2 \tan\left(\frac{x}{2}\right)}{3} + \frac{1}{3}}{8 a \tan\left(\frac{x}{2}\right)^4 + 8 a \tan\left(\frac{x}{2}\right)^3} - \frac{\tan\left(\frac{x}{2}\right)^2}{8 a} + \frac{\tan\left(\frac{x}{2}\right)^3}{24 a} - \frac{3 \ln\left(\tan\left(\frac{x}{2}\right)\right)}{2 a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(sin(x)^5*(a + a/sin(x))),x)`

[Out] $(7*\tan(x/2))/(8*a) - (6*\tan(x/2)^2 - (2*\tan(x/2))/3 + 23*\tan(x/2)^3 + 1/3)/(8*a*\tan(x/2)^3 + 8*a*\tan(x/2)^4) - \tan(x/2)^2/(8*a) + \tan(x/2)^3/(24*a) - (3*\log(\tan(x/2)))/(2*a)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^5(x)}{\csc(x)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(x)**5/(a+a*csc(x)),x)`

[Out] `Integral(csc(x)**5/(csc(x) + 1), x)/a`

$$3.2 \quad \int \frac{\csc^4(x)}{a+a \csc(x)} dx$$

Optimal. Leaf size=44

$$\frac{2 \cot(x)}{a} - \frac{3 \tanh^{-1}(\cos(x))}{2a} + \frac{\cot(x) \csc^2(x)}{a \csc(x) + a} - \frac{3 \cot(x) \csc(x)}{2a}$$

[Out] $-3/2*\operatorname{arctanh}(\cos(x))/a+2*\cot(x)/a-3/2*\cot(x)*\csc(x)/a+\cot(x)*\csc(x)^2/(a+a*\csc(x))$

Rubi [A] time = 0.07, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3818, 3787, 3767, 8, 3768, 3770}

$$\frac{2 \cot(x)}{a} - \frac{3 \tanh^{-1}(\cos(x))}{2a} + \frac{\cot(x) \csc^2(x)}{a \csc(x) + a} - \frac{3 \cot(x) \csc(x)}{2a}$$

Antiderivative was successfully verified.

[In] `Int[Csc[x]^4/(a + a*Csc[x]),x]`

[Out] $(-3*\operatorname{ArcTanh}[\cos(x)])/(2*a) + (2*\cot(x))/a - (3*\cot(x)*\csc(x))/(2*a) + (\cot(x)*\csc(x)^2)/(a + a*\csc(x))$

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 3767

`Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]`

Rule 3768

`Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x])*(b*Csc[c + d*x])^(n - 1)/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]`

Rule 3770

`Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

Rule 3787

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3818

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)/(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.)), x_Symbol] := Simp[(d^2*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 2))/(f*(a +
b*Csc[e + f*x])), x] - Dist[d^2/(a*b), Int[(d*Csc[e + f*x])^(n - 2)*(b*(n
- 2) - a*(n - 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[
a^2 - b^2, 0] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned} \int \frac{\csc^4(x)}{a + a \csc(x)} dx &= \frac{\cot(x) \csc^2(x)}{a + a \csc(x)} - \frac{\int \csc^2(x)(2a - 3a \csc(x)) dx}{a^2} \\ &= \frac{\cot(x) \csc^2(x)}{a + a \csc(x)} - \frac{2 \int \csc^2(x) dx}{a} + \frac{3 \int \csc^3(x) dx}{a} \\ &= -\frac{3 \cot(x) \csc(x)}{2a} + \frac{\cot(x) \csc^2(x)}{a + a \csc(x)} + \frac{3 \int \csc(x) dx}{2a} + \frac{2 \text{Subst}(\int 1 dx, x, \cot(x))}{a} \\ &= -\frac{3 \tanh^{-1}(\cos(x))}{2a} + \frac{2 \cot(x)}{a} - \frac{3 \cot(x) \csc(x)}{2a} + \frac{\cot(x) \csc^2(x)}{a + a \csc(x)} \end{aligned}$$

Mathematica [A] time = 0.35, size = 83, normalized size = 1.89

$$\frac{-4 \tan\left(\frac{x}{2}\right) + 4 \cot\left(\frac{x}{2}\right) - \csc^2\left(\frac{x}{2}\right) + \sec^2\left(\frac{x}{2}\right) + 12 \log\left(\sin\left(\frac{x}{2}\right)\right) - 12 \log\left(\cos\left(\frac{x}{2}\right)\right) - \frac{16 \sin\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)}}{8a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^4/(a + a*Csc[x]), x]
```

```
[Out] (4*Cot[x/2] - Csc[x/2]^2 - 12*Log[Cos[x/2]] + 12*Log[Sin[x/2]] + Sec[x/2]^2
- (16*Sin[x/2])/(Cos[x/2] + Sin[x/2]) - 4*Tan[x/2])/(8*a)
```

fricas [B] time = 0.47, size = 134, normalized size = 3.05

$$\frac{8 \cos(x)^3 + 6 \cos(x)^2 - 3 (\cos(x)^3 + \cos(x)^2 + (\cos(x)^2 - 1) \sin(x) - \cos(x) - 1) \log\left(\frac{1}{2} \cos(x) + \frac{1}{2}\right) + 3 (\cos(x) - 1) \log\left(\frac{1}{2} \cos(x) + \frac{1}{2}\right)}{4 (a \cos(x)^3 + a \cos(x)^2 - a \cos(x) - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^4/(a+a*csc(x)),x, algorithm="fricas")

[Out] $\frac{1}{4}*(8*\cos(x)^3 + 6*\cos(x)^2 - 3*(\cos(x)^3 + \cos(x)^2 + (\cos(x)^2 - 1)*\sin(x) - \cos(x) - 1)*\log(1/2*\cos(x) + 1/2) + 3*(\cos(x)^3 + \cos(x)^2 + (\cos(x)^2 - 1)*\sin(x) - \cos(x) - 1)*\log(-1/2*\cos(x) + 1/2) - 2*(4*\cos(x)^2 + \cos(x) - 2)*\sin(x) - 6*\cos(x) - 4)/(a*\cos(x)^3 + a*\cos(x)^2 - a*\cos(x) + (a*\cos(x)^2 - a)*\sin(x) - a)$

giac [A] time = 0.37, size = 73, normalized size = 1.66

$$\frac{3 \log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{2a} + \frac{a \tan\left(\frac{1}{2}x\right)^2 - 4a \tan\left(\frac{1}{2}x\right)}{8a^2} + \frac{2}{a\left(\tan\left(\frac{1}{2}x\right) + 1\right)} - \frac{18 \tan\left(\frac{1}{2}x\right)^2 - 4 \tan\left(\frac{1}{2}x\right) + 1}{8a \tan\left(\frac{1}{2}x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^4/(a+a*csc(x)),x, algorithm="giac")

[Out] $\frac{3}{2}*\log(\text{abs}(\tan(1/2*x)))/a + 1/8*(a*\tan(1/2*x)^2 - 4*a*\tan(1/2*x))/a^2 + 2/(a*(\tan(1/2*x) + 1)) - 1/8*(18*\tan(1/2*x)^2 - 4*\tan(1/2*x) + 1)/(a*\tan(1/2*x)^2)$

maple [A] time = 0.27, size = 67, normalized size = 1.52

$$\frac{\tan^2\left(\frac{x}{2}\right)}{8a} - \frac{\tan\left(\frac{x}{2}\right)}{2a} - \frac{1}{8a \tan\left(\frac{x}{2}\right)^2} + \frac{1}{2a \tan\left(\frac{x}{2}\right)} + \frac{3 \ln\left(\tan\left(\frac{x}{2}\right)\right)}{2a} + \frac{2}{a\left(\tan\left(\frac{x}{2}\right) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(x)^4/(a+a*csc(x)),x)

[Out] $\frac{1}{8}/a*\tan(1/2*x)^2 - 1/2/a*\tan(1/2*x) - 1/8/a/\tan(1/2*x)^2 + 1/2/a/\tan(1/2*x) + 3/2/a*\ln(\tan(1/2*x)) + 2/a/(\tan(1/2*x) + 1)$

maxima [B] time = 0.33, size = 97, normalized size = 2.20

$$-\frac{\frac{4 \sin(x)}{\cos(x)+1} - \frac{\sin(x)^2}{(\cos(x)+1)^2}}{8a} + \frac{\frac{3 \sin(x)}{\cos(x)+1} + \frac{20 \sin(x)^2}{(\cos(x)+1)^2} - 1}{8\left(\frac{a \sin(x)^2}{(\cos(x)+1)^2} + \frac{a \sin(x)^3}{(\cos(x)+1)^3}\right)} + \frac{3 \log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^4/(a+a*csc(x)),x, algorithm="maxima")

[Out] $-1/8*(4*\sin(x)/(\cos(x) + 1) - \sin(x)^2/(\cos(x) + 1)^2)/a + 1/8*(3*\sin(x)/(\cos(x) + 1) + 20*\sin(x)^2/(\cos(x) + 1)^2 - 1)/(a*\sin(x)^2/(\cos(x) + 1)^2 + a*\sin(x)^3/(\cos(x) + 1)^3) + 3/2*\log(\sin(x)/(\cos(x) + 1))/a$

mupad [B] time = 0.26, size = 69, normalized size = 1.57

$$\frac{10 \tan\left(\frac{x}{2}\right)^2 + \frac{3 \tan\left(\frac{x}{2}\right)}{2} - \frac{1}{2}}{4 a \tan\left(\frac{x}{2}\right)^3 + 4 a \tan\left(\frac{x}{2}\right)^2} - \frac{\tan\left(\frac{x}{2}\right)}{2 a} + \frac{\tan\left(\frac{x}{2}\right)^2}{8 a} + \frac{3 \ln\left(\tan\left(\frac{x}{2}\right)\right)}{2 a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(sin(x)^4*(a + a/sin(x))),x)`

[Out] $((3*\tan(x/2))/2 + 10*\tan(x/2)^2 - 1/2)/(4*a*\tan(x/2)^2 + 4*a*\tan(x/2)^3) - \tan(x/2)/(2*a) + \tan(x/2)^2/(8*a) + (3*\log(\tan(x/2)))/(2*a)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^4(x)}{\csc(x)+1} dx$$

a

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(x)**4/(a+a*csc(x)),x)`

[Out] `Integral(csc(x)**4/(csc(x) + 1), x)/a`

$$3.3 \quad \int \frac{\csc^3(x)}{a+a \csc(x)} dx$$

Optimal. Leaf size=27

$$-\frac{\cot(x)}{a} + \frac{\tanh^{-1}(\cos(x))}{a} - \frac{\cot(x)}{a \csc(x) + a}$$

[Out] arctanh(cos(x))/a-cot(x)/a-cot(x)/(a+a*csc(x))

Rubi [A] time = 0.09, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3790, 3789, 3770, 3794}

$$-\frac{\cot(x)}{a} + \frac{\tanh^{-1}(\cos(x))}{a} - \frac{\cot(x)}{a \csc(x) + a}$$

Antiderivative was successfully verified.

[In] Int[Csc[x]^3/(a + a*Csc[x]),x]

[Out] ArcTanh[Cos[x]]/a - Cot[x]/a - Cot[x]/(a + a*Csc[x])

Rule 3770

Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3789

Int[csc[(e_.) + (f_.)*(x_.)]^2/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]

Rule 3790

Int[csc[(e_.) + (f_.)*(x_.)]^3/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := -Simp[Cot[e + f*x]/(b*f), x] - Dist[a/b, Int[Csc[e + f*x]^2/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]

Rule 3794

Int[csc[(e_.) + (f_.)*(x_.)]/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\csc^3(x)}{a + a \csc(x)} dx &= -\frac{\cot(x)}{a} - \int \frac{\csc^2(x)}{a + a \csc(x)} dx \\
&= -\frac{\cot(x)}{a} - \frac{\int \csc(x) dx}{a} + \int \frac{\csc(x)}{a + a \csc(x)} dx \\
&= \frac{\tanh^{-1}(\cos(x))}{a} - \frac{\cot(x)}{a} - \frac{\cot(x)}{a + a \csc(x)}
\end{aligned}$$

Mathematica [B] time = 0.16, size = 63, normalized size = 2.33

$$\frac{\tan\left(\frac{x}{2}\right) - \cot\left(\frac{x}{2}\right) - 2 \log\left(\sin\left(\frac{x}{2}\right)\right) + 2 \log\left(\cos\left(\frac{x}{2}\right)\right) + \frac{4 \sin\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)}}{2a}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[x]^3/(a + a*Csc[x]),x]

[Out] (-Cot[x/2] + 2*Log[Cos[x/2]] - 2*Log[Sin[x/2]] + (4*Sin[x/2])/(Cos[x/2] + Sin[x/2]) + Tan[x/2])/(2*a)

fricas [B] time = 0.52, size = 91, normalized size = 3.37

$$\frac{4 \cos(x)^2 + (\cos(x)^2 - (\cos(x) + 1) \sin(x) - 1) \log\left(\frac{1}{2} \cos(x) + \frac{1}{2}\right) - (\cos(x)^2 - (\cos(x) + 1) \sin(x) - 1) \log\left(-\frac{1}{2} \cos(x) + \frac{1}{2}\right) + 2*(2*\cos(x) + 1)*\sin(x) + 2*\cos(x) - 2)}{2(a \cos(x)^2 - (a \cos(x) + a) \sin(x) - a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^3/(a+a*csc(x)),x, algorithm="fricas")

[Out] 1/2*(4*cos(x)^2 + (cos(x)^2 - (cos(x) + 1)*sin(x) - 1)*log(1/2*cos(x) + 1/2) - (cos(x)^2 - (cos(x) + 1)*sin(x) - 1)*log(-1/2*cos(x) + 1/2) + 2*(2*cos(x) + 1)*sin(x) + 2*cos(x) - 2)/(a*cos(x)^2 - (a*cos(x) + a)*sin(x) - a)

giac [A] time = 0.56, size = 53, normalized size = 1.96

$$-\frac{\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{a} + \frac{\tan\left(\frac{1}{2}x\right)}{2a} + \frac{\tan\left(\frac{1}{2}x\right)^2 - 4 \tan\left(\frac{1}{2}x\right) - 1}{2\left(\tan\left(\frac{1}{2}x\right)^2 + \tan\left(\frac{1}{2}x\right)\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^3/(a+a*csc(x)),x, algorithm="giac")

[Out] $-\log(\text{abs}(\tan(1/2*x)))/a + 1/2*\tan(1/2*x)/a + 1/2*(\tan(1/2*x)^2 - 4*\tan(1/2*x) - 1)/((\tan(1/2*x)^2 + \tan(1/2*x))*a)$

maple [A] time = 0.23, size = 45, normalized size = 1.67

$$\frac{\tan\left(\frac{x}{2}\right)}{2a} - \frac{1}{2a \tan\left(\frac{x}{2}\right)} - \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a} - \frac{2}{a\left(\tan\left(\frac{x}{2}\right) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(x)^3/(a+a*csc(x)),x)

[Out] $1/2/a*\tan(1/2*x)-1/2/a/\tan(1/2*x)-1/a*\ln(\tan(1/2*x))-2/a/(\tan(1/2*x)+1)$

maxima [B] time = 0.33, size = 68, normalized size = 2.52

$$-\frac{\frac{5 \sin(x)}{\cos(x)+1} + 1}{2\left(\frac{a \sin(x)}{\cos(x)+1} + \frac{a \sin(x)^2}{(\cos(x)+1)^2}\right)} - \frac{\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a} + \frac{\sin(x)}{2a(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^3/(a+a*csc(x)),x, algorithm="maxima")

[Out] $-1/2*(5*\sin(x)/(\cos(x) + 1) + 1)/(a*\sin(x)/(\cos(x) + 1) + a*\sin(x)^2/(\cos(x) + 1)^2) - \log(\sin(x)/(\cos(x) + 1))/a + 1/2*\sin(x)/(a*(\cos(x) + 1))$

mupad [B] time = 0.25, size = 49, normalized size = 1.81

$$\frac{\tan\left(\frac{x}{2}\right)}{2a} - \frac{5 \tan\left(\frac{x}{2}\right) + 1}{2a \tan\left(\frac{x}{2}\right)^2 + 2a \tan\left(\frac{x}{2}\right)} - \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(x)^3*(a + a/sin(x))),x)

[Out] $\tan(x/2)/(2*a) - (5*\tan(x/2) + 1)/(2*a*\tan(x/2) + 2*a*\tan(x/2)^2) - \log(\tan(x/2))/a$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^3(x)}{\csc(x)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3/(a+a*csc(x)),x)
```

```
[Out] Integral(csc(x)**3/(csc(x) + 1), x)/a
```

$$3.4 \quad \int \frac{\csc^2(x)}{a+a \csc(x)} dx$$

Optimal. Leaf size=20

$$\frac{\cot(x)}{a \csc(x) + a} - \frac{\tanh^{-1}(\cos(x))}{a}$$

[Out] `-arctanh(cos(x))/a+cot(x)/(a+a*csc(x))`

Rubi [A] time = 0.06, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3789, 3770, 3794}

$$\frac{\cot(x)}{a \csc(x) + a} - \frac{\tanh^{-1}(\cos(x))}{a}$$

Antiderivative was successfully verified.

[In] `Int[Csc[x]^2/(a + a*Csc[x]),x]`

[Out] `-(ArcTanh[Cos[x]]/a) + Cot[x]/(a + a*Csc[x])`

Rule 3770

`Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

Rule 3789

`Int[csc[(e_.) + (f_.)*(x_.)]^2/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]`

Rule 3794

`Int[csc[(e_.) + (f_.)*(x_.)]/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]`

Rubi steps

$$\int \frac{\csc^2(x)}{a + a \csc(x)} dx = \frac{\int \csc(x) dx}{a} - \int \frac{\csc(x)}{a + a \csc(x)} dx$$

$$= -\frac{\tanh^{-1}(\cos(x))}{a} + \frac{\cot(x)}{a + a \csc(x)}$$

Mathematica [B] time = 0.05, size = 44, normalized size = 2.20

$$\frac{\log\left(\sin\left(\frac{x}{2}\right)\right) - \log\left(\cos\left(\frac{x}{2}\right)\right) - \frac{2 \sin\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)}}{a}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[x]^2/(a + a*Csc[x]),x]

[Out] (-Log[Cos[x/2]] + Log[Sin[x/2]] - (2*Sin[x/2]))/(Cos[x/2] + Sin[x/2])/a

fricas [B] time = 0.49, size = 53, normalized size = 2.65

$$\frac{(\cos(x) + \sin(x) + 1) \log\left(\frac{1}{2} \cos(x) + \frac{1}{2}\right) - (\cos(x) + \sin(x) + 1) \log\left(-\frac{1}{2} \cos(x) + \frac{1}{2}\right) - 2 \cos(x) + 2 \sin(x) - 2}{2(a \cos(x) + a \sin(x) + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^2/(a+a*csc(x)),x, algorithm="fricas")

[Out] -1/2*((cos(x) + sin(x) + 1)*log(1/2*cos(x) + 1/2) - (cos(x) + sin(x) + 1)*log(-1/2*cos(x) + 1/2) - 2*cos(x) + 2*sin(x) - 2)/(a*cos(x) + a*sin(x) + a)

giac [A] time = 0.60, size = 24, normalized size = 1.20

$$\frac{\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{a} + \frac{2}{a\left(\tan\left(\frac{1}{2}x\right) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^2/(a+a*csc(x)),x, algorithm="giac")

[Out] log(abs(tan(1/2*x)))/a + 2/(a*(tan(1/2*x) + 1))

maple [A] time = 0.25, size = 24, normalized size = 1.20

$$\frac{2}{a \left(\tan\left(\frac{x}{2}\right) + 1 \right)} + \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csc(x)^2/(a+a*csc(x)),x)`

[Out] `2/a/(tan(1/2*x)+1)+1/a*ln(tan(1/2*x))`

maxima [A] time = 0.32, size = 31, normalized size = 1.55

$$\frac{\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a} + \frac{2}{a + \frac{a \sin(x)}{\cos(x)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(x)^2/(a+a*csc(x)),x, algorithm="maxima")`

[Out] `log(sin(x)/(cos(x) + 1))/a + 2/(a + a*sin(x)/(cos(x) + 1))`

mupad [B] time = 0.18, size = 23, normalized size = 1.15

$$\frac{2}{a \left(\tan\left(\frac{x}{2}\right) + 1 \right)} + \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(sin(x)^2*(a + a/sin(x))),x)`

[Out] `2/(a*(tan(x/2) + 1)) + log(tan(x/2))/a`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^2(x)}{\csc(x)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(x)**2/(a+a*csc(x)),x)`

[Out] `Integral(csc(x)**2/(csc(x) + 1), x)/a`

$$3.5 \quad \int \frac{\csc(x)}{a+a \csc(x)} dx$$

Optimal. Leaf size=12

$$-\frac{\cot(x)}{a \csc(x) + a}$$

[Out] $-\cot(x)/(a+a*\csc(x))$

Rubi [A] time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3794}

$$-\frac{\cot(x)}{a \csc(x) + a}$$

Antiderivative was successfully verified.

[In] Int[Csc[x]/(a + a*Csc[x]),x]

[Out] -(Cot[x]/(a + a*Csc[x]))

Rule 3794

Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\int \frac{\csc(x)}{a + a \csc(x)} dx = -\frac{\cot(x)}{a + a \csc(x)}$$

Mathematica [B] time = 0.02, size = 26, normalized size = 2.17

$$\frac{2 \sin\left(\frac{x}{2}\right)}{a \left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[x]/(a + a*Csc[x]),x]

[Out] (2*Sin[x/2])/(a*(Cos[x/2] + Sin[x/2]))

fricas [A] time = 0.50, size = 22, normalized size = 1.83

$$-\frac{\cos(x) - \sin(x) + 1}{a \cos(x) + a \sin(x) + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)/(a+a*csc(x)),x, algorithm="fricas")

[Out] -(cos(x) - sin(x) + 1)/(a*cos(x) + a*sin(x) + a)

giac [A] time = 0.45, size = 13, normalized size = 1.08

$$-\frac{2}{a \left(\tan\left(\frac{1}{2}x\right) + 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)/(a+a*csc(x)),x, algorithm="giac")

[Out] -2/(a*(tan(1/2*x) + 1))

maple [A] time = 0.20, size = 14, normalized size = 1.17

$$-\frac{2}{a \left(\tan\left(\frac{x}{2}\right) + 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(x)/(a+a*csc(x)),x)

[Out] -2/a/(tan(1/2*x)+1)

maxima [A] time = 0.34, size = 16, normalized size = 1.33

$$-\frac{2}{a + \frac{a \sin(x)}{\cos(x)+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)/(a+a*csc(x)),x, algorithm="maxima")

[Out] -2/(a + a*sin(x)/(cos(x) + 1))

mupad [B] time = 0.17, size = 13, normalized size = 1.08

$$-\frac{2}{a \left(\tan\left(\frac{x}{2}\right) + 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(sin(x)*(a + a/sin(x))),x)`

[Out] `-2/(a*(tan(x/2) + 1))`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc(x)}{\csc(x)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(x)/(a+a*csc(x)),x)`

[Out] `Integral(csc(x)/(csc(x) + 1), x)/a`

$$3.6 \quad \int \frac{1}{a+a \csc(c+dx)} dx$$

Optimal. Leaf size=28

$$\frac{\cot(c+dx)}{d(a \csc(c+dx)+a)} + \frac{x}{a}$$

[Out] x/a+cot(d*x+c)/d/(a+a*csc(d*x+c))

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3777, 8}

$$\frac{\cot(c+dx)}{d(a \csc(c+dx)+a)} + \frac{x}{a}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Csc[c + d*x])^(-1), x]

[Out] x/a + Cot[c + d*x]/(d*(a + a*Csc[c + d*x]))

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3777

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.))^(n_), x_Symbol] := -Simp[(Cot[c + d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)), Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \frac{1}{a+a \csc(c+dx)} dx &= \frac{\cot(c+dx)}{d(a+a \csc(c+dx))} + \frac{\int a dx}{a^2} \\ &= \frac{x}{a} + \frac{\cot(c+dx)}{d(a+a \csc(c+dx))} \end{aligned}$$

Mathematica [A] time = 0.09, size = 47, normalized size = 1.68

$$\frac{-\frac{2 \sin\left(\frac{1}{2}(c+dx)\right)}{\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)} + c + dx}{ad}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Csc[c + d*x])^(-1),x]

[Out] (c + d*x - (2*Sin[(c + d*x)/2]))/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])/(a*d)

fricas [A] time = 0.56, size = 54, normalized size = 1.93

$$\frac{dx + (dx + 1) \cos(dx + c) + (dx - 1) \sin(dx + c) + 1}{ad \cos(dx + c) + ad \sin(dx + c) + ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*csc(d*x+c)),x, algorithm="fricas")

[Out] (d*x + (d*x + 1)*cos(d*x + c) + (d*x - 1)*sin(d*x + c) + 1)/(a*d*cos(d*x + c) + a*d*sin(d*x + c) + a*d)

giac [A] time = 0.42, size = 32, normalized size = 1.14

$$\frac{\frac{dx+c}{a} + \frac{2}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*csc(d*x+c)),x, algorithm="giac")

[Out] ((d*x + c)/a + 2/(a*(tan(1/2*d*x + 1/2*c) + 1)))/d

maple [A] time = 0.52, size = 41, normalized size = 1.46

$$\frac{2}{ad \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1 \right)} + \frac{2 \arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+a*csc(d*x+c)),x)

[Out] $2/a/d/(\tan(1/2*d*x+1/2*c)+1)+2/a/d*\arctan(\tan(1/2*d*x+1/2*c))$

maxima [A] time = 0.42, size = 50, normalized size = 1.79

$$\frac{2 \left(\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a} + \frac{1}{a + \frac{a \sin(dx+c)}{\cos(dx+c)+1}} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+a*csc(d*x+c)),x, algorithm="maxima")`

[Out] $2*(\arctan(\sin(d*x + c)/(\cos(d*x + c) + 1)))/a + 1/(a + a*\sin(d*x + c)/(\cos(d*x + c) + 1)))/d$

mupad [B] time = 0.22, size = 27, normalized size = 0.96

$$\frac{x}{a} + \frac{2}{a d \left(\tan\left(\frac{c}{2} + \frac{d x}{2}\right) + 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a + a/sin(c + d*x)),x)`

[Out] $x/a + 2/(a*d*(\tan(c/2 + (d*x)/2) + 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\csc(c+dx)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+a*csc(d*x+c)),x)`

[Out] `Integral(1/(csc(c + d*x) + 1), x)/a`

$$3.7 \quad \int \frac{\sin(x)}{a+a \csc(x)} dx$$

Optimal. Leaf size=25

$$-\frac{x}{a} - \frac{2 \cos(x)}{a} + \frac{\cos(x)}{a \csc(x) + a}$$

[Out] $-x/a-2*\cos(x)/a+\cos(x)/(a+a*\csc(x))$

Rubi [A] time = 0.05, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3819, 3787, 2638, 8}

$$-\frac{x}{a} - \frac{2 \cos(x)}{a} + \frac{\cos(x)}{a \csc(x) + a}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sin}[x]/(a + a*\text{Csc}[x]), x]$

[Out] $-(x/a) - (2*\text{Cos}[x])/a + \text{Cos}[x]/(a + a*\text{Csc}[x])$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 2638

$\text{Int}[\text{sin}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{Cos}[c + d*x]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 3787

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^n*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] \rightarrow \text{Dist}[a, \text{Int}[(d*\text{Csc}[e + f*x])^n, x], x] + \text{Dist}[b/d, \text{Int}[(d*\text{Csc}[e + f*x])^{n+1}, x], x] /; \text{FreeQ}[\{a, b, d, e, f, n\}, x]$

Rule 3819

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^n/(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] \rightarrow \text{Simp}[(\text{Cot}[e + f*x]*(d*\text{Csc}[e + f*x])^n)/(f*(a + b*\text{Csc}[e + f*x])), x] - \text{Dist}[1/a^2, \text{Int}[(d*\text{Csc}[e + f*x])^n*(a*(n-1) - b*n*\text{Csc}[e + f*x]), x], x] /; \text{FreeQ}[\{a, b, d, e, f\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[n, 0]$

Rubi steps

$$\begin{aligned} \int \frac{\sin(x)}{a + a \csc(x)} dx &= \frac{\cos(x)}{a + a \csc(x)} - \frac{\int (-2a + a \csc(x)) \sin(x) dx}{a^2} \\ &= \frac{\cos(x)}{a + a \csc(x)} - \frac{\int 1 dx}{a} + \frac{2 \int \sin(x) dx}{a} \\ &= -\frac{x}{a} - \frac{2 \cos(x)}{a} + \frac{\cos(x)}{a + a \csc(x)} \end{aligned}$$

Mathematica [A] time = 0.08, size = 32, normalized size = 1.28

$$-\frac{x + \cos(x) - \frac{2 \sin\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)}}{a}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]/(a + a*Csc[x]),x]

[Out] -((x + Cos[x] - (2*Sin[x/2]))/(Cos[x/2] + Sin[x/2]))/a)

fricas [A] time = 0.61, size = 35, normalized size = 1.40

$$-\frac{(x + 2) \cos(x) + \cos(x)^2 + (x + \cos(x) - 1) \sin(x) + x + 1}{a \cos(x) + a \sin(x) + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(a+a*csc(x)),x, algorithm="fricas")

[Out] -((x + 2)*cos(x) + cos(x)^2 + (x + cos(x) - 1)*sin(x) + x + 1)/(a*cos(x) + a*sin(x) + a)

giac [A] time = 0.53, size = 44, normalized size = 1.76

$$-\frac{x}{a} - \frac{2 \left(\tan\left(\frac{1}{2}x\right)^2 + \tan\left(\frac{1}{2}x\right) + 2 \right)}{\left(\tan\left(\frac{1}{2}x\right)^3 + \tan\left(\frac{1}{2}x\right)^2 + \tan\left(\frac{1}{2}x\right) + 1 \right) a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(a+a*csc(x)),x, algorithm="giac")

[Out] $-x/a - 2*(\tan(1/2*x)^2 + \tan(1/2*x) + 2)/((\tan(1/2*x)^3 + \tan(1/2*x)^2 + \tan(1/2*x) + 1)*a)$

maple [A] time = 0.40, size = 40, normalized size = 1.60

$$-\frac{2}{a\left(\tan\left(\frac{x}{2}\right)+1\right)} - \frac{2}{a\left(\tan^2\left(\frac{x}{2}\right)+1\right)} - \frac{2\arctan\left(\tan\left(\frac{x}{2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(x)/(a+a*csc(x)),x)`

[Out] $-2/a/(\tan(1/2*x)+1)-2/a/(\tan(1/2*x)^2+1)-2/a*\arctan(\tan(1/2*x))$

maxima [B] time = 0.43, size = 78, normalized size = 3.12

$$-\frac{2\left(\frac{\sin(x)}{\cos(x)+1} + \frac{\sin(x)^2}{(\cos(x)+1)^2} + 2\right)}{a + \frac{a\sin(x)}{\cos(x)+1} + \frac{a\sin(x)^2}{(\cos(x)+1)^2} + \frac{a\sin(x)^3}{(\cos(x)+1)^3}} - \frac{2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)/(a+a*csc(x)),x, algorithm="maxima")`

[Out] $-2*(\sin(x)/(\cos(x) + 1) + \sin(x)^2/(\cos(x) + 1)^2 + 2)/(a + a*\sin(x)/(\cos(x) + 1) + a*\sin(x)^2/(\cos(x) + 1)^2 + a*\sin(x)^3/(\cos(x) + 1)^3) - 2*\arctan(\sin(x)/(\cos(x) + 1))/a$

mupad [B] time = 0.24, size = 46, normalized size = 1.84

$$-\frac{x}{a} - \frac{2\tan\left(\frac{x}{2}\right)^2 + 2\tan\left(\frac{x}{2}\right) + 4}{a\left(\tan\left(\frac{x}{2}\right)^2 + 1\right)\left(\tan\left(\frac{x}{2}\right) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(x)/(a + a/sin(x)),x)`

[Out] $-x/a - (2*\tan(x/2) + 2*\tan(x/2)^2 + 4)/(a*(\tan(x/2)^2 + 1)*(\tan(x/2) + 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin(x)}{\csc(x)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a+a*csc(x)),x)
```

```
[Out] Integral(sin(x)/(csc(x) + 1), x)/a
```

$$3.8 \quad \int \frac{\sin^2(x)}{a+a \csc(x)} dx$$

Optimal. Leaf size=40

$$\frac{3x}{2a} + \frac{2 \cos(x)}{a} - \frac{3 \sin(x) \cos(x)}{2a} + \frac{\sin(x) \cos(x)}{a \csc(x) + a}$$

[Out] $3/2*x/a+2*\cos(x)/a-3/2*\cos(x)*\sin(x)/a+\cos(x)*\sin(x)/(a+a*\csc(x))$

Rubi [A] time = 0.06, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3819, 3787, 2635, 8, 2638}

$$\frac{3x}{2a} + \frac{2 \cos(x)}{a} - \frac{3 \sin(x) \cos(x)}{2a} + \frac{\sin(x) \cos(x)}{a \csc(x) + a}$$

Antiderivative was successfully verified.

[In] Int[Sin[x]^2/(a + a*Csc[x]),x]

[Out] $(3*x)/(2*a) + (2*\cos[x])/a - (3*\cos[x]*\sin[x])/(2*a) + (\cos[x]*\sin[x])/(a + a*\csc[x])$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x] * (b*SIN[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2638

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_)])*(d_.)^(n_.)*(csc[(e_.) + (f_.)*(x_)])*(b_.) + (a_.), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]

Rule 3819

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(f*(a + b*Csc[e + f*x])), x] - Dist[1/a^2, Int[(d*Csc[e + f*x])^n*(a*(n - 1) - b*n*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \frac{\sin^2(x)}{a + a \csc(x)} dx &= \frac{\cos(x) \sin(x)}{a + a \csc(x)} - \frac{\int (-3a + 2a \csc(x)) \sin^2(x) dx}{a^2} \\ &= \frac{\cos(x) \sin(x)}{a + a \csc(x)} - \frac{2 \int \sin(x) dx}{a} + \frac{3 \int \sin^2(x) dx}{a} \\ &= \frac{2 \cos(x)}{a} - \frac{3 \cos(x) \sin(x)}{2a} + \frac{\cos(x) \sin(x)}{a + a \csc(x)} + \frac{3 \int 1 dx}{2a} \\ &= \frac{3x}{2a} + \frac{2 \cos(x)}{a} - \frac{3 \cos(x) \sin(x)}{2a} + \frac{\cos(x) \sin(x)}{a + a \csc(x)} \end{aligned}$$

Mathematica [A] time = 0.13, size = 42, normalized size = 1.05

$$-\frac{-6x + \sin(2x) - 4 \cos(x) + \frac{8 \sin\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)}}{4a}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]^2/(a + a*Csc[x]),x]

[Out] -1/4*(-6*x - 4*Cos[x] + (8*Sin[x/2]))/(Cos[x/2] + Sin[x/2]) + Sin[2*x])/a

fricas [A] time = 0.52, size = 53, normalized size = 1.32

$$\frac{\cos(x)^3 + 3(x + 1) \cos(x) + 2 \cos(x)^2 - (\cos(x)^2 - 3x - \cos(x) + 2) \sin(x) + 3x + 2}{2(a \cos(x) + a \sin(x) + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^2/(a+a*csc(x)),x, algorithm="fricas")

[Out] 1/2*(cos(x)^3 + 3*(x + 1)*cos(x) + 2*cos(x)^2 - (cos(x)^2 - 3*x - cos(x) + 2)*sin(x) + 3*x + 2)/(a*cos(x) + a*sin(x) + a)

giac [A] time = 0.69, size = 56, normalized size = 1.40

$$\frac{3x}{2a} + \frac{\tan\left(\frac{1}{2}x\right)^3 + 2 \tan\left(\frac{1}{2}x\right)^2 - \tan\left(\frac{1}{2}x\right) + 2}{\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right)^2 a} + \frac{2}{a\left(\tan\left(\frac{1}{2}x\right) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^2/(a+a*csc(x)),x, algorithm="giac")

[Out] 3/2*x/a + (tan(1/2*x)^3 + 2*tan(1/2*x)^2 - tan(1/2*x) + 2)/((tan(1/2*x)^2 + 1)^2*a) + 2/(a*(tan(1/2*x) + 1))

maple [B] time = 0.52, size = 100, normalized size = 2.50

$$\frac{2}{a\left(\tan\left(\frac{x}{2}\right) + 1\right)} + \frac{\tan^3\left(\frac{x}{2}\right)}{a\left(\tan^2\left(\frac{x}{2}\right) + 1\right)^2} + \frac{2\left(\tan^2\left(\frac{x}{2}\right)\right)}{a\left(\tan^2\left(\frac{x}{2}\right) + 1\right)^2} - \frac{\tan\left(\frac{x}{2}\right)}{a\left(\tan^2\left(\frac{x}{2}\right) + 1\right)^2} + \frac{2}{a\left(\tan^2\left(\frac{x}{2}\right) + 1\right)^2} + \frac{3 \arctan\left(\tan\left(\frac{x}{2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)^2/(a+a*csc(x)),x)

[Out] 2/a/(tan(1/2*x)+1)+1/a/(tan(1/2*x)^2+1)^2*tan(1/2*x)^3+2/a/(tan(1/2*x)^2+1)^2*tan(1/2*x)^2-1/a/(tan(1/2*x)^2+1)^2*tan(1/2*x)+2/a/(tan(1/2*x)^2+1)^2+3/a*arctan(tan(1/2*x))

maxima [B] time = 0.43, size = 128, normalized size = 3.20

$$\frac{\frac{\sin(x)}{\cos(x)+1} + \frac{5 \sin(x)^2}{(\cos(x)+1)^2} + \frac{3 \sin(x)^3}{(\cos(x)+1)^3} + \frac{3 \sin(x)^4}{(\cos(x)+1)^4} + 4}{a + \frac{a \sin(x)}{\cos(x)+1} + \frac{2a \sin(x)^2}{(\cos(x)+1)^2} + \frac{2a \sin(x)^3}{(\cos(x)+1)^3} + \frac{a \sin(x)^4}{(\cos(x)+1)^4} + \frac{a \sin(x)^5}{(\cos(x)+1)^5}} + \frac{3 \arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^2/(a+a*csc(x)),x, algorithm="maxima")

[Out] (sin(x))/(cos(x) + 1) + 5*sin(x)^2/(cos(x) + 1)^2 + 3*sin(x)^3/(cos(x) + 1)^3 + 3*sin(x)^4/(cos(x) + 1)^4 + 4/(a + a*sin(x)/(cos(x) + 1) + 2*a*sin(x)^2/(cos(x) + 1)^2 + 2*a*sin(x)^3/(cos(x) + 1)^3 + a*sin(x)^4/(cos(x) + 1)^4 + a*sin(x)^5/(cos(x) + 1)^5) + 3*arctan(sin(x)/(cos(x) + 1))/a

mupad [B] time = 0.27, size = 59, normalized size = 1.48

$$\frac{3x}{2a} + \frac{3 \tan\left(\frac{x}{2}\right)^4 + 3 \tan\left(\frac{x}{2}\right)^3 + 5 \tan\left(\frac{x}{2}\right)^2 + \tan\left(\frac{x}{2}\right) + 4}{a\left(\tan\left(\frac{x}{2}\right)^2 + 1\right)^2 \left(\tan\left(\frac{x}{2}\right) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(x)^2/(a + a/sin(x)),x)`

[Out] $(3*x)/(2*a) + (\tan(x/2) + 5*\tan(x/2)^2 + 3*\tan(x/2)^3 + 3*\tan(x/2)^4 + 4)/(a*(\tan(x/2)^2 + 1)^2*(\tan(x/2) + 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin^2(x)}{\csc(x)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)**2/(a+a*csc(x)),x)`

[Out] `Integral(sin(x)**2/(csc(x) + 1), x)/a`

3.9 $\int \frac{\sin^3(x)}{a+a \csc(x)} dx$

Optimal. Leaf size=53

$$-\frac{3x}{2a} + \frac{4 \cos^3(x)}{3a} - \frac{4 \cos(x)}{a} + \frac{3 \sin(x) \cos(x)}{2a} + \frac{\sin^2(x) \cos(x)}{a \csc(x) + a}$$

[Out] $-3/2*x/a-4*\cos(x)/a+4/3*\cos(x)^3/a+3/2*\cos(x)*\sin(x)/a+\cos(x)*\sin(x)^2/(a+a*\csc(x))$

Rubi [A] time = 0.07, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3819, 3787, 2633, 2635, 8}

$$-\frac{3x}{2a} + \frac{4 \cos^3(x)}{3a} - \frac{4 \cos(x)}{a} + \frac{3 \sin(x) \cos(x)}{2a} + \frac{\sin^2(x) \cos(x)}{a \csc(x) + a}$$

Antiderivative was successfully verified.

[In] `Int[Sin[x]^3/(a + a*Csc[x]),x]`

[Out] $(-3*x)/(2*a) - (4*\cos[x])/a + (4*\cos[x]^3)/(3*a) + (3*\cos[x]*\sin[x])/(2*a) + (\cos[x]*\sin[x]^2)/(a + a*\csc[x])$

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 2633

`Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]`

Rule 2635

`Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x])*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]`

Rule 3787

`Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[`

$(d*\text{Csc}[e + f*x])^{(n + 1)}, x], x] /; \text{FreeQ}[\{a, b, d, e, f, n\}, x]$

Rule 3819

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^{(n_.)}/(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> \text{Simp}[(\text{Cot}[e + f*x]*(d*\text{Csc}[e + f*x])^n)/(f*(a + b*\text{Csc}[e + f*x])), x] - \text{Dist}[1/a^2, \text{Int}[(d*\text{Csc}[e + f*x])^n*(a*(n - 1) - b*n*\text{Csc}[e + f*x]), x], x] /; \text{FreeQ}[\{a, b, d, e, f\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[n, 0]$

Rubi steps

$$\begin{aligned} \int \frac{\sin^3(x)}{a + a \csc(x)} dx &= \frac{\cos(x) \sin^2(x)}{a + a \csc(x)} - \frac{\int (-4a + 3a \csc(x)) \sin^3(x) dx}{a^2} \\ &= \frac{\cos(x) \sin^2(x)}{a + a \csc(x)} - \frac{3 \int \sin^2(x) dx}{a} + \frac{4 \int \sin^3(x) dx}{a} \\ &= \frac{3 \cos(x) \sin(x)}{2a} + \frac{\cos(x) \sin^2(x)}{a + a \csc(x)} - \frac{3 \int 1 dx}{2a} - \frac{4 \text{Subst}\left(\int (1 - x^2) dx, x, \cos(x)\right)}{a} \\ &= -\frac{3x}{2a} - \frac{4 \cos(x)}{a} + \frac{4 \cos^3(x)}{3a} + \frac{3 \cos(x) \sin(x)}{2a} + \frac{\cos(x) \sin^2(x)}{a + a \csc(x)} \end{aligned}$$

Mathematica [A] time = 0.17, size = 49, normalized size = 0.92

$$\frac{-21 \cos(x) + \cos(3x) + 3 \left(-6x + \sin(2x) + \frac{8 \sin(\frac{x}{2})}{\sin(\frac{x}{2}) + \cos(\frac{x}{2})} \right)}{12a}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]^3/(a + a*Csc[x]),x]

[Out] (-21*Cos[x] + Cos[3*x] + 3*(-6*x + (8*Sin[x/2])/(Cos[x/2] + Sin[x/2]) + Sin[2*x]))/(12*a)

fricas [A] time = 0.45, size = 70, normalized size = 1.32

$$\frac{2 \cos(x)^4 - \cos(x)^3 - 3(3x + 5) \cos(x) - 12 \cos(x)^2 + (2 \cos(x)^3 + 3 \cos(x)^2 - 9x - 9 \cos(x) + 6) \sin(x) - 9}{6(a \cos(x) + a \sin(x) + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^3/(a+a*csc(x)),x, algorithm="fricas")

[Out] $\frac{1}{6}*(2*\cos(x)^4 - \cos(x)^3 - 3*(3*x + 5)*\cos(x) - 12*\cos(x)^2 + (2*\cos(x)^3 + 3*\cos(x)^2 - 9*x - 9*\cos(x) + 6)*\sin(x) - 9*x - 6)/(a*\cos(x) + a*\sin(x) + a)$

giac [A] time = 0.29, size = 67, normalized size = 1.26

$$\frac{\frac{3x}{2a} - \frac{2}{a\left(\tan\left(\frac{1}{2}x\right) + 1\right)} - \frac{3 \tan\left(\frac{1}{2}x\right)^5 + 6 \tan\left(\frac{1}{2}x\right)^4 + 24 \tan\left(\frac{1}{2}x\right)^2 - 3 \tan\left(\frac{1}{2}x\right) + 10}{3\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right)^3} a}{a\left(\tan\left(\frac{x}{2}\right) + 1\right) a\left(\tan^2\left(\frac{x}{2}\right) + 1\right)^3 - a\left(\tan^2\left(\frac{x}{2}\right) + 1\right)^3 + a\left(\tan^2\left(\frac{x}{2}\right) + 1\right)^3 - 3a\left(\tan^2\left(\frac{x}{2}\right) + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^3/(a+a*csc(x)),x, algorithm="giac")

[Out] $-3/2*x/a - 2/(a*(\tan(1/2*x) + 1)) - 1/3*(3*\tan(1/2*x)^5 + 6*\tan(1/2*x)^4 + 24*\tan(1/2*x)^2 - 3*\tan(1/2*x) + 10)/((\tan(1/2*x)^2 + 1)^3*a)$

maple [B] time = 0.51, size = 121, normalized size = 2.28

$$\frac{2}{a\left(\tan\left(\frac{x}{2}\right) + 1\right)} - \frac{\tan^5\left(\frac{x}{2}\right)}{a\left(\tan^2\left(\frac{x}{2}\right) + 1\right)^3} - \frac{2\left(\tan^4\left(\frac{x}{2}\right)\right)}{a\left(\tan^2\left(\frac{x}{2}\right) + 1\right)^3} - \frac{8\left(\tan^2\left(\frac{x}{2}\right)\right)}{a\left(\tan^2\left(\frac{x}{2}\right) + 1\right)^3} + \frac{\tan\left(\frac{x}{2}\right)}{a\left(\tan^2\left(\frac{x}{2}\right) + 1\right)^3} - \frac{10}{3a\left(\tan^2\left(\frac{x}{2}\right) + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)^3/(a+a*csc(x)),x)

[Out] $-2/a/(\tan(1/2*x)+1) - 1/a/(\tan(1/2*x)^2+1)^3*\tan(1/2*x)^5 - 2/a/(\tan(1/2*x)^2+1)^3*\tan(1/2*x)^4 - 8/a/(\tan(1/2*x)^2+1)^3*\tan(1/2*x)^2 + 1/a/(\tan(1/2*x)^2+1)^3*\tan(1/2*x) - 10/3/a/(\tan(1/2*x)^2+1)^3 - 3/a*\arctan(\tan(1/2*x))$

maxima [B] time = 0.43, size = 180, normalized size = 3.40

$$\frac{\frac{7 \sin(x)}{\cos(x)+1} + \frac{39 \sin(x)^2}{(\cos(x)+1)^2} + \frac{24 \sin(x)^3}{(\cos(x)+1)^3} + \frac{24 \sin(x)^4}{(\cos(x)+1)^4} + \frac{9 \sin(x)^5}{(\cos(x)+1)^5} + \frac{9 \sin(x)^6}{(\cos(x)+1)^6} + 16}{3\left(a + \frac{a \sin(x)}{\cos(x)+1} + \frac{3a \sin(x)^2}{(\cos(x)+1)^2} + \frac{3a \sin(x)^3}{(\cos(x)+1)^3} + \frac{3a \sin(x)^4}{(\cos(x)+1)^4} + \frac{3a \sin(x)^5}{(\cos(x)+1)^5} + \frac{a \sin(x)^6}{(\cos(x)+1)^6} + \frac{a \sin(x)^7}{(\cos(x)+1)^7}\right)} - \frac{3 \arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^3/(a+a*csc(x)),x, algorithm="maxima")

[Out] $-1/3*(7*\sin(x)/(\cos(x) + 1) + 39*\sin(x)^2/(\cos(x) + 1)^2 + 24*\sin(x)^3/(\cos(x) + 1)^3 + 24*\sin(x)^4/(\cos(x) + 1)^4 + 9*\sin(x)^5/(\cos(x) + 1)^5 + 9*\sin(x)^6/(\cos(x) + 1)^6 + 16)/(a + a*\sin(x)/(\cos(x) + 1) + 3*a*\sin(x)^2/(\cos(x) + 1)^2 + 3*a*\sin(x)^3/(\cos(x) + 1)^3 + 3*a*\sin(x)^4/(\cos(x) + 1)^4 + 3*a*$

$\sin(x)^5/(\cos(x) + 1)^5 + a*\sin(x)^6/(\cos(x) + 1)^6 + a*\sin(x)^7/(\cos(x) + 1)^7) - 3*\arctan(\sin(x)/(\cos(x) + 1))/a$

mupad [B] time = 0.35, size = 78, normalized size = 1.47

$$-\frac{3x}{2a} - \frac{3 \tan\left(\frac{x}{2}\right)^6 + 3 \tan\left(\frac{x}{2}\right)^5 + 8 \tan\left(\frac{x}{2}\right)^4 + 8 \tan\left(\frac{x}{2}\right)^3 + 13 \tan\left(\frac{x}{2}\right)^2 + \frac{7 \tan\left(\frac{x}{2}\right)}{3} + \frac{16}{3}}{a \left(\tan\left(\frac{x}{2}\right)^2 + 1 \right)^3 \left(\tan\left(\frac{x}{2}\right) + 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(x)^3/(a + a/sin(x)),x)`

[Out] `-(3*x)/(2*a) - ((7*tan(x/2))/3 + 13*tan(x/2)^2 + 8*tan(x/2)^3 + 8*tan(x/2)^4 + 3*tan(x/2)^5 + 3*tan(x/2)^6 + 16/3)/(a*(tan(x/2)^2 + 1)^3*(tan(x/2) + 1))`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin^3(x)}{\csc(x)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)**3/(a+a*csc(x)),x)`

[Out] `Integral(sin(x)**3/(csc(x) + 1), x)/a`

3.10 $\int \frac{\sin^4(x)}{a+a \csc(x)} dx$

Optimal. Leaf size=66

$$\frac{15x}{8a} - \frac{4 \cos^3(x)}{3a} + \frac{4 \cos(x)}{a} - \frac{5 \sin^3(x) \cos(x)}{4a} - \frac{15 \sin(x) \cos(x)}{8a} + \frac{\sin^3(x) \cos(x)}{a \csc(x) + a}$$

[Out] 15/8*x/a+4*cos(x)/a-4/3*cos(x)^3/a-15/8*cos(x)*sin(x)/a-5/4*cos(x)*sin(x)^3/a+cos(x)*sin(x)^3/(a+a*csc(x))

Rubi [A] time = 0.07, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3819, 3787, 2635, 8, 2633}

$$\frac{15x}{8a} - \frac{4 \cos^3(x)}{3a} + \frac{4 \cos(x)}{a} - \frac{5 \sin^3(x) \cos(x)}{4a} - \frac{15 \sin(x) \cos(x)}{8a} + \frac{\sin^3(x) \cos(x)}{a \csc(x) + a}$$

Antiderivative was successfully verified.

[In] Int[Sin[x]^4/(a + a*Csc[x]),x]

[Out] (15*x)/(8*a) + (4*Cos[x])/a - (4*Cos[x]^3)/(3*a) - (15*Cos[x]*Sin[x])/(8*a) - (5*Cos[x]*Sin[x]^3)/(4*a) + (Cos[x]*Sin[x]^3)/(a + a*Csc[x])

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2633

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x] * (b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3787

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol] := Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[

$(d*\text{Csc}[e + f*x])^{(n + 1)}, x], x] /; \text{FreeQ}[\{a, b, d, e, f, n\}, x]$

Rule 3819

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^{(n_.)}/(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> \text{Simp}[(\text{Cot}[e + f*x]*(d*\text{Csc}[e + f*x])^n)/(f*(a + b*\text{Csc}[e + f*x])), x] - \text{Dist}[1/a^2, \text{Int}[(d*\text{Csc}[e + f*x])^n*(a*(n - 1) - b*n*\text{Csc}[e + f*x]), x], x] /; \text{FreeQ}[\{a, b, d, e, f\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& \text{LtQ}[n, 0]$

Rubi steps

$$\begin{aligned} \int \frac{\sin^4(x)}{a + a \csc(x)} dx &= \frac{\cos(x) \sin^3(x)}{a + a \csc(x)} - \frac{\int (-5a + 4a \csc(x)) \sin^4(x) dx}{a^2} \\ &= \frac{\cos(x) \sin^3(x)}{a + a \csc(x)} - \frac{4 \int \sin^3(x) dx}{a} + \frac{5 \int \sin^4(x) dx}{a} \\ &= -\frac{5 \cos(x) \sin^3(x)}{4a} + \frac{\cos(x) \sin^3(x)}{a + a \csc(x)} + \frac{15 \int \sin^2(x) dx}{4a} + \frac{4 \text{Subst}\left(\int (1 - x^2) dx, x, \cos(x)\right)}{a} \\ &= \frac{4 \cos(x)}{a} - \frac{4 \cos^3(x)}{3a} - \frac{15 \cos(x) \sin(x)}{8a} - \frac{5 \cos(x) \sin^3(x)}{4a} + \frac{\cos(x) \sin^3(x)}{a + a \csc(x)} + \frac{15 \int 1 dx}{8a} \\ &= \frac{15x}{8a} + \frac{4 \cos(x)}{a} - \frac{4 \cos^3(x)}{3a} - \frac{15 \cos(x) \sin(x)}{8a} - \frac{5 \cos(x) \sin^3(x)}{4a} + \frac{\cos(x) \sin^3(x)}{a + a \csc(x)} \end{aligned}$$

Mathematica [A] time = 0.25, size = 57, normalized size = 0.86

$$\frac{168 \cos(x) - 8 \cos(3x) + 3 \left(60x - 16 \sin(2x) + \sin(4x) - \frac{64 \sin\left(\frac{x}{2}\right)}{\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)} \right)}{96a}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]^4/(a + a*Csc[x]),x]

[Out] (168*Cos[x] - 8*Cos[3*x] + 3*(60*x - (64*Sin[x/2])/(Cos[x/2] + Sin[x/2]) - 16*Sin[2*x] + Sin[4*x]))/(96*a)

fricas [A] time = 0.59, size = 81, normalized size = 1.23

$$\frac{6 \cos(x)^5 + 8 \cos(x)^4 - 25 \cos(x)^3 - 45(x + 1) \cos(x) - 48 \cos(x)^2 - (6 \cos(x)^4 - 2 \cos(x)^3 - 27 \cos(x)^2 + \dots)}{24(a \cos(x) + a \sin(x) + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^4/(a+a*csc(x)),x, algorithm="fricas")

[Out] $-1/24*(6*\cos(x)^5 + 8*\cos(x)^4 - 25*\cos(x)^3 - 45*(x + 1)*\cos(x) - 48*\cos(x)^2 - (6*\cos(x)^4 - 2*\cos(x)^3 - 27*\cos(x)^2 + 45*x + 21*\cos(x) - 24)*\sin(x) - 45*x - 24)/(a*\cos(x) + a*\sin(x) + a)$

giac [A] time = 0.34, size = 91, normalized size = 1.38

$$\frac{15x}{8a} + \frac{2}{a\left(\tan\left(\frac{1}{2}x\right) + 1\right)} + \frac{21 \tan\left(\frac{1}{2}x\right)^7 + 24 \tan\left(\frac{1}{2}x\right)^6 + 45 \tan\left(\frac{1}{2}x\right)^5 + 120 \tan\left(\frac{1}{2}x\right)^4 - 45 \tan\left(\frac{1}{2}x\right)^3 + 136 \tan\left(\frac{1}{2}x\right)^2 - 21 \tan\left(\frac{1}{2}x\right) + 40}{12\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right)^4 a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^4/(a+a*csc(x)),x, algorithm="giac")

[Out] $15/8*x/a + 2/(a*(\tan(1/2*x) + 1)) + 1/12*(21*\tan(1/2*x)^7 + 24*\tan(1/2*x)^6 + 45*\tan(1/2*x)^5 + 120*\tan(1/2*x)^4 - 45*\tan(1/2*x)^3 + 136*\tan(1/2*x)^2 - 21*\tan(1/2*x) + 40)/((\tan(1/2*x)^2 + 1)^4*a)$

maple [B] time = 0.43, size = 185, normalized size = 2.80

$$\frac{2}{a\left(\tan\left(\frac{x}{2}\right) + 1\right)} + \frac{7\left(\tan^7\left(\frac{x}{2}\right)\right)}{4a\left(\tan^2\left(\frac{x}{2}\right) + 1\right)^4} + \frac{2\left(\tan^6\left(\frac{x}{2}\right)\right)}{a\left(\tan^2\left(\frac{x}{2}\right) + 1\right)^4} + \frac{15\left(\tan^5\left(\frac{x}{2}\right)\right)}{4a\left(\tan^2\left(\frac{x}{2}\right) + 1\right)^4} + \frac{10\left(\tan^4\left(\frac{x}{2}\right)\right)}{a\left(\tan^2\left(\frac{x}{2}\right) + 1\right)^4} - \frac{15\left(\tan^3\left(\frac{x}{2}\right)\right)}{4a\left(\tan^2\left(\frac{x}{2}\right) + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)^4/(a+a*csc(x)),x)

[Out] $2/a/(\tan(1/2*x)+1)+7/4/a/(\tan(1/2*x)^2+1)^4*\tan(1/2*x)^7+2/a/(\tan(1/2*x)^2+1)^4*\tan(1/2*x)^6+15/4/a/(\tan(1/2*x)^2+1)^4*\tan(1/2*x)^5+10/a/(\tan(1/2*x)^2+1)^4*\tan(1/2*x)^4-15/4/a/(\tan(1/2*x)^2+1)^4*\tan(1/2*x)^3+34/3/a/(\tan(1/2*x)^2+1)^4*\tan(1/2*x)^2-7/4/a/(\tan(1/2*x)^2+1)^4*\tan(1/2*x)+10/3/a/(\tan(1/2*x)^2+1)^4+15/4/a*\arctan(\tan(1/2*x))$

maxima [B] time = 0.43, size = 230, normalized size = 3.48

$$\frac{19 \sin(x)}{\cos(x)+1} + \frac{211 \sin(x)^2}{(\cos(x)+1)^2} + \frac{91 \sin(x)^3}{(\cos(x)+1)^3} + \frac{219 \sin(x)^4}{(\cos(x)+1)^4} + \frac{165 \sin(x)^5}{(\cos(x)+1)^5} + \frac{165 \sin(x)^6}{(\cos(x)+1)^6} + \frac{45 \sin(x)^7}{(\cos(x)+1)^7} + \frac{45 \sin(x)^8}{(\cos(x)+1)^8} + 64$$

$$12\left(a + \frac{a \sin(x)}{\cos(x)+1} + \frac{4a \sin(x)^2}{(\cos(x)+1)^2} + \frac{4a \sin(x)^3}{(\cos(x)+1)^3} + \frac{6a \sin(x)^4}{(\cos(x)+1)^4} + \frac{6a \sin(x)^5}{(\cos(x)+1)^5} + \frac{4a \sin(x)^6}{(\cos(x)+1)^6} + \frac{4a \sin(x)^7}{(\cos(x)+1)^7} + \frac{a \sin(x)^8}{(\cos(x)+1)^8} + \frac{a \sin(x)^9}{(\cos(x)+1)^9}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^4/(a+a*csc(x)),x, algorithm="maxima")

[Out] $1/12*(19*\sin(x)/(\cos(x) + 1) + 211*\sin(x)^2/(\cos(x) + 1)^2 + 91*\sin(x)^3/(\cos(x) + 1)^3 + 219*\sin(x)^4/(\cos(x) + 1)^4 + 165*\sin(x)^5/(\cos(x) + 1)^5 + 165*\sin(x)^6/(\cos(x) + 1)^6 + 45*\sin(x)^7/(\cos(x) + 1)^7 + 45*\sin(x)^8/(\cos(x) + 1)^8 + 64)/(a + a*\sin(x)/(\cos(x) + 1) + 4*a*\sin(x)^2/(\cos(x) + 1)^2 + 4*a*\sin(x)^3/(\cos(x) + 1)^3 + 6*a*\sin(x)^4/(\cos(x) + 1)^4 + 6*a*\sin(x)^5/(\cos(x) + 1)^5 + 4*a*\sin(x)^6/(\cos(x) + 1)^6 + 4*a*\sin(x)^7/(\cos(x) + 1)^7 + a*\sin(x)^8/(\cos(x) + 1)^8 + a*\sin(x)^9/(\cos(x) + 1)^9) + 15/4*\arctan(\sin(x)/(\cos(x) + 1)))/a$

mupad [B] time = 0.40, size = 93, normalized size = 1.41

$$\frac{15x}{8a} + \frac{\frac{15 \tan\left(\frac{x}{2}\right)^8}{4} + \frac{15 \tan\left(\frac{x}{2}\right)^7}{4} + \frac{55 \tan\left(\frac{x}{2}\right)^6}{4} + \frac{55 \tan\left(\frac{x}{2}\right)^5}{4} + \frac{73 \tan\left(\frac{x}{2}\right)^4}{4} + \frac{91 \tan\left(\frac{x}{2}\right)^3}{12} + \frac{211 \tan\left(\frac{x}{2}\right)^2}{12} + \frac{19 \tan\left(\frac{x}{2}\right)}{12} + \frac{16}{3}}{a \left(\tan\left(\frac{x}{2}\right)^2 + 1 \right)^4 \left(\tan\left(\frac{x}{2}\right) + 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(x)^4/(a + a/sin(x)),x)`

[Out] $(15*x)/(8*a) + ((19*\tan(x/2))/12 + (211*\tan(x/2)^2)/12 + (91*\tan(x/2)^3)/12 + (73*\tan(x/2)^4)/4 + (55*\tan(x/2)^5)/4 + (55*\tan(x/2)^6)/4 + (15*\tan(x/2)^7)/4 + (15*\tan(x/2)^8)/4 + 16/3)/(a*(\tan(x/2)^2 + 1)^4*(\tan(x/2) + 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin^4(x)}{\csc(x)+1} dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)**4/(a+a*csc(x)),x)`

[Out] `Integral(sin(x)**4/(csc(x) + 1), x)/a`

$$3.11 \quad \int \frac{1}{(a+a \csc(c+dx))^2} dx$$

Optimal. Leaf size=57

$$\frac{4 \cot(c+dx)}{3a^2 d(\csc(c+dx)+1)} + \frac{x}{a^2} + \frac{\cot(c+dx)}{3d(a \csc(c+dx)+a)^2}$$

[Out] $x/a^2 + 4/3 * \cot(d*x+c)/a^2/d/(1+\csc(d*x+c)) + 1/3 * \cot(d*x+c)/d/(a+a*\csc(d*x+c))^2$

Rubi [A] time = 0.07, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3777, 3919, 3794}

$$\frac{4 \cot(c+dx)}{3a^2 d(\csc(c+dx)+1)} + \frac{x}{a^2} + \frac{\cot(c+dx)}{3d(a \csc(c+dx)+a)^2}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Csc[c + d*x])^(-2), x]

[Out] $x/a^2 + (4*\cot[c + d*x])/(3*a^2*d*(1 + \csc[c + d*x])) + \cot[c + d*x]/(3*d*(a + a*\csc[c + d*x])^2)$

Rule 3777

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.))^n], x_Symbol] :> -Simp[(Cot[c + d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)), Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]

Rule 3794

Int[csc[(e_.) + (f_.)*(x_.)]/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3919

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{(a + a \csc(c + dx))^2} dx &= \frac{\cot(c + dx)}{3d(a + a \csc(c + dx))^2} - \frac{\int \frac{-3a + a \csc(c + dx)}{a + a \csc(c + dx)} dx}{3a^2} \\ &= \frac{x}{a^2} + \frac{\cot(c + dx)}{3d(a + a \csc(c + dx))^2} - \frac{4 \int \frac{\csc(c + dx)}{a + a \csc(c + dx)} dx}{3a} \\ &= \frac{x}{a^2} + \frac{\cot(c + dx)}{3d(a + a \csc(c + dx))^2} + \frac{4 \cot(c + dx)}{3d(a^2 + a^2 \csc(c + dx))} \end{aligned}$$

Mathematica [A] time = 0.33, size = 108, normalized size = 1.89

$$\frac{3(3c + 3dx - 4) \cos\left(\frac{1}{2}(c + dx)\right) + (-3c - 3dx + 10) \cos\left(\frac{3}{2}(c + dx)\right) + 6 \sin\left(\frac{1}{2}(c + dx)\right) ((c + dx) \cos(c + dx) + 2)}{6a^2d \left(\sin\left(\frac{1}{2}(c + dx)\right) + \cos\left(\frac{1}{2}(c + dx)\right)\right)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Csc[c + d*x])^(-2), x]

[Out] (3*(-4 + 3*c + 3*d*x)*Cos[(c + d*x)/2] + (10 - 3*c - 3*d*x)*Cos[(3*(c + d*x))/2] + 6*(-3 + 2*c + 2*d*x + (c + d*x)*Cos[c + d*x])*Sin[(c + d*x)/2])/(6*a^2*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3)

fricas [B] time = 0.47, size = 124, normalized size = 2.18

$$\frac{(3dx - 5) \cos(dx + c)^2 - 6dx - (3dx + 4) \cos(dx + c) - (6dx + (3dx + 5) \cos(dx + c) + 1) \sin(dx + c) + 1}{3(a^2d \cos(dx + c)^2 - a^2d \cos(dx + c) - 2a^2d - (a^2d \cos(dx + c) + 2a^2d) \sin(dx + c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*csc(d*x+c))^2,x, algorithm="fricas")

[Out] 1/3*((3*d*x - 5)*cos(d*x + c)^2 - 6*d*x - (3*d*x + 4)*cos(d*x + c) - (6*d*x + (3*d*x + 5)*cos(d*x + c) + 1)*sin(d*x + c) + 1)/(a^2*d*cos(d*x + c)^2 - a^2*d*cos(d*x + c) - 2*a^2*d - (a^2*d*cos(d*x + c) + 2*a^2*d)*sin(d*x + c))

giac [A] time = 0.74, size = 60, normalized size = 1.05

$$\frac{\frac{3(dx+c)}{a^2} + \frac{2\left(3 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 9 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 4\right)}{a^2\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*csc(d*x+c))^2,x, algorithm="giac")

[Out] 1/3*(3*(d*x + c)/a^2 + 2*(3*tan(1/2*d*x + 1/2*c)^2 + 9*tan(1/2*d*x + 1/2*c) + 4)/(a^2*(tan(1/2*d*x + 1/2*c) + 1)^3)/d

maple [A] time = 0.71, size = 83, normalized size = 1.46

$$-\frac{4}{3d a^2 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^3} + \frac{2}{d a^2 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^2} + \frac{2}{d a^2 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)} + \frac{2 \arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{d a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+a*csc(d*x+c))^2,x)

[Out] -4/3/d/a^2/(tan(1/2*d*x+1/2*c)+1)^3+2/d/a^2/(tan(1/2*d*x+1/2*c)+1)^2+2/d/a^2/(tan(1/2*d*x+1/2*c)+1)+2/d/a^2*arctan(tan(1/2*d*x+1/2*c))

maxima [B] time = 0.43, size = 142, normalized size = 2.49

$$\frac{2 \left(\frac{\frac{9 \sin(dx+c)}{\cos(dx+c)+1} + \frac{3 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + 4}{a^2 + \frac{3a^2 \sin(dx+c)}{\cos(dx+c)+1} + \frac{3a^2 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{a^2 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}} + \frac{3 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2} \right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*csc(d*x+c))^2,x, algorithm="maxima")

[Out] 2/3*((9*sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 4)/(a^2 + 3*a^2*sin(d*x + c)/(cos(d*x + c) + 1) + 3*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c)^3/(cos(d*x + c) + 1)^3) + 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d

mupad [B] time = 0.41, size = 52, normalized size = 0.91

$$\frac{x}{a^2} + \frac{2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 + 6 \tan\left(\frac{c}{2} + \frac{dx}{2}\right) + \frac{8}{3}}{a^2 d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right) + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + a/sin(c + d*x))^2,x)

[Out] $x/a^2 + (6*\tan(c/2 + (d*x)/2) + 2*\tan(c/2 + (d*x)/2)^2 + 8/3)/(a^2*d*(\tan(c/2 + (d*x)/2) + 1)^3)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\frac{\csc^2(c+dx)+2 \csc(c+dx)+1}{a^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*csc(d*x+c))**2,x)

[Out] Integral(1/(csc(c + d*x)**2 + 2*csc(c + d*x) + 1), x)/a**2

$$3.12 \quad \int \frac{1}{(a+a \csc(c+dx))^3} dx$$

Optimal. Leaf size=88

$$\frac{22 \cot(c+dx)}{15d(a^3 \csc(c+dx) + a^3)} + \frac{x}{a^3} + \frac{7 \cot(c+dx)}{15ad(a \csc(c+dx) + a)^2} + \frac{\cot(c+dx)}{5d(a \csc(c+dx) + a)^3}$$

[Out] $x/a^3 + 1/5 * \cot(d*x+c)/d / (a+a*\csc(d*x+c))^3 + 7/15 * \cot(d*x+c)/a/d / (a+a*\csc(d*x+c))^2 + 22/15 * \cot(d*x+c)/d / (a^3+a^3*\csc(d*x+c))$

Rubi [A] time = 0.11, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3777, 3922, 3919, 3794}

$$\frac{22 \cot(c+dx)}{15d(a^3 \csc(c+dx) + a^3)} + \frac{x}{a^3} + \frac{7 \cot(c+dx)}{15ad(a \csc(c+dx) + a)^2} + \frac{\cot(c+dx)}{5d(a \csc(c+dx) + a)^3}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Csc[c + d*x])^(-3), x]

[Out] $x/a^3 + \text{Cot}[c + d*x]/(5*d*(a + a*\text{Csc}[c + d*x])^3) + (7*\text{Cot}[c + d*x])/(15*a*d*(a + a*\text{Csc}[c + d*x])^2) + (22*\text{Cot}[c + d*x])/(15*d*(a^3 + a^3*\text{Csc}[c + d*x]))$

Rule 3777

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.))^n, x_Symbol] :> -Simp[(Cot[c + d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)), Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]

Rule 3794

Int[csc[(e_.) + (f_.)*(x_.)]/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> -Simp[Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3919

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] :> Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -

a*d, 0]

Rule 3922

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(a + a \csc(c + dx))^3} dx &= \frac{\cot(c + dx)}{5d(a + a \csc(c + dx))^3} - \frac{\int \frac{-5a + 2a \csc(c + dx)}{(a + a \csc(c + dx))^2} dx}{5a^2} \\ &= \frac{\cot(c + dx)}{5d(a + a \csc(c + dx))^3} + \frac{7 \cot(c + dx)}{15ad(a + a \csc(c + dx))^2} + \frac{\int \frac{15a^2 - 7a^2 \csc(c + dx)}{a + a \csc(c + dx)} dx}{15a^4} \\ &= \frac{x}{a^3} + \frac{\cot(c + dx)}{5d(a + a \csc(c + dx))^3} + \frac{7 \cot(c + dx)}{15ad(a + a \csc(c + dx))^2} - \frac{22 \int \frac{\csc(c + dx)}{a + a \csc(c + dx)} dx}{15a^2} \\ &= \frac{x}{a^3} + \frac{\cot(c + dx)}{5d(a + a \csc(c + dx))^3} + \frac{7 \cot(c + dx)}{15ad(a + a \csc(c + dx))^2} + \frac{22 \cot(c + dx)}{15d(a^3 + a^3 \csc(c + dx))} \end{aligned}$$

Mathematica [A] time = 1.04, size = 123, normalized size = 1.40

$$\frac{2 \sin\left(\frac{1}{2}(c+dx)\right)(-51 \sin(c+dx)+16 \cos(2(c+dx))-38)}{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^5} - \frac{13}{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{3}{\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)^4} + \frac{15c + 15dx}{15a^3d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Csc[c + d*x])^(-3), x]

[Out] (15*c + 15*d*x + 3/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 - 13/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (2*Sin[(c + d*x)/2]*(-38 + 16*Cos[2*(c + d*x)]) - 51*Sin[c + d*x]))/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5)/(15*a^3*d)

fricas [B] time = 0.52, size = 181, normalized size = 2.06

$$\frac{(15dx + 32) \cos(dx + c)^3 + (45dx - 19) \cos(dx + c)^2 - 60dx - 6(5dx + 9) \cos(dx + c) + ((15dx - 32) \cos(dx + c) - 15a^3d \cos(dx + c)^3 + 3a^3d \cos(dx + c)^2 - 2a^3d \cos(dx + c) - 4a^3d + (a^3d \cos(dx + c))^2)}{15a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*csc(d*x+c))^3,x, algorithm="fricas")

[Out] 1/15*((15*d*x + 32)*cos(d*x + c)^3 + (45*d*x - 19)*cos(d*x + c)^2 - 60*d*x - 6*(5*d*x + 9)*cos(d*x + c) + ((15*d*x - 32)*cos(d*x + c)^2 - 60*d*x - 3*(10*d*x + 17)*cos(d*x + c) + 3)*sin(d*x + c) - 3)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d + (a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) - 4*a^3*d)*sin(d*x + c))

giac [A] time = 0.45, size = 86, normalized size = 0.98

$$\frac{\frac{15(dx+c)}{a^3} + \frac{2\left(15 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^4 + 75 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 145 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 95 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 22\right)}{a^3\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right)^5}}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*csc(d*x+c))^3,x, algorithm="giac")

[Out] 1/15*(15*(d*x + c)/a^3 + 2*(15*tan(1/2*d*x + 1/2*c)^4 + 75*tan(1/2*d*x + 1/2*c)^3 + 145*tan(1/2*d*x + 1/2*c)^2 + 95*tan(1/2*d*x + 1/2*c) + 22)/(a^3*(tan(1/2*d*x + 1/2*c) + 1)^5)/d

maple [A] time = 0.79, size = 125, normalized size = 1.42

$$-\frac{4}{d a^3 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^4} + \frac{8}{5d a^3 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^5} + \frac{4}{3d a^3 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^3} + \frac{2}{d a^3 \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 1\right)^2} + \frac{1}{d a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+a*csc(d*x+c))^3,x)

[Out] -4/d/a^3/(tan(1/2*d*x+1/2*c)+1)^4+8/5/d/a^3/(tan(1/2*d*x+1/2*c)+1)^5+4/3/d/a^3/(tan(1/2*d*x+1/2*c)+1)^3+2/d/a^3/(tan(1/2*d*x+1/2*c)+1)^2+2/d/a^3/(tan(1/2*d*x+1/2*c)+1)+2/d/a^3*arctan(tan(1/2*d*x+1/2*c))

maxima [B] time = 0.44, size = 228, normalized size = 2.59

$$2 \left(\frac{\frac{95 \sin(dx+c)}{\cos(dx+c)+1} + \frac{145 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{75 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{15 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + 22}{a^3 + \frac{5a^3 \sin(dx+c)}{\cos(dx+c)+1} + \frac{10a^3 \sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{10a^3 \sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{5a^3 \sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{a^3 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}} + \frac{15 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3} \right)$$

$$15d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*csc(d*x+c))^3,x, algorithm="maxima")

[Out] 2/15*((95*sin(d*x + c)/(cos(d*x + c) + 1) + 145*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 75*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 15*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 22)/(a^3 + 5*a^3*sin(d*x + c)/(cos(d*x + c) + 1) + 10*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 10*a^3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5) + 15*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3/d

mupad [B] time = 2.18, size = 78, normalized size = 0.89

$$\frac{x}{a^3} + \frac{2 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^4 + 10 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3 + \frac{58 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2}{3} + \frac{38 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{3} + \frac{44}{15}}{a^3 d \left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right) + 1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + a/sin(c + d*x))^3,x)

[Out] x/a^3 + ((38*tan(c/2 + (d*x)/2))/3 + (58*tan(c/2 + (d*x)/2)^2)/3 + 10*tan(c/2 + (d*x)/2)^3 + 2*tan(c/2 + (d*x)/2)^4 + 44/15)/(a^3*d*(tan(c/2 + (d*x)/2) + 1)^5)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\frac{\csc^3(c+dx)+3 \csc^2(c+dx)+3 \csc(c+dx)+1}{a^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*csc(d*x+c))**3,x)

[Out] Integral(1/(csc(c + d*x)**3 + 3*csc(c + d*x)**2 + 3*csc(c + d*x) + 1), x)/a**3

3.13 $\int (a + a \csc(x))^{5/2} dx$

Optimal. Leaf size=65

$$-2a^{5/2} \tan^{-1} \left(\frac{\sqrt{a} \cot(x)}{\sqrt{a \csc(x) + a}} \right) - \frac{14a^3 \cot(x)}{3\sqrt{a \csc(x) + a}} - \frac{2}{3} a^2 \cot(x) \sqrt{a \csc(x) + a}$$

[Out] $-2*a^{(5/2)}*\arctan(\cot(x)*a^{(1/2)}/(a+a*\csc(x))^{(1/2)})-14/3*a^3*\cot(x)/(a+a*\csc(x))^{(1/2)}-2/3*a^2*\cot(x)*(a+a*\csc(x))^{(1/2)}$

Rubi [A] time = 0.09, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3775, 3915, 3774, 203, 3792}

$$-\frac{14a^3 \cot(x)}{3\sqrt{a \csc(x) + a}} - \frac{2}{3} a^2 \cot(x) \sqrt{a \csc(x) + a} - 2a^{5/2} \tan^{-1} \left(\frac{\sqrt{a} \cot(x)}{\sqrt{a \csc(x) + a}} \right)$$

Antiderivative was successfully verified.

[In] Int[(a + a*Csc[x])^(5/2), x]

[Out] $-2*a^{(5/2)}*ArcTan[(Sqrt[a]*Cot[x])/Sqrt[a + a*Csc[x]]] - (14*a^3*Cot[x])/(3*Sqrt[a + a*Csc[x]]) - (2*a^2*Cot[x]*Sqrt[a + a*Csc[x]])/3$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3774

Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3775

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] := -Simp[(b^2*Cot[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[a/(n - 1), Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3792


```
Int[csc[(e_.) + (f_.)*(x_.)]*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol]
:> Simp[(-2*b*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, e, f}, x]
&& EqQ[a^2 - b^2, 0]
```

Rule 3915

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)]*(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.)), x_Symbol]
:> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned} \int (a + a \csc(x))^{5/2} dx &= -\frac{2}{3}a^2 \cot(x)\sqrt{a + a \csc(x)} + \frac{1}{3}(2a) \int \sqrt{a + a \csc(x)} \left(\frac{3a}{2} + \frac{7}{2}a \csc(x) \right) dx \\ &= -\frac{2}{3}a^2 \cot(x)\sqrt{a + a \csc(x)} + a^2 \int \sqrt{a + a \csc(x)} dx + \frac{1}{3}(7a^2) \int \csc(x)\sqrt{a + a \csc(x)} dx \\ &= -\frac{14a^3 \cot(x)}{3\sqrt{a + a \csc(x)}} - \frac{2}{3}a^2 \cot(x)\sqrt{a + a \csc(x)} - (2a^3) \text{Subst} \left(\int \frac{1}{a + x^2} dx, x, \frac{a \cot(x)}{\sqrt{a + a \csc(x)}} \right) \\ &= -2a^{5/2} \tan^{-1} \left(\frac{\sqrt{a} \cot(x)}{\sqrt{a + a \csc(x)}} \right) - \frac{14a^3 \cot(x)}{3\sqrt{a + a \csc(x)}} - \frac{2}{3}a^2 \cot(x)\sqrt{a + a \csc(x)} \end{aligned}$$

Mathematica [A] time = 1.90, size = 80, normalized size = 1.23

$$\frac{2a^2 \sqrt{a(\csc(x) + 1)} \left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right) \right) \left(\sqrt{\csc(x) - 1} (\csc(x) + 8) + 3 \tan^{-1} \left(\sqrt{\csc(x) - 1} \right) \right)}{3\sqrt{\csc(x) - 1} \left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right) \right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Csc[x])^(5/2), x]
```

```
[Out] (-2*a^2*Sqrt[a*(1 + Csc[x])]*(3*ArcTan[Sqrt[-1 + Csc[x]]] + Sqrt[-1 + Csc[x]])*(8 + Csc[x]))*(Cos[x/2] - Sin[x/2])/(3*Sqrt[-1 + Csc[x]]*(Cos[x/2] + Sin[x/2]))
```

fricas [B] time = 0.56, size = 318, normalized size = 4.89

$$\left[\frac{3 \left(a^2 \cos(x)^2 - a^2 - \left(a^2 \cos(x) + a^2 \right) \sin(x) \right) \sqrt{-a} \log \left(\frac{2 a \cos(x)^2 - 2 \left(\cos(x)^2 + (\cos(x) + 1) \sin(x) - 1 \right) \sqrt{-a} \sqrt{\frac{a \sin(x) + a}{\sin(x)}} + a \cos(x) - (2 a \cos(x) + a) \sin(x) - a}{\cos(x) + \sin(x) + 1} \right)}{3 \left(\cos(x)^2 - (\cos(x) + 1) \sin(x) - 1 \right)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*csc(x))^(5/2),x, algorithm="fricas")

[Out] [1/3*(3*(a^2*cos(x)^2 - a^2 - (a^2*cos(x) + a^2)*sin(x))*sqrt(-a)*log((2*a*cos(x)^2 - 2*(cos(x)^2 + (cos(x) + 1)*sin(x) - 1)*sqrt(-a)*sqrt((a*sin(x) + a)/sin(x)) + a*cos(x) - (2*a*cos(x) + a)*sin(x) - a)/(cos(x) + sin(x) + 1)) + 2*(8*a^2*cos(x)^2 + a^2*cos(x) - 7*a^2 + (8*a^2*cos(x) + 7*a^2)*sin(x))*sqrt((a*sin(x) + a)/sin(x)))/(cos(x)^2 - (cos(x) + 1)*sin(x) - 1), 2/3*(3*(a^2*cos(x)^2 - a^2 - (a^2*cos(x) + a^2)*sin(x))*sqrt(a)*arctan(-sqrt(a)*sqrt((a*sin(x) + a)/sin(x))*(cos(x) - sin(x) + 1)/(a*cos(x) + a*sin(x) + a)) + (8*a^2*cos(x)^2 + a^2*cos(x) - 7*a^2 + (8*a^2*cos(x) + 7*a^2)*sin(x))*sqrt((a*sin(x) + a)/sin(x)))/(cos(x)^2 - (cos(x) + 1)*sin(x) - 1)]

giac [B] time = 1.01, size = 250, normalized size = 3.85

$$\left(a^2 \sqrt{|a|} + a|a|^{\frac{3}{2}} \right) \arctan \left(\frac{\sqrt{2} \left(\sqrt{2} \sqrt{|a|} + 2 \sqrt{a \tan \left(\frac{1}{2} x \right)} \right)}{2 \sqrt{|a|}} \right) + \left(a^2 \sqrt{|a|} + a|a|^{\frac{3}{2}} \right) \arctan \left(-\frac{\sqrt{2} \left(\sqrt{2} \sqrt{|a|} - 2 \sqrt{a \tan \left(\frac{1}{2} x \right)} \right)}{2 \sqrt{|a|}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*csc(x))^(5/2),x, algorithm="giac")

[Out] (a^2*sqrt(abs(a)) + a*abs(a)^(3/2))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(a)) + 2*sqrt(a*tan(1/2*x)))/sqrt(abs(a))) + (a^2*sqrt(abs(a)) + a*abs(a)^(3/2))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(a)) - 2*sqrt(a*tan(1/2*x)))/sqrt(abs(a))) + 1/2*(a^2*sqrt(abs(a)) - a*abs(a)^(3/2))*log(a*tan(1/2*x) + sqrt(2)*sqrt(a*tan(1/2*x))*sqrt(abs(a)) + abs(a)) - 1/2*(a^2*sqrt(abs(a)) - a*abs(a)^(3/2))*log(a*tan(1/2*x) - sqrt(2)*sqrt(a*tan(1/2*x))*sqrt(abs(a)) + abs(a)) + 1/6*sqrt(2)*(sqrt(a*tan(1/2*x))*a^2*tan(1/2*x) + 15*sqrt(a*tan(1/2*x))*a^2) - 1/6*sqrt(2)*(15*a^4*tan(1/2*x) + a^4)/(sqrt(a*tan(1/2*x))*a*tan(1/2*x))

maple [B] time = 0.92, size = 535, normalized size = 8.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*csc(x))^(5/2),x)`

[Out] $\frac{1}{6} * (-3 * \sin(x) * \cos(x) * (-(-1 + \cos(x)) / \sin(x))^{3/2} * \ln(-2^{1/2} * (-(-1 + \cos(x)) / \sin(x))^{1/2} * \sin(x) + \sin(x) - \cos(x) + 1) / (2^{1/2} * (-(-1 + \cos(x)) / \sin(x))^{1/2} * \sin(x) - \sin(x) + \cos(x) - 1)) - 12 * \sin(x) * \cos(x) * (-(-1 + \cos(x)) / \sin(x))^{3/2} * \arctan(2^{1/2} * (-(-1 + \cos(x)) / \sin(x))^{1/2} + 1) - 12 * \sin(x) * \cos(x) * (-(-1 + \cos(x)) / \sin(x))^{3/2} * \arctan(2^{1/2} * (-(-1 + \cos(x)) / \sin(x))^{1/2} - 1) - 3 * \sin(x) * \cos(x) * (-(-1 + \cos(x)) / \sin(x))^{3/2} * \ln(-2^{1/2} * (-(-1 + \cos(x)) / \sin(x))^{1/2} * \sin(x) - \sin(x) + \cos(x) - 1) / (2^{1/2} * (-(-1 + \cos(x)) / \sin(x))^{1/2} * \sin(x) + \sin(x) - \cos(x) + 1)) - 3 * \sin(x) * (-(-1 + \cos(x)) / \sin(x))^{3/2} * \ln(-2^{1/2} * (-(-1 + \cos(x)) / \sin(x))^{1/2} * \sin(x) - \sin(x) + \cos(x) - 1) / (2^{1/2} * (-(-1 + \cos(x)) / \sin(x))^{1/2} * \sin(x) + \sin(x) - \cos(x) + 1)) - 12 * \sin(x) * (-(-1 + \cos(x)) / \sin(x))^{3/2} * \arctan(2^{1/2} * (-(-1 + \cos(x)) / \sin(x))^{1/2} + 1) - 12 * \sin(x) * (-(-1 + \cos(x)) / \sin(x))^{3/2} * \arctan(2^{1/2} * (-(-1 + \cos(x)) / \sin(x))^{1/2} - 1) - 3 * \sin(x) * (-(-1 + \cos(x)) / \sin(x))^{3/2} * \ln(-2^{1/2} * (-(-1 + \cos(x)) / \sin(x))^{1/2} * \sin(x) - \sin(x) + \cos(x) - 1) / (2^{1/2} * (-(-1 + \cos(x)) / \sin(x))^{1/2} * \sin(x) + \sin(x) - \cos(x) + 1)) + 16 * \sin(x) * \cos(x) * 2^{1/2} - 16 * \cos(x)^2 * 2^{1/2} - 14 * \sin(x) * 2^{1/2} + 2 * \cos(x) * 2^{1/2} + 14 * 2^{1/2} * \sin(x) * (a * (1 + \sin(x)) / \sin(x))^{5/2} / (\cos(x)^2 * \sin(x) - \cos(x)^3 + 2 * \cos(x) * \sin(x) + 3 * \cos(x)^2 - 4 * \sin(x) + 2 * \cos(x) - 4) * 2^{1/2})$

maxima [B] time = 0.47, size = 417, normalized size = 6.42

$$\frac{1}{22} \sqrt{2} a^{\frac{5}{2}} \left(\frac{\sin(x)}{\cos(x) + 1} \right)^{\frac{11}{2}} + \frac{5}{18} \sqrt{2} a^{\frac{5}{2}} \left(\frac{\sin(x)}{\cos(x) + 1} \right)^{\frac{9}{2}} + \frac{9}{14} \sqrt{2} a^{\frac{5}{2}} \left(\frac{\sin(x)}{\cos(x) + 1} \right)^{\frac{7}{2}} + \frac{1}{2} \sqrt{2} a^{\frac{5}{2}} \left(\frac{\sin(x)}{\cos(x) + 1} \right)^{\frac{5}{2}} - \frac{2}{3} \sqrt{2} a^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*csc(x))^(5/2),x, algorithm="maxima")`

[Out] $\frac{1}{22} * \sqrt{2} * a^{5/2} * (\sin(x) / (\cos(x) + 1))^{11/2} + \frac{5}{18} * \sqrt{2} * a^{5/2} * (\sin(x) / (\cos(x) + 1))^{9/2} + \frac{9}{14} * \sqrt{2} * a^{5/2} * (\sin(x) / (\cos(x) + 1))^{7/2} + \frac{1}{2} * \sqrt{2} * a^{5/2} * (\sin(x) / (\cos(x) + 1))^{5/2} - \frac{2}{3} * \sqrt{2} * a^{5/2} * (\sin(x) / (\cos(x) + 1))^{3/2} + \sqrt{2} * (\sqrt{2} * \arctan(1/2 * \sqrt{2} * (\sqrt{2} + 2 * \sqrt{\sin(x) / (\cos(x) + 1)}))) + \sqrt{2} * \arctan(-1/2 * \sqrt{2} * (\sqrt{2} - 2 * \sqrt{\sin(x) / (\cos(x) + 1)}))) * a^{5/2} - 2 * \sqrt{2} * a^{5/2} * \sqrt{\sin(x) / (\cos(x) + 1)} - 1/1386 * (693 * \sqrt{2} * a^{5/2} * \sin(x) / (\cos(x) + 1) + 1155 * \sqrt{2} * a^{5/2} * \sin(x)^2 / (\cos(x) + 1)^2 + 1386 * \sqrt{2} * a^{5/2} * \sin(x)^3 / (\cos(x) + 1)^3 + 990 * \sqrt{2} * a^{5/2} * \sin(x)^4 / (\cos(x) + 1)^4 + 385 * \sqrt{2} * a^{5/2} * \sin(x)^5 / (\cos(x) + 1)^5 + 63 * \sqrt{2} * a^{5/2} * \sin(x)^6 / (\cos(x) + 1)^6) / \sqrt{\sin(x) / (\cos(x) + 1)} - 1/42 * (7 * \sqrt{2} * a^{5/2} * \sin(x) / (\cos(x) + 1) + 105 * \sqrt{2} * a^{5/2} * \sin(x)^2 / (\cos(x) + 1)^2 - 210 * \sqrt{2} * a^{5/2} * \sin(x)^3 / (\cos(x) + 1)^3 - 70 * \sqrt{2} * a^{5/2} * \sin(x)^4 / (\cos(x) + 1)^4 - 21 * \sqrt{2} * a^{5/2} * \sin(x)$

$\frac{1}{(\cos(x) + 1)^5} - \frac{3\sqrt{2}a^{5/2}\sin(x)^6}{(\cos(x) + 1)^6} / \frac{\sin(x)}{(\cos(x) + 1)^{5/2}}$

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(a + \frac{a}{\sin(x)} \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + a/sin(x))^(5/2), x)`

[Out] `int((a + a/sin(x))^(5/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \csc(x) + a)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*csc(x))**(5/2), x)`

[Out] `Integral((a*csc(x) + a)**(5/2), x)`

3.14 $\int (a + a \csc(x))^{3/2} dx$

Optimal. Leaf size=44

$$-2a^{3/2} \tan^{-1} \left(\frac{\sqrt{a} \cot(x)}{\sqrt{a \csc(x) + a}} \right) - \frac{2a^2 \cot(x)}{\sqrt{a \csc(x) + a}}$$

[Out] $-2*a^{(3/2)}*\arctan(\cot(x)*a^{(1/2)}/(a+a*\csc(x))^{(1/2)})-2*a^2*\cot(x)/(a+a*\csc(x))^{(1/2)}$

Rubi [A] time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3775, 21, 3774, 203}

$$-\frac{2a^2 \cot(x)}{\sqrt{a \csc(x) + a}} - 2a^{3/2} \tan^{-1} \left(\frac{\sqrt{a} \cot(x)}{\sqrt{a \csc(x) + a}} \right)$$

Antiderivative was successfully verified.

[In] Int[(a + a*Csc[x])^(3/2), x]

[Out] $-2*a^{(3/2)}*ArcTan[(Sqrt[a]*Cot[x])/Sqrt[a + a*Csc[x]]] - (2*a^2*Cot[x])/Sqrt[a + a*Csc[x]]$

Rule 21

Int[(u_)*((a_) + (b_)*(v_))^(m_)*((c_) + (d_)*(v_))^(n_), x_Symbol] := Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x, a + b*x])

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3774

Int[Sqrt[csc[(c_) + (d_)*(x_)]*(b_) + (a_)], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3775

```
Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.))^(n_.), x_Symbol] := -Simp[(b^2*Co
t[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[a/(n - 1),
Int[(a + b*Csc[c + d*x])^(n - 2)*(a*(n - 1) + b*(3*n - 4)*Csc[c + d*x]), x]
, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && Integer
Q[2*n]
```

Rubi steps

$$\begin{aligned} \int (a + a \csc(x))^{3/2} dx &= -\frac{2a^2 \cot(x)}{\sqrt{a + a \csc(x)}} + (2a) \int \frac{\frac{a}{2} + \frac{1}{2}a \csc(x)}{\sqrt{a + a \csc(x)}} dx \\ &= -\frac{2a^2 \cot(x)}{\sqrt{a + a \csc(x)}} + a \int \sqrt{a + a \csc(x)} dx \\ &= -\frac{2a^2 \cot(x)}{\sqrt{a + a \csc(x)}} - (2a^2) \text{Subst} \left(\int \frac{1}{a + x^2} dx, x, \frac{a \cot(x)}{\sqrt{a + a \csc(x)}} \right) \\ &= -2a^{3/2} \tan^{-1} \left(\frac{\sqrt{a} \cot(x)}{\sqrt{a + a \csc(x)}} \right) - \frac{2a^2 \cot(x)}{\sqrt{a + a \csc(x)}} \end{aligned}$$

Mathematica [A] time = 0.09, size = 69, normalized size = 1.57

$$-\frac{2a\sqrt{a(\csc(x)+1)} \left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right) \right) \left(\sqrt{\csc(x)-1} + \tan^{-1}\left(\sqrt{\csc(x)-1}\right) \right)}{\sqrt{\csc(x)-1} \left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Csc[x])^(3/2), x]

[Out] (-2*a*(ArcTan[Sqrt[-1 + Csc[x]]] + Sqrt[-1 + Csc[x]])*Sqrt[a*(1 + Csc[x])]*(Cos[x/2] - Sin[x/2]))/(Sqrt[-1 + Csc[x]]*(Cos[x/2] + Sin[x/2]))

fricas [B] time = 0.46, size = 212, normalized size = 4.82

$$\left[\frac{(a \cos(x) + a \sin(x) + a)\sqrt{-a} \log \left(\frac{2a \cos(x)^2 - 2(\cos(x)^2 + (\cos(x)+1) \sin(x) - 1)\sqrt{-a} \sqrt{\frac{a \sin(x)+a}{\sin(x)}} + a \cos(x) - (2a \cos(x)+a) \sin(x) - a}{\cos(x)+\sin(x)+1} \right)}{\cos(x) + \sin(x) + 1} \right] - 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*csc(x))^(3/2),x, algorithm="fricas")
```

```
[Out] [(a*cos(x) + a*sin(x) + a)*sqrt(-a)*log((2*a*cos(x)^2 - 2*(cos(x)^2 + (cos(x) + 1)*sin(x) - 1)*sqrt(-a)*sqrt((a*sin(x) + a)/sin(x)) + a*cos(x) - (2*a*cos(x) + a)*sin(x) - a)/(cos(x) + sin(x) + 1)) - 2*(a*cos(x) - a*sin(x) + a)*sqrt((a*sin(x) + a)/sin(x)))/(cos(x) + sin(x) + 1), 2*((a*cos(x) + a*sin(x) + a)*sqrt(a)*arctan(-sqrt(a)*sqrt((a*sin(x) + a)/sin(x))*(cos(x) - sin(x) + 1)/(a*cos(x) + a*sin(x) + a)) - (a*cos(x) - a*sin(x) + a)*sqrt((a*sin(x) + a)/sin(x)))/(cos(x) + sin(x) + 1)]
```

giac [B] time = 1.16, size = 195, normalized size = 4.43

$$\sqrt{2} \sqrt{a \tan\left(\frac{1}{2} x\right)} a - \frac{\sqrt{2} a^2}{\sqrt{a \tan\left(\frac{1}{2} x\right)}} + \left(a \sqrt{|a|} + |a|^{\frac{3}{2}}\right) \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} \sqrt{|a|} + 2 \sqrt{a \tan\left(\frac{1}{2} x\right)}\right)}{2 \sqrt{|a|}}\right) + \left(a \sqrt{|a|} + |a|^{\frac{3}{2}}\right) \arctan\left(\frac{\sqrt{2} \left(\sqrt{2} \sqrt{|a|} + 2 \sqrt{a \tan\left(\frac{1}{2} x\right)}\right)}{2 \sqrt{|a|}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*csc(x))^(3/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*sqrt(a*tan(1/2*x))*a - sqrt(2)*a^2/sqrt(a*tan(1/2*x)) + (a*sqrt(abs(a)) + abs(a)^(3/2))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(a)) + 2*sqrt(a*tan(1/2*x)))/sqrt(abs(a))) + (a*sqrt(abs(a)) + abs(a)^(3/2))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(a)) - 2*sqrt(a*tan(1/2*x)))/sqrt(abs(a))) + 1/2*(a*sqrt(abs(a)) - abs(a)^(3/2))*log(a*tan(1/2*x) + sqrt(2)*sqrt(a*tan(1/2*x))*sqrt(abs(a)) + abs(a)) - 1/2*(a*sqrt(abs(a)) - abs(a)^(3/2))*log(a*tan(1/2*x) - sqrt(2)*sqrt(a*tan(1/2*x))*sqrt(abs(a)) + abs(a))
```

maple [B] time = 0.63, size = 273, normalized size = 6.20

$$\left(\sqrt{\frac{-1+\cos(x)}{\sin(x)}} \ln\left(\frac{\sqrt{2} \sqrt{\frac{-1+\cos(x)}{\sin(x)}} \sin(x)+\sin(x)-\cos(x)+1}{\sqrt{2} \sqrt{\frac{-1+\cos(x)}{\sin(x)}} \sin(x)-\sin(x)+\cos(x)-1}\right) \sin(x) + 4 \sqrt{\frac{-1+\cos(x)}{\sin(x)}} \arctan\left(\sqrt{2} \sqrt{\frac{-1+\cos(x)}{\sin(x)}} + 1\right) \sin(x) + 4 \sqrt{\frac{-1+\cos(x)}{\sin(x)}} \arctan\left(\sqrt{2} \sqrt{\frac{-1+\cos(x)}{\sin(x)}} + 1\right) \sin(x) + 4 \sqrt{\frac{-1+\cos(x)}{\sin(x)}} \arctan\left(\sqrt{2} \sqrt{\frac{-1+\cos(x)}{\sin(x)}} + 1\right) \sin(x) + 4 \sqrt{\frac{-1+\cos(x)}{\sin(x)}} \arctan\left(\sqrt{2} \sqrt{\frac{-1+\cos(x)}{\sin(x)}} + 1\right) \sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*csc(x))^(3/2),x)
```

```
[Out] -1/2*((-(-1+cos(x))/sin(x))^(1/2)*ln(-(2^(1/2))*(-(-1+cos(x))/sin(x))^(1/2)*sin(x)+sin(x)-cos(x)+1)/(2^(1/2))*(-(-1+cos(x))/sin(x))^(1/2)*sin(x)-sin(x)+cos(x)-1))*sin(x)+4*(-(-1+cos(x))/sin(x))^(1/2)*arctan(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)+1)*sin(x)+4*(-(-1+cos(x))/sin(x))^(1/2)*arctan(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)-1)*sin(x)+(-(-1+cos(x))/sin(x))^(1/2)*ln(-(2^(1/2))*(-(-1+cos(x))/sin(x))^(1/2))*sin(x)+(-(-1+cos(x))/sin(x))^(1/2)*ln(-(2^(1/2))*(-(-1+cos(x))/sin(x))^(1/2))*sin(x)
```

$$\begin{aligned} & (-(-1+\cos(x))/\sin(x))^{1/2} \sin(x) - \sin(x) + \cos(x) - 1 / (2^{1/2} * (-(-1+\cos(x))/ \\ & \sin(x))^{1/2} \sin(x) + \sin(x) - \cos(x) + 1) * \sin(x) - 2 * \sin(x) * 2^{1/2} - 2 * \cos(x) * 2^{1/2} \\ & + 2 * 2^{1/2} * \sin(x) * (a * (1 + \sin(x)) / \sin(x))^{3/2} / (\cos(x) * \sin(x) + \cos(x) - 2 * \\ & \sin(x) + \cos(x) - 2) * 2^{1/2} \end{aligned}$$

maxima [B] time = 0.45, size = 200, normalized size = 4.55

$$\sqrt{2} \left(\sqrt{2} \arctan \left(\frac{1}{2} \sqrt{2} \left(\sqrt{2} + 2 \sqrt{\frac{\sin(x)}{\cos(x) + 1}} \right) \right) + \sqrt{2} \arctan \left(-\frac{1}{2} \sqrt{2} \left(\sqrt{2} - 2 \sqrt{\frac{\sin(x)}{\cos(x) + 1}} \right) \right) \right) a^{\frac{3}{2}} - \frac{1}{5} \sqrt{2} \left(a^{\frac{3}{2}} \left(\frac{\sin(x)}{\cos(x) + 1} \right)^{\frac{5}{2}} + 5 a^{\frac{3}{2}} \left(\frac{\sin(x)}{\cos(x) + 1} \right)^{\frac{3}{2}} + 10 a^{\frac{3}{2}} \sqrt{\frac{\sin(x)}{\cos(x) + 1}} \right) - \frac{1}{5} (5 \sqrt{2} a^{\frac{3}{2}} \sin(x) / (\cos(x) + 1) - 15 \sqrt{2} a^{\frac{3}{2}} \sin^2(x) / (\cos(x) + 1)^2 - 5 \sqrt{2} a^{\frac{3}{2}} \sin^3(x) / (\cos(x) + 1)^3 - \sqrt{2} a^{\frac{3}{2}} \sin^4(x) / (\cos(x) + 1)^4) / (\sin(x) / (\cos(x) + 1))^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*csc(x))^(3/2),x, algorithm="maxima")

[Out] sqrt(2)*(sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(sin(x)/(cos(x) + 1)))) + sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(sin(x)/(cos(x) + 1)))))*a^(3/2) - 1/5*sqrt(2)*(a^(3/2)*(sin(x)/(cos(x) + 1))^(5/2) + 5*a^(3/2)*(sin(x)/(cos(x) + 1))^(3/2) + 10*a^(3/2)*sqrt(sin(x)/(cos(x) + 1))) - 1/5*(5*sqrt(2)*a^(3/2)*sin(x)/(cos(x) + 1) - 15*sqrt(2)*a^(3/2)*sin(x)^2/(cos(x) + 1)^2 - 5*sqrt(2)*a^(3/2)*sin(x)^3/(cos(x) + 1)^3 - sqrt(2)*a^(3/2)*sin(x)^4/(cos(x) + 1)^4)/(sin(x)/(cos(x) + 1))^(3/2)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(a + \frac{a}{\sin(x)} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/sin(x))^(3/2),x)

[Out] int((a + a/sin(x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \csc(x) + a)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*csc(x))**(3/2),x)

[Out] Integral((a*csc(x) + a)**(3/2), x)

3.15 $\int \sqrt{a + a \csc(x)} dx$

Optimal. Leaf size=26

$$-2\sqrt{a} \tan^{-1} \left(\frac{\sqrt{a} \cot(x)}{\sqrt{a \csc(x) + a}} \right)$$

[Out] $-2*\arctan(\cot(x)*a^{(1/2)/(a+a*\csc(x))^{(1/2)})*a^{(1/2)}$

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3774, 203}

$$-2\sqrt{a} \tan^{-1} \left(\frac{\sqrt{a} \cot(x)}{\sqrt{a \csc(x) + a}} \right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Csc[x]], x]

[Out] $-2*\text{Sqrt}[a]*\text{ArcTan}[(\text{Sqrt}[a]*\text{Cot}[x])/\text{Sqrt}[a + a*\text{Csc}[x]]]$

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3774

Int[Sqrt[csc[(c_) + (d_)*(x_)]*(b_) + (a_)], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned} \int \sqrt{a + a \csc(x)} dx &= - \left((2a) \text{Subst} \left(\int \frac{1}{a + x^2} dx, x, \frac{a \cot(x)}{\sqrt{a + a \csc(x)}} \right) \right) \\ &= -2\sqrt{a} \tan^{-1} \left(\frac{\sqrt{a} \cot(x)}{\sqrt{a + a \csc(x)}} \right) \end{aligned}$$

Mathematica [A] time = 0.05, size = 32, normalized size = 1.23

$$-\frac{2a \cot(x) \tan^{-1} \left(\sqrt{\csc(x) - 1} \right)}{\sqrt{\csc(x) - 1} \sqrt{a(\csc(x) + 1)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + a*Csc[x]],x]

[Out] (-2*a*ArcTan[Sqrt[-1 + Csc[x]]]*Cot[x])/(Sqrt[-1 + Csc[x]]*Sqrt[a*(1 + Csc[x])])

fricas [B] time = 0.55, size = 120, normalized size = 4.62

$$\left[\sqrt{-a} \log \left(\frac{2a \cos(x)^2 - 2(\cos(x)^2 + (\cos(x) + 1) \sin(x) - 1) \sqrt{-a} \sqrt{\frac{a \sin(x) + a}{\sin(x)}} + a \cos(x) - (2a \cos(x) + a) \sin(x)}{\cos(x) + \sin(x) + 1} \right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*csc(x))^(1/2),x, algorithm="fricas")

[Out] [sqrt(-a)*log((2*a*cos(x)^2 - 2*(cos(x)^2 + (cos(x) + 1)*sin(x) - 1)*sqrt(-a)*sqrt((a*sin(x) + a)/sin(x)) + a*cos(x) - (2*a*cos(x) + a)*sin(x) - a)/(cos(x) + sin(x) + 1)), 2*sqrt(a)*arctan(-sqrt(a)*sqrt((a*sin(x) + a)/sin(x))*(cos(x) - sin(x) + 1)/(a*cos(x) + a*sin(x) + a))]

giac [B] time = 1.06, size = 353, normalized size = 13.58

$$\frac{1}{4} \sqrt{2} \left(\frac{2 \sqrt{2} \left(a \sqrt{|a|} \operatorname{sgn} \left(\tan \left(\frac{1}{2} x \right)^3 + \tan \left(\frac{1}{2} x \right)^2 + \tan \left(\frac{1}{2} x \right) + 1 \right) + |a|^{\frac{3}{2}} \operatorname{sgn} \left(\tan \left(\frac{1}{2} x \right)^3 + \tan \left(\frac{1}{2} x \right)^2 + \tan \left(\frac{1}{2} x \right) + 1 \right)}{a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*csc(x))^(1/2),x, algorithm="giac")

[Out] 1/4*sqrt(2)*(2*sqrt(2)*(a*sqrt(abs(a))*sgn(tan(1/2*x)^3 + tan(1/2*x)^2 + tan(1/2*x) + 1) + abs(a)^(3/2)*sgn(tan(1/2*x)^3 + tan(1/2*x)^2 + tan(1/2*x) + 1))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(a)) + 2*sqrt(a*tan(1/2*x)))/sqrt(abs(a)))/a + 2*sqrt(2)*(a*sqrt(abs(a))*sgn(tan(1/2*x)^3 + tan(1/2*x)^2 + tan(1/2*x) + 1) + abs(a)^(3/2)*sgn(tan(1/2*x)^3 + tan(1/2*x)^2 + tan(1/2*x) + 1))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(a)) - 2*sqrt(a*tan(1/2*x)))/sqrt(abs(a)))/a + sqrt(2)*(a*sqrt(abs(a))*sgn(tan(1/2*x)^3 + tan(1/2*x)^2 + tan(1/2*x) + 1) - abs(a)^(3/2)*sgn(tan(1/2*x)^3 + tan(1/2*x)^2 + tan(1/2*x) + 1))*log(a*tan(1/2*x) + sqrt(2)*sqrt(a*tan(1/2*x))*sqrt(abs(a)) + abs(a))/a - sqrt(2)*(a*sqrt(abs(a))*sgn(tan(1/2*x)^3 + tan(1/2*x)^2 + tan(1/2*x) + 1)

- abs(a)^(3/2)*sgn(tan(1/2*x)^3 + tan(1/2*x)^2 + tan(1/2*x) + 1))*log(a*tan(1/2*x) - sqrt(2)*sqrt(a*tan(1/2*x))*sqrt(abs(a) + abs(a))/a)*sgn(sin(x))

maple [B] time = 0.72, size = 199, normalized size = 7.65

$$\frac{\sqrt{2} \sqrt{\frac{a(1+\sin(x))}{\sin(x)}} \sin(x) \sqrt{\frac{-1+\cos(x)}{\sin(x)}} \left(\ln \left(-\frac{\sqrt{2} \sqrt{\frac{-1+\cos(x)}{\sin(x)}} \sin(x) + \sin(x) - \cos(x) + 1}{\sqrt{2} \sqrt{\frac{-1+\cos(x)}{\sin(x)}} \sin(x) - \sin(x) + \cos(x) - 1} \right) + 4 \arctan \left(\sqrt{2} \sqrt{\frac{-1+\cos(x)}{\sin(x)}} + 1 \right) \right)}{2 - 2 \cos(x) + 2 \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*csc(x))^(1/2),x)

[Out] 1/2*2^(1/2)*(a*(1+sin(x))/sin(x))^(1/2)*sin(x)*(-(-1+cos(x))/sin(x))^(1/2)*(ln(-(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)*sin(x)+sin(x)-cos(x)+1)/(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)*sin(x)-sin(x)+cos(x)-1))+4*arctan(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)+1)+4*arctan(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)-1)+ln(-(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)*sin(x)-sin(x)+cos(x)-1)/(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)*sin(x)+sin(x)-cos(x)+1)))/(1-cos(x)+sin(x))

maxima [B] time = 0.44, size = 148, normalized size = 5.69

$$-\frac{2}{3} \sqrt{2} \sqrt{a} \left(\frac{\sin(x)}{\cos(x) + 1} \right)^{\frac{3}{2}} + \sqrt{2} \left(\sqrt{2} \arctan \left(\frac{1}{2} \sqrt{2} \left(\sqrt{2} + 2 \sqrt{\frac{\sin(x)}{\cos(x) + 1}} \right) \right) + \sqrt{2} \arctan \left(-\frac{1}{2} \sqrt{2} \left(\sqrt{2} - 2 \sqrt{\frac{\sin(x)}{\cos(x) + 1}} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*csc(x))^(1/2),x, algorithm="maxima")

[Out] -2/3*sqrt(2)*sqrt(a)*(sin(x)/(cos(x) + 1))^(3/2) + sqrt(2)*(sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(sin(x)/(cos(x) + 1)))) + sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(sin(x)/(cos(x) + 1)))))*sqrt(a) - 2*sqrt(2)*sqrt(a)*sqrt(sin(x)/(cos(x) + 1)) + 2/3*(3*sqrt(2)*sqrt(a)*sin(x)/(cos(x) + 1) + sqrt(2)*sqrt(a)*sin(x)^2/(cos(x) + 1)^2)/sqrt(sin(x)/(cos(x) + 1))

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{a + \frac{a}{\sin(x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/sin(x))^(1/2),x)

```
[Out] int((a + a/sin(x))^(1/2), x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \sqrt{a \csc(x) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*csc(x))**(1/2),x)
```

```
[Out] Integral(sqrt(a*csc(x) + a), x)
```

$$3.16 \quad \int \frac{1}{\sqrt{a+a \csc(x)}} dx$$

Optimal. Leaf size=62

$$\frac{\sqrt{2} \tan^{-1}\left(\frac{\sqrt{a} \cot(x)}{\sqrt{2} \sqrt{a \csc(x)+a}}\right)}{\sqrt{a}} - \frac{2 \tan^{-1}\left(\frac{\sqrt{a} \cot(x)}{\sqrt{a \csc(x)+a}}\right)}{\sqrt{a}}$$

[Out] $-2*\arctan(\cot(x)*a^{(1/2)/(a+a*\csc(x))^{(1/2)})/a^{(1/2)}+\arctan(1/2*\cot(x)*a^{(1/2)*2^{(1/2)/(a+a*\csc(x))^{(1/2)}}*2^{(1/2)/a^{(1/2)}$

Rubi [A] time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3776, 3774, 203, 3795}

$$\frac{\sqrt{2} \tan^{-1}\left(\frac{\sqrt{a} \cot(x)}{\sqrt{2} \sqrt{a \csc(x)+a}}\right)}{\sqrt{a}} - \frac{2 \tan^{-1}\left(\frac{\sqrt{a} \cot(x)}{\sqrt{a \csc(x)+a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a + a*Csc[x]],x]

[Out] $(-2*\text{ArcTan}[(\text{Sqrt}[a]*\text{Cot}[x])/(\text{Sqrt}[a + a*\text{Csc}[x]])]/\text{Sqrt}[a] + (\text{Sqrt}[2]*\text{ArcTan}[(\text{Sqrt}[a]*\text{Cot}[x])/(\text{Sqrt}[2]*\text{Sqrt}[a + a*\text{Csc}[x]])])/(\text{Sqrt}[a])$

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3774

Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3776

Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[c + d*x]], x], x] - Dist[b/a, Int[Csc[c + d*x]/Sqrt[a + b*Csc[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3795

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol]
:> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{a + a \csc(x)}} dx &= \frac{\int \sqrt{a + a \csc(x)} dx}{a} - \int \frac{\csc(x)}{\sqrt{a + a \csc(x)}} dx \\ &= -\left(2 \operatorname{Subst}\left(\int \frac{1}{a + x^2} dx, x, \frac{a \cot(x)}{\sqrt{a + a \csc(x)}}\right)\right) + 2 \operatorname{Subst}\left(\int \frac{1}{2a + x^2} dx, x, \frac{a \cot(x)}{\sqrt{a + a \csc(x)}}\right) \\ &= -\frac{2 \tan^{-1}\left(\frac{\sqrt{a} \cot(x)}{\sqrt{a + a \csc(x)}}\right)}{\sqrt{a}} + \frac{\sqrt{2} \tan^{-1}\left(\frac{\sqrt{a} \cot(x)}{\sqrt{2} \sqrt{a + a \csc(x)}}\right)}{\sqrt{a}} \end{aligned}$$

Mathematica [A] time = 0.14, size = 54, normalized size = 0.87

$$\frac{\cot(x) \left(\sqrt{2} \tan^{-1} \left(\frac{\sqrt{\csc(x)-1}}{\sqrt{2}} \right) - 2 \tan^{-1} \left(\sqrt{\csc(x)-1} \right) \right)}{\sqrt{\csc(x)-1} \sqrt{a(\csc(x)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + a*Csc[x]], x]
```

```
[Out] ((-2*ArcTan[Sqrt[-1 + Csc[x]]] + Sqrt[2]*ArcTan[Sqrt[-1 + Csc[x]]/Sqrt[2]])*Cot[x])/(Sqrt[-1 + Csc[x]]*Sqrt[a*(1 + Csc[x])])
```

fricas [A] time = 0.54, size = 219, normalized size = 3.53

$$\frac{\sqrt{2} a \sqrt{-\frac{1}{a}} \log\left(\frac{\sqrt{2} \sqrt{\frac{a \sin(x)+a}{\sin(x)}} \sqrt{-\frac{1}{a}} \sin(x)+\cos(x)}{\sin(x)+1}\right) - \sqrt{-a} \log\left(\frac{2 a \cos(x)^2+2(\cos(x)^2+(\cos(x)+1) \sin(x)-1) \sqrt{-a} \sqrt{\frac{a \sin(x)+a}{\sin(x)}}+a \cos(x)-\sqrt{-a}}{\cos(x)+\sin(x)+1}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*csc(x))^(1/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(2)*a*sqrt(-1/a)*log((sqrt(2)*sqrt((a*sin(x) + a)/sin(x))*sqrt(-1/a)*sin(x) + cos(x))/(sin(x) + 1)) - sqrt(-a)*log((2*a*cos(x)^2 + 2*(cos(x)^2 +
```

$(\cos(x) + 1) \cdot \sin(x) - 1) \cdot \sqrt{-a} \cdot \sqrt{(a \cdot \sin(x) + a) / \sin(x)} + a \cdot \cos(x) - (2 \cdot a \cdot \cos(x) + a) \cdot \sin(x) - a) / (\cos(x) + \sin(x) + 1) / a, -2 \cdot (\sqrt{2} \cdot \sqrt{a}) \cdot \arctan(\sqrt{2} \cdot \sqrt{(a \cdot \sin(x) + a) / \sin(x)}) \cdot \sin(x) / (\sqrt{a} \cdot (\cos(x) + \sin(x) + 1))) - \sqrt{a} \cdot \arctan(-\sqrt{a} \cdot \sqrt{(a \cdot \sin(x) + a) / \sin(x)}) \cdot (\cos(x) - \sin(x) + 1) / (a \cdot \cos(x) + a \cdot \sin(x) + a)) / a]$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a \csc(x) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*csc(x))^(1/2),x, algorithm="giac")

[Out] integrate(1/sqrt(a*csc(x) + a), x)

maple [B] time = 0.74, size = 221, normalized size = 3.56

$$\frac{\left(4\sqrt{2} \arctan\left(\sqrt{-\frac{-1+\cos(x)}{\sin(x)}}\right) - \ln\left(-\frac{\sqrt{2} \sqrt{-\frac{-1+\cos(x)}{\sin(x)}} \sin(x) + \sin(x) - \cos(x) + 1}{\sqrt{2} \sqrt{-\frac{-1+\cos(x)}{\sin(x)}} \sin(x) - \sin(x) + \cos(x) - 1}\right) - 4 \arctan\left(\sqrt{2} \sqrt{-\frac{-1+\cos(x)}{\sin(x)}} + 1\right) - 4 \arctan\left(\sqrt{2} \sqrt{-\frac{-1+\cos(x)}{\sin(x)}} - 1\right) \right)}{4 \sqrt{\frac{a(1+\sin(x))}{\sin(x)}} \sin(x) \sqrt{\frac{a(1+\sin(x))}{\sin(x)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+a*csc(x))^(1/2),x)

[Out] $-1/4 \cdot (4 \cdot 2^{(1/2)} \cdot \arctan((-(1+\cos(x))/\sin(x))^{(1/2)}) - \ln(-(2^{(1/2)} \cdot (-(1+\cos(x))/\sin(x))^{(1/2)} \cdot \sin(x) + \sin(x) - \cos(x) + 1) / (2^{(1/2)} \cdot (-(1+\cos(x))/\sin(x))^{(1/2)} \cdot \sin(x) - \sin(x) + \cos(x) - 1))) - 4 \cdot \arctan(2^{(1/2)} \cdot (-(1+\cos(x))/\sin(x))^{(1/2)} + 1) - 4 \cdot \arctan(2^{(1/2)} \cdot (-(1+\cos(x))/\sin(x))^{(1/2)} - 1) - \ln(-(2^{(1/2)} \cdot (-(1+\cos(x))/\sin(x))^{(1/2)} \cdot \sin(x) - \sin(x) + \cos(x) - 1) / (2^{(1/2)} \cdot (-(1+\cos(x))/\sin(x))^{(1/2)} \cdot \sin(x) + \sin(x) - \cos(x) + 1))) \cdot (1 - \cos(x) + \sin(x)) / (a \cdot (1 + \sin(x)) / \sin(x))^{(1/2)} / \sin(x) / (-(1+\cos(x))/\sin(x))^{(1/2)} \cdot 2^{(1/2)})$

maxima [A] time = 0.45, size = 83, normalized size = 1.34

$$\frac{\sqrt{2} \left(\sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} \left(\sqrt{2} + 2 \sqrt{\frac{\sin(x)}{\cos(x)+1}} \right) \right) + \sqrt{2} \arctan\left(-\frac{1}{2} \sqrt{2} \left(\sqrt{2} - 2 \sqrt{\frac{\sin(x)}{\cos(x)+1}} \right) \right) \right)}{\sqrt{a}} - \frac{2 \sqrt{2} \arctan\left(\sqrt{\frac{\sin(x)}{\cos(x)+1}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*csc(x))^(1/2),x, algorithm="maxima")

```
[Out] sqrt(2)*(sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*sqrt(sin(x)/(cos(x) + 1)))
) + sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*sqrt(sin(x)/(cos(x) + 1)))))/s
qrt(a) - 2*sqrt(2)*arctan(sqrt(sin(x)/(cos(x) + 1)))/sqrt(a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{a + \frac{a}{\sin(x)}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + a/sin(x))^(1/2), x)
```

```
[Out] int(1/(a + a/sin(x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a \csc(x) + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*csc(x))**(1/2), x)
```

```
[Out] Integral(1/sqrt(a*csc(x) + a), x)
```


$$3.17 \quad \int \frac{1}{(a+a \csc(x))^{3/2}} dx$$

Optimal. Leaf size=81

$$-\frac{2 \tan^{-1}\left(\frac{\sqrt{a} \cot(x)}{\sqrt{a \csc(x)+a}}\right)}{a^{3/2}} + \frac{5 \tan^{-1}\left(\frac{\sqrt{a} \cot(x)}{\sqrt{2} \sqrt{a \csc(x)+a}}\right)}{2\sqrt{2} a^{3/2}} + \frac{\cot(x)}{2(a \csc(x) + a)^{3/2}}$$

[Out] $-2*\arctan(\cot(x)*a^{(1/2)}/(a+a*\csc(x))^{(1/2)})/a^{(3/2)}+1/2*\cot(x)/(a+a*\csc(x))^{(3/2)}+5/4*\arctan(1/2*\cot(x)*a^{(1/2)}*2^{(1/2)}/(a+a*\csc(x))^{(1/2)})/a^{(3/2)}*2^{(1/2)}$

Rubi [A] time = 0.10, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3777, 3920, 3774, 203, 3795}

$$-\frac{2 \tan^{-1}\left(\frac{\sqrt{a} \cot(x)}{\sqrt{a \csc(x)+a}}\right)}{a^{3/2}} + \frac{5 \tan^{-1}\left(\frac{\sqrt{a} \cot(x)}{\sqrt{2} \sqrt{a \csc(x)+a}}\right)}{2\sqrt{2} a^{3/2}} + \frac{\cot(x)}{2(a \csc(x) + a)^{3/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Csc}[x])^{(-3/2)}, x]$

[Out] $(-2*\text{ArcTan}[(\text{Sqrt}[a]*\text{Cot}[x])/(\text{Sqrt}[a + a*\text{Csc}[x]])])/a^{(3/2)} + (5*\text{ArcTan}[(\text{Sqrt}[a]*\text{Cot}[x])/(\text{Sqrt}[2]*\text{Sqrt}[a + a*\text{Csc}[x]])])/(2*\text{Sqrt}[2]*a^{(3/2)}) + \text{Cot}[x]/(2*(a + a*\text{Csc}[x])^{(3/2)})$

Rule 203

$\text{Int}[(a + (b_*)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTan}[(\text{Rt}[b, 2]*x)/(\text{Rt}[a, 2])])]/(\text{Rt}[a, 2]*\text{Rt}[b, 2]), x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{PosQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 3774

$\text{Int}[\text{Sqrt}[\csc[(c_*) + (d_*)*(x_)]*(b_*) + (a_*)], x_Symbol] \rightarrow \text{Dist}[(-2*b)/d, \text{Subst}[\text{Int}[1/(a + x^2), x], x, (b*\text{Cot}[c + d*x])/(\text{Sqrt}[a + b*\csc[c + d*x]])], x] /; \text{FreeQ}\{a, b, c, d\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rule 3777

$\text{Int}[(\csc[(c_*) + (d_*)*(x_)]*(b_*) + (a_*)^n), x_Symbol] \rightarrow -\text{Simp}[(\text{Cot}[c + d*x]*(a + b*\csc[c + d*x])^n)/(d*(2*n + 1)), x] + \text{Dist}[1/(a^2*(2*n + 1)), \text{Int}[(a + b*\csc[c + d*x])^{(n + 1)}*(a*(2*n + 1) - b*(n + 1)*\csc[c + d*x]), x]$

, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]

Rule 3795

Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3920

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(a + a \csc(x))^{3/2}} dx &= \frac{\cot(x)}{2(a + a \csc(x))^{3/2}} - \frac{\int \frac{-2a + \frac{1}{2}a \csc(x)}{\sqrt{a + a \csc(x)}} dx}{2a^2} \\
 &= \frac{\cot(x)}{2(a + a \csc(x))^{3/2}} + \frac{\int \sqrt{a + a \csc(x)} dx}{a^2} - \frac{5 \int \frac{\csc(x)}{\sqrt{a + a \csc(x)}} dx}{4a} \\
 &= \frac{\cot(x)}{2(a + a \csc(x))^{3/2}} - \frac{2 \operatorname{Subst}\left(\int \frac{1}{a+x^2} dx, x, \frac{a \cot(x)}{\sqrt{a + a \csc(x)}}\right)}{a} + \frac{5 \operatorname{Subst}\left(\int \frac{1}{2a+x^2} dx, x, \frac{a \cot(x)}{\sqrt{a + a \csc(x)}}\right)}{2a} \\
 &= -\frac{2 \tan^{-1}\left(\frac{\sqrt{a} \cot(x)}{\sqrt{a + a \csc(x)}}\right)}{a^{3/2}} + \frac{5 \tan^{-1}\left(\frac{\sqrt{a} \cot(x)}{\sqrt{2} \sqrt{a + a \csc(x)}}\right)}{2\sqrt{2} a^{3/2}} + \frac{\cot(x)}{2(a + a \csc(x))^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.43, size = 129, normalized size = 1.59

$$\frac{\left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\right) \left(-2 \csc(x) + 8\sqrt{\csc(x) - 1} (\csc(x) + 1) \tan^{-1}\left(\sqrt{\csc(x) - 1}\right) - 5\sqrt{2} \sqrt{\csc(x) - 1} \csc(x) \left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)\right)}{4a(\csc(x) - 1)\sqrt{a(\csc(x) + 1)} \left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right)\right)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Csc[x])^(-3/2), x]

```
[Out] -1/4*((Cos[x/2] - Sin[x/2])*(2 - 2*Csc[x] + 8*ArcTan[Sqrt[-1 + Csc[x]]])*Sqrt[-1 + Csc[x]]*(1 + Csc[x]) - 5*Sqrt[2]*ArcTan[Sqrt[-1 + Csc[x]]/Sqrt[2]]*Sqrt[-1 + Csc[x]]*Csc[x]*(Cos[x/2] + Sin[x/2])^2)/(a*(-1 + Csc[x])*Sqrt[a*(1 + Csc[x])]*(Cos[x/2] + Sin[x/2]))
```

fricas [B] time = 0.61, size = 427, normalized size = 5.27

$$\frac{5\sqrt{2}\left(\cos(x)^2 - (\cos(x) + 2)\sin(x) - \cos(x) - 2\right)\sqrt{-a} \log\left(-\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\sin(x)+a}{\sin(x)}}\sin(x)-a\cos(x)}{\sin(x)+1}\right) + 4\left(\cos(x)^2 - (\cos(x) + 2)\sin(x) - \cos(x) - 2\right)\sqrt{-a} \arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\sin(x)+a}{\sin(x)}}\sin(x)-a\cos(x)}{\sin(x)+1}\right)}{4\left(\cos(x)^2 - (\cos(x) + 2)\sin(x) - \cos(x) - 2\right)\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*csc(x))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(5*sqrt(2)*(cos(x)^2 - (cos(x) + 2)*sin(x) - cos(x) - 2)*sqrt(-a)*log(-(sqrt(2)*sqrt(-a)*sqrt((a*sin(x) + a)/sin(x))*sin(x) - a*cos(x))/(sin(x) + 1)) + 4*(cos(x)^2 - (cos(x) + 2)*sin(x) - cos(x) - 2)*sqrt(-a)*log((2*a*cos(x)^2 + 2*(cos(x)^2 + (cos(x) + 1)*sin(x) - 1)*sqrt(-a)*sqrt((a*sin(x) + a)/sin(x)) + a*cos(x) - (2*a*cos(x) + a)*sin(x) - a)/(cos(x) + sin(x) + 1)) + 2*(cos(x)^2 + (cos(x) + 1)*sin(x) - 1)*sqrt((a*sin(x) + a)/sin(x)))/(a^2*cos(x)^2 - a^2*cos(x) - 2*a^2 - (a^2*cos(x) + 2*a^2)*sin(x)), 1/2*(5*sqrt(2)*(cos(x)^2 - (cos(x) + 2)*sin(x) - cos(x) - 2)*sqrt(a)*arctan(sqrt(2)*sqrt(a)*sqrt((a*sin(x) + a)/sin(x))*(cos(x) + 1)/(a*cos(x) + a*sin(x) + a)) + 4*(cos(x)^2 - (cos(x) + 2)*sin(x) - cos(x) - 2)*sqrt(a)*arctan(-sqrt(a)*sqrt((a*sin(x) + a)/sin(x))*(cos(x) - sin(x) + 1)/(a*cos(x) + a*sin(x) + a)) - (cos(x)^2 + (cos(x) + 1)*sin(x) - 1)*sqrt((a*sin(x) + a)/sin(x)))/(a^2*cos(x)^2 - a^2*cos(x) - 2*a^2 - (a^2*cos(x) + 2*a^2)*sin(x))]
```

giac [B] time = 1.93, size = 304, normalized size = 3.75

$$\frac{1}{2} \left(\frac{5\sqrt{2} \arctan\left(\frac{\sqrt{a \tan\left(\frac{1}{2}x\right)}}{\sqrt{a}}\right)}{a^2 \operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right) + 1\right)} - \frac{2\left(a\sqrt{|a|} + |a|^{\frac{3}{2}}\right) \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|a|} + 2\sqrt{a \tan\left(\frac{1}{2}x\right)}\right)}{2\sqrt{|a|}}\right)}{a^3 \operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right) + 1\right)} - \frac{2\left(a\sqrt{|a|} + |a|^{\frac{3}{2}}\right) \arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|a|} - 2\sqrt{a \tan\left(\frac{1}{2}x\right)}\right)}{2\sqrt{|a|}}\right)}{a^3 \operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right) + 1\right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*csc(x))^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(5*sqrt(2)*arctan(sqrt(a*tan(1/2*x))/sqrt(a))/(a^(3/2)*sgn(tan(1/2*x)
+ 1)) - 2*(a*sqrt(abs(a)) + abs(a)^(3/2))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(
abs(a)) + 2*sqrt(a*tan(1/2*x)))/sqrt(abs(a)))/(a^3*sgn(tan(1/2*x) + 1)) - 2
*(a*sqrt(abs(a)) + abs(a)^(3/2))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(a))
- 2*sqrt(a*tan(1/2*x)))/sqrt(abs(a)))/(a^3*sgn(tan(1/2*x) + 1)) - (a*sqrt(a
bs(a)) - abs(a)^(3/2))*log(a*tan(1/2*x) + sqrt(2)*sqrt(a*tan(1/2*x))*sqrt(a
bs(a)) + abs(a))/(a^3*sgn(tan(1/2*x) + 1)) + (a*sqrt(abs(a)) - abs(a)^(3/2)
)*log(a*tan(1/2*x) - sqrt(2)*sqrt(a*tan(1/2*x))*sqrt(abs(a)) + abs(a))/(a^3
*sgn(tan(1/2*x) + 1)) + sqrt(2)*(sqrt(a*tan(1/2*x))*a*tan(1/2*x) - sqrt(a*t
an(1/2*x))*a)/((a*tan(1/2*x) + a)^2*a*sgn(tan(1/2*x) + 1))*sgn(tan(1/2*x))
```

maple [B] time = 0.81, size = 1141, normalized size = 14.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*csc(x))^(3/2),x)
```

```
[Out] -1/8*(-1+cos(x))*(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)*sin(x)+4*ln(-(2^(1/2)
*(-(-1+cos(x))/sin(x))^(1/2)*sin(x)+sin(x)-cos(x)+1)/(2^(1/2)*(-(-1+cos(x))
/sin(x))^(1/2)*sin(x)-sin(x)+cos(x)-1))+16*arctan(2^(1/2)*(-(-1+cos(x))/sin
(x))^(1/2)+1)+10*2^(1/2)*cos(x)*sin(x)*arctan((-(-1+cos(x))/sin(x))^(1/2))-
2^(1/2)*cos(x)*sin(x)*(-(-1+cos(x))/sin(x))^(3/2)+2^(1/2)*cos(x)*sin(x)*(-(-
1+cos(x))/sin(x))^(1/2)+4*ln(-(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)*sin(x)-
sin(x)+cos(x)-1)/(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)*sin(x)+sin(x)-cos(x)+
1))+16*arctan(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)-1)+2^(1/2)*cos(x)^2*(-(-1
+cos(x))/sin(x))^(3/2)-2*cos(x)*sin(x)*ln(-(2^(1/2)*(-(-1+cos(x))/sin(x))^(
1/2)*sin(x)-sin(x)+cos(x)-1)/(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)*sin(x)+si
n(x)-cos(x)+1))-2*cos(x)*sin(x)*ln(-(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)*si
n(x)+sin(x)-cos(x)+1)/(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)*sin(x)-sin(x)+co
s(x)-1))-8*cos(x)*sin(x)*arctan(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)+1)-8*co
s(x)*sin(x)*arctan(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)-1)+10*2^(1/2)*cos(x)
*arctan((-(-1+cos(x))/sin(x))^(1/2))-20*2^(1/2)*sin(x)*arctan((-(-1+cos(x))
/sin(x))^(1/2))+10*2^(1/2)*cos(x)^2*arctan((-(-1+cos(x))/sin(x))^(1/2))-2^(
1/2)*sin(x)*(-(-1+cos(x))/sin(x))^(3/2)-2^(1/2)*cos(x)^2*(-(-1+cos(x))/sin(
x))^(1/2)-20*2^(1/2)*arctan((-(-1+cos(x))/sin(x))^(1/2))-2*cos(x)^2*ln(-(2^(
1/2)*(-(-1+cos(x))/sin(x))^(1/2)*sin(x)-sin(x)+cos(x)-1)/(2^(1/2)*(-(-1+co
s(x))/sin(x))^(1/2)*sin(x)+sin(x)-cos(x)+1))-2*cos(x)^2*ln(-(2^(1/2)*(-(-1+
cos(x))/sin(x))^(1/2)*sin(x)+sin(x)-cos(x)+1)/(2^(1/2)*(-(-1+cos(x))/sin(x)
)^(1/2)*sin(x)-sin(x)+cos(x)-1))-8*cos(x)^2*arctan(2^(1/2)*(-(-1+cos(x))/si
n(x))^(1/2)+1)-8*cos(x)^2*arctan(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)-1)-2*c
os(x)*ln(-(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)*sin(x)-sin(x)+cos(x)-1)/(2^(
1/2)*(-(-1+cos(x))/sin(x))^(1/2)*sin(x)+sin(x)-cos(x)+1))-2*cos(x)*ln(-(2^(
1/2)*(-(-1+cos(x))/sin(x))^(1/2)*sin(x)+sin(x)-cos(x)+1)/(2^(1/2)*(-(-1+cos
(x))/sin(x))^(1/2)*sin(x)-sin(x)+cos(x)-1))-8*cos(x)*arctan(2^(1/2)*(-(-1+c
```

$\cos(x)/\sin(x))^{(1/2)+1}-8*\cos(x)*\arctan(2^{(1/2)*(-(-1+\cos(x))/\sin(x))^{(1/2)}-1)+4*\sin(x)*\ln(-2^{(1/2)*(-(-1+\cos(x))/\sin(x))^{(1/2)*\sin(x)-\sin(x)+\cos(x)-1)/(2^{(1/2)*(-(-1+\cos(x))/\sin(x))^{(1/2)*\sin(x)+\sin(x)-\cos(x)+1)}+4*\sin(x)*\ln(-2^{(1/2)*(-(-1+\cos(x))/\sin(x))^{(1/2)*\sin(x)+\sin(x)-\cos(x)+1)/(2^{(1/2)*(-(-1+\cos(x))/\sin(x))^{(1/2)*\sin(x)-\sin(x)+\cos(x)-1)}+16*\sin(x)*\arctan(2^{(1/2)*(-(-1+\cos(x))/\sin(x))^{(1/2)+1}+16*\sin(x)*\arctan(2^{(1/2)*(-(-1+\cos(x))/\sin(x))^{(1/2)-1}-2^{(1/2)*(-(-1+\cos(x))/\sin(x))^{(3/2)+2^{(1/2)*(-(-1+\cos(x))/\sin(x))^{(1/2)})/(a*(1+\sin(x))/\sin(x))^{(3/2)/\sin(x)^3/(-(-1+\cos(x))/\sin(x))^{(3/2)}*2^{(1/2)}$

maxima [B] time = 0.45, size = 150, normalized size = 1.85

$$\frac{\sqrt{2} \left(\frac{\sin(x)}{\cos(x)+1} \right)^{\frac{3}{2}} - \sqrt{2} \sqrt{\frac{\sin(x)}{\cos(x)+1}}}{2 \left(a^{\frac{3}{2}} + \frac{2a^{\frac{3}{2}} \sin(x)}{\cos(x)+1} + \frac{a^{\frac{3}{2}} \sin(x)^2}{(\cos(x)+1)^2} \right)} + \frac{\sqrt{2} \left(\sqrt{2} \arctan \left(\frac{1}{2} \sqrt{2} \left(\sqrt{2} + 2 \sqrt{\frac{\sin(x)}{\cos(x)+1}} \right) \right) + \sqrt{2} \arctan \left(-\frac{1}{2} \sqrt{2} \left(\sqrt{2} - 2 \sqrt{\frac{\sin(x)}{\cos(x)+1}} \right) \right) \right)}{a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*csc(x))^(3/2),x, algorithm="maxima")

[Out] $-1/2*(\sqrt{2}*(\sin(x)/(\cos(x) + 1))^{(3/2)} - \sqrt{2}*\sqrt{\sin(x)/(\cos(x) + 1)})) / (a^{(3/2)} + 2*a^{(3/2)*\sin(x)/(\cos(x) + 1)} + a^{(3/2)*\sin(x)^2/(\cos(x) + 1)^2}) + \sqrt{2}*(\sqrt{2}*\arctan(1/2*\sqrt{2}*(\sqrt{2} + 2*\sqrt{\sin(x)/(\cos(x) + 1)}))) + \sqrt{2}*\arctan(-1/2*\sqrt{2}*(\sqrt{2} - 2*\sqrt{\sin(x)/(\cos(x) + 1)}))) / a^{(3/2)} - 5/2*\sqrt{2}*\arctan(\sqrt{\sin(x)/(\cos(x) + 1)}) / a^{(3/2)}$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a + \frac{a}{\sin(x)}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + a/sin(x))^(3/2),x)

[Out] int(1/(a + a/sin(x))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \csc(x) + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*csc(x))**(3/2),x)
```

```
[Out] Integral((a*csc(x) + a)**(-3/2), x)
```

$$3.18 \quad \int \frac{1}{(a+a \csc(x))^{5/2}} dx$$

Optimal. Leaf size=100

$$-\frac{2 \tan^{-1}\left(\frac{\sqrt{a} \cot(x)}{\sqrt{a \csc(x)+a}}\right)}{a^{5/2}} + \frac{43 \tan^{-1}\left(\frac{\sqrt{a} \cot(x)}{\sqrt{2} \sqrt{a \csc(x)+a}}\right)}{16\sqrt{2} a^{5/2}} + \frac{11 \cot(x)}{16a(a \csc(x) + a)^{3/2}} + \frac{\cot(x)}{4(a \csc(x) + a)^{5/2}}$$

[Out] $-2*\arctan(\cot(x)*a^{(1/2)/(a+a*\csc(x))^{(1/2)})/a^{(5/2)}+1/4*\cot(x)/(a+a*\csc(x))^{(5/2)}+11/16*\cot(x)/a/(a+a*\csc(x))^{(3/2)}+43/32*\arctan(1/2*\cot(x)*a^{(1/2)*2^{(1/2)/(a+a*\csc(x))^{(1/2)})/a^{(5/2)}*2^{(1/2)}$

Rubi [A] time = 0.15, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3777, 3922, 3920, 3774, 203, 3795}

$$-\frac{2 \tan^{-1}\left(\frac{\sqrt{a} \cot(x)}{\sqrt{a \csc(x)+a}}\right)}{a^{5/2}} + \frac{43 \tan^{-1}\left(\frac{\sqrt{a} \cot(x)}{\sqrt{2} \sqrt{a \csc(x)+a}}\right)}{16\sqrt{2} a^{5/2}} + \frac{11 \cot(x)}{16a(a \csc(x) + a)^{3/2}} + \frac{\cot(x)}{4(a \csc(x) + a)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Csc[x])^(-5/2), x]

[Out] $(-2*\text{ArcTan}[(\text{Sqrt}[a]*\text{Cot}[x])/\text{Sqrt}[a + a*\text{Csc}[x]])/a^{(5/2)} + (43*\text{ArcTan}[(\text{Sqrt}[a]*\text{Cot}[x])/(\text{Sqrt}[2]*\text{Sqrt}[a + a*\text{Csc}[x]])])/(16*\text{Sqrt}[2]*a^{(5/2)}) + \text{Cot}[x]/(4*(a + a*\text{Csc}[x])^{(5/2)}) + (11*\text{Cot}[x])/(16*a*(a + a*\text{Csc}[x])^{(3/2)})$

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 3774

Int[Sqrt[csc[(c_) + (d_)*(x_)]*(b_) + (a_)], x_Symbol] := Dist[(-2*b)/d, Subst[Int[1/(a + x^2), x], x, (b*Cot[c + d*x])/Sqrt[a + b*Csc[c + d*x]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]

Rule 3777

Int[(csc[(c_) + (d_)*(x_)]*(b_) + (a_))^(n_), x_Symbol] := -Simp[(Cot[c + d*x]*(a + b*Csc[c + d*x])^n)/(d*(2*n + 1)), x] + Dist[1/(a^2*(2*n + 1)), Int[(a + b*Csc[c + d*x])^(n + 1)*(a*(2*n + 1) - b*(n + 1)*Csc[c + d*x]), x]

, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]

Rule 3795

Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]

Rule 3920

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]

Rule 3922

Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_.)), x_Symbol] :> -Simp[((b*c - a*d)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(b*f*(2*m + 1)), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*m]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + a \csc(x))^{5/2}} dx &= \frac{\cot(x)}{4(a + a \csc(x))^{5/2}} - \frac{\int \frac{-4a + \frac{3}{2}a \csc(x)}{(a + a \csc(x))^{3/2}} dx}{4a^2} \\
&= \frac{\cot(x)}{4(a + a \csc(x))^{5/2}} + \frac{11 \cot(x)}{16a(a + a \csc(x))^{3/2}} + \frac{\int \frac{8a^2 - \frac{11}{4}a^2 \csc(x)}{\sqrt{a + a \csc(x)}} dx}{8a^4} \\
&= \frac{\cot(x)}{4(a + a \csc(x))^{5/2}} + \frac{11 \cot(x)}{16a(a + a \csc(x))^{3/2}} + \frac{\int \sqrt{a + a \csc(x)} dx}{a^3} - \frac{43 \int \frac{\csc(x)}{\sqrt{a + a \csc(x)}} dx}{32a^2} \\
&= \frac{\cot(x)}{4(a + a \csc(x))^{5/2}} + \frac{11 \cot(x)}{16a(a + a \csc(x))^{3/2}} - \frac{2 \text{Subst}\left(\int \frac{1}{a+x^2} dx, x, \frac{a \cot(x)}{\sqrt{a + a \csc(x)}}\right)}{a^2} + \frac{43 \text{Subst}\left(\int \frac{1}{\sqrt{a+x^2}} dx, x, \frac{a \cot(x)}{\sqrt{a + a \csc(x)}}\right)}{16\sqrt{2}a^{5/2}} \\
&= -\frac{2 \tan^{-1}\left(\frac{\sqrt{a} \cot(x)}{\sqrt{a + a \csc(x)}}\right)}{a^{5/2}} + \frac{43 \tan^{-1}\left(\frac{\sqrt{a} \cot(x)}{\sqrt{2} \sqrt{a + a \csc(x)}}\right)}{16\sqrt{2}a^{5/2}} + \frac{\cot(x)}{4(a + a \csc(x))^{5/2}} + \frac{11 \cot(x)}{16a(a + a \csc(x))^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.51, size = 139, normalized size = 1.39

$$\frac{\csc^2(x) \left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right) \right) \left(8 \sin(x) + 15 \cos(2x) - 64 \sqrt{\csc(x) - 1} \left(\sin\left(\frac{x}{2}\right) + \cos\left(\frac{x}{2}\right) \right)^4 \tan^{-1}\left(\sqrt{\csc(x) - 1}\right) + 43 \sqrt{2} \sqrt{\csc(x) - 1} \left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right) \right) \right)}{32(a(\csc(x) + 1))^{5/2} \left(\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Csc[x])^(-5/2), x]

[Out] (Csc[x]^2*(Cos[x/2] + Sin[x/2])*(7 + 15*Cos[2*x] - 64*ArcTan[Sqrt[-1 + Csc[x]]]*Sqrt[-1 + Csc[x]]*(Cos[x/2] + Sin[x/2])^4 + 43*Sqrt[2]*ArcTan[Sqrt[-1 + Csc[x]]/Sqrt[2]]*Sqrt[-1 + Csc[x]]*(Cos[x/2] + Sin[x/2])^4 + 8*Sin[x]))/(32*(a*(1 + Csc[x]))^(5/2)*(Cos[x/2] - Sin[x/2]))

fricas [B] time = 0.48, size = 546, normalized size = 5.46

$$\left[\frac{43 \sqrt{2} \left(\cos(x)^3 + 3 \cos(x)^2 + (\cos(x)^2 - 2 \cos(x) - 4) \sin(x) - 2 \cos(x) - 4 \right) \sqrt{-a} \log \left(-\frac{\sqrt{2} \sqrt{-a} \sqrt{\frac{a \sin(x) + a}{\sin(x)}} \sin(x)}{\sin(x) + 1} \right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*csc(x))^(5/2),x, algorithm="fricas")

[Out] [-1/32*(43*sqrt(2)*(cos(x)^3 + 3*cos(x)^2 + (cos(x)^2 - 2*cos(x) - 4)*sin(x) - 2*cos(x) - 4)*sqrt(-a)*log(-(sqrt(2)*sqrt(-a)*sqrt((a*sin(x) + a)/sin(x)))*sin(x) - a*cos(x))/(sin(x) + 1)) + 32*(cos(x)^3 + 3*cos(x)^2 + (cos(x)^2 - 2*cos(x) - 4)*sin(x) - 2*cos(x) - 4)*sqrt(-a)*log((2*a*cos(x)^2 + 2*(cos(x)^2 + (cos(x) + 1)*sin(x) - 1)*sqrt(-a)*sqrt((a*sin(x) + a)/sin(x)) + a*cos(x) - (2*a*cos(x) + a)*sin(x) - a)/(cos(x) + sin(x) + 1)) - 2*(15*cos(x)^3 + 4*cos(x)^2 - (15*cos(x)^2 + 11*cos(x) - 4)*sin(x) - 15*cos(x) - 4)*sqrt((a*sin(x) + a)/sin(x)))/(a^3*cos(x)^3 + 3*a^3*cos(x)^2 - 2*a^3*cos(x) - 4*a^3 + (a^3*cos(x)^2 - 2*a^3*cos(x) - 4*a^3)*sin(x)), 1/16*(43*sqrt(2)*(cos(x)^3 + 3*cos(x)^2 + (cos(x)^2 - 2*cos(x) - 4)*sin(x) - 2*cos(x) - 4)*sqrt(a)*arctan(sqrt(2)*sqrt(a)*sqrt((a*sin(x) + a)/sin(x))*(cos(x) + 1)/(a*cos(x) + a*sin(x) + a)) + 32*(cos(x)^3 + 3*cos(x)^2 + (cos(x)^2 - 2*cos(x) - 4)*sin(x) - 2*cos(x) - 4)*sqrt(a)*arctan(-sqrt(a)*sqrt((a*sin(x) + a)/sin(x))*(cos(x) - sin(x) + 1)/(a*cos(x) + a*sin(x) + a)) + (15*cos(x)^3 + 4*cos(x)^2 - (15*cos(x)^2 + 11*cos(x) - 4)*sin(x) - 15*cos(x) - 4)*sqrt((a*sin(x) + a)/sin(x)))/(a^3*cos(x)^3 + 3*a^3*cos(x)^2 - 2*a^3*cos(x) - 4*a^3 + (a^3*cos(x)^2 - 2*a^3*cos(x) - 4*a^3)*sin(x))]

giac [B] time = 1.62, size = 348, normalized size = 3.48

$$\frac{1}{16} \left(\frac{43\sqrt{2} \arctan\left(\frac{\sqrt{a \tan\left(\frac{1}{2}x\right)}}{\sqrt{a}}\right)}{a^{\frac{5}{2}} \operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right) + 1\right)} - \frac{16\left(a\sqrt{|a|} + |a|^{\frac{3}{2}}\right) \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|a|} + 2\sqrt{a \tan\left(\frac{1}{2}x\right)}\right)}{2\sqrt{|a|}}\right)}{a^4 \operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right) + 1\right)} - \frac{16\left(a\sqrt{|a|} + |a|^{\frac{3}{2}}\right) \arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\sqrt{|a|} - 2\sqrt{a \tan\left(\frac{1}{2}x\right)}\right)}{2\sqrt{|a|}}\right)}{a^4 \operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right) + 1\right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*csc(x))^(5/2),x, algorithm="giac")

[Out] -1/16*(43*sqrt(2)*arctan(sqrt(a*tan(1/2*x))/sqrt(a))/(a^(5/2)*sgn(tan(1/2*x) + 1)) - 16*(a*sqrt(abs(a)) + abs(a)^(3/2))*arctan(1/2*sqrt(2)*(sqrt(2)*sqrt(abs(a)) + 2*sqrt(a*tan(1/2*x)))/sqrt(abs(a)))/(a^4*sgn(tan(1/2*x) + 1)) - 16*(a*sqrt(abs(a)) + abs(a)^(3/2))*arctan(-1/2*sqrt(2)*(sqrt(2)*sqrt(abs(a)) - 2*sqrt(a*tan(1/2*x)))/sqrt(abs(a)))/(a^4*sgn(tan(1/2*x) + 1)) - 8*(a*sqrt(abs(a)) - abs(a)^(3/2))*log(a*tan(1/2*x) + sqrt(2)*sqrt(a*tan(1/2*x))*sqrt(abs(a)) + abs(a))/(a^4*sgn(tan(1/2*x) + 1)) + 8*(a*sqrt(abs(a)) - abs(a)^(3/2))*log(a*tan(1/2*x) - sqrt(2)*sqrt(a*tan(1/2*x))*sqrt(abs(a)) + abs(a))/(a^4*sgn(tan(1/2*x) + 1)) + sqrt(2)*(11*sqrt(a*tan(1/2*x))*a^3*tan(1/2*x)^3 + 19*sqrt(a*tan(1/2*x))*a^3*tan(1/2*x)^2 - 19*sqrt(a*tan(1/2*x))*a^3*tan(1/2*x) - 11*sqrt(a*tan(1/2*x))*a^3)/((a*tan(1/2*x) + a)^4*a^2*sgn(tan(1/2*x) + 1))*sgn(tan(1/2*x))

maple [B] time = 0.61, size = 1961, normalized size = 19.61

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(a+a*\text{csc}(x))^{5/2}, x)$

[Out]
$$\begin{aligned} & -1/128*(-1+\cos(x))^2*(-11*2^{(1/2)}*(-(-1+\cos(x))/\sin(x))^{(1/2)}*\sin(x)-19*\sin(x)*\cos(x)^2*(-(-1+\cos(x))/\sin(x))^{(3/2)}*2^{(1/2)}-11*\sin(x)*\cos(x)^2*(-(-1+\cos(x))/\sin(x))^{(1/2)}*2^{(1/2)}+11*\sin(x)*\cos(x)^2*(-(-1+\cos(x))/\sin(x))^{(7/2)}*2^{(1/2)}+19*\sin(x)*\cos(x)^2*(-(-1+\cos(x))/\sin(x))^{(5/2)}*2^{(1/2)}-172*\sin(x)*\cos(x)^2*2^{(1/2)}*\arctan((-(-1+\cos(x))/\sin(x))^{(1/2)})-11*\cos(x)^3*(-(-1+\cos(x))/\sin(x))^{(7/2)}*2^{(1/2)}-11*\cos(x)^2*(-(-1+\cos(x))/\sin(x))^{(7/2)}*2^{(1/2)}+32*\sin(x)*\cos(x)^2*\ln(-(2^{(1/2)}*(-(-1+\cos(x))/\sin(x))^{(1/2)}*\sin(x)+\sin(x)-\cos(x)+1)/(2^{(1/2)}*(-(-1+\cos(x))/\sin(x))^{(1/2)}*\sin(x)-\sin(x)+\cos(x)-1))+128*\sin(x)*\cos(x)^2*\arctan(2^{(1/2)}*(-(-1+\cos(x))/\sin(x))^{(1/2)}-1)-128*\ln(-(2^{(1/2)}*(-(-1+\cos(x))/\sin(x))^{(1/2)}*\sin(x)+\sin(x)-\cos(x)+1)/(2^{(1/2)}*(-(-1+\cos(x))/\sin(x))^{(1/2)}*\sin(x)-\sin(x)+\cos(x)-1))-512*\arctan(2^{(1/2)}*(-(-1+\cos(x))/\sin(x))^{(1/2)}+1)-344*2^{(1/2)}*\cos(x)*\sin(x)*\arctan((-(-1+\cos(x))/\sin(x))^{(1/2)})-38*2^{(1/2)}*\cos(x)*\sin(x)*(-(-1+\cos(x))/\sin(x))^{(3/2)}-22*2^{(1/2)}*\cos(x)*\sin(x)*(-(-1+\cos(x))/\sin(x))^{(1/2)}+38*\sin(x)*\cos(x)*(-(-1+\cos(x))/\sin(x))^{(5/2)}*2^{(1/2)}+22*\sin(x)*\cos(x)*(-(-1+\cos(x))/\sin(x))^{(7/2)}*2^{(1/2)}-128*\ln(-(2^{(1/2)}*(-(-1+\cos(x))/\sin(x))^{(1/2)}*\sin(x)-\sin(x)+\cos(x)-1)/(2^{(1/2)}*(-(-1+\cos(x))/\sin(x))^{(1/2)}*\sin(x)+\sin(x)-\cos(x)+1))-512*\arctan(2^{(1/2)}*(-(-1+\cos(x))/\sin(x))^{(1/2)}-1)+32*\sin(x)*\cos(x)^2*\ln(-(2^{(1/2)}*(-(-1+\cos(x))/\sin(x))^{(1/2)}*\sin(x)-\sin(x)+\cos(x)-1)/(2^{(1/2)}*(-(-1+\cos(x))/\sin(x))^{(1/2)}*\sin(x)+\sin(x)-\cos(x)+1))+128*\sin(x)*\cos(x)^2*\arctan(2^{(1/2)}*(-(-1+\cos(x))/\sin(x))^{(1/2)}+1)+11*\sin(x)*(-(-1+\cos(x))/\sin(x))^{(7/2)}*2^{(1/2)}-19*\cos(x)^3*(-(-1+\cos(x))/\sin(x))^{(5/2)}*2^{(1/2)}+11*\cos(x)*(-(-1+\cos(x))/\sin(x))^{(7/2)}*2^{(1/2)}-19*\cos(x)^2*(-(-1+\cos(x))/\sin(x))^{(5/2)}*2^{(1/2)}+19*\sin(x)*(-(-1+\cos(x))/\sin(x))^{(5/2)}*2^{(1/2)}+19*\cos(x)^3*(-(-1+\cos(x))/\sin(x))^{(3/2)}*2^{(1/2)}+19*\cos(x)*(-(-1+\cos(x))/\sin(x))^{(5/2)}*2^{(1/2)}+11*\cos(x)^3*(-(-1+\cos(x))/\sin(x))^{(1/2)}*2^{(1/2)}-19*\cos(x)*(-(-1+\cos(x))/\sin(x))^{(3/2)}*2^{(1/2)}+172*\cos(x)^3*2^{(1/2)}*\arctan((-(-1+\cos(x))/\sin(x))^{(1/2)})-11*\cos(x)*(-(-1+\cos(x))/\sin(x))^{(1/2)}*2^{(1/2)}+19*2^{(1/2)}*\cos(x)^2*(-(-1+\cos(x))/\sin(x))^{(3/2)}+64*\cos(x)*\sin(x)*\ln(-(2^{(1/2)}*(-(-1+\cos(x))/\sin(x))^{(1/2)}*\sin(x)-\sin(x)+\cos(x)-1)/(2^{(1/2)}*(-(-1+\cos(x))/\sin(x))^{(1/2)}*\sin(x)+\sin(x)-\cos(x)+1))+64*\cos(x)*\sin(x)*\ln(-(2^{(1/2)}*(-(-1+\cos(x))/\sin(x))^{(1/2)}*\sin(x)+\sin(x)-\cos(x)+1)/(2^{(1/2)}*(-(-1+\cos(x))/\sin(x))^{(1/2)}*\sin(x)-\sin(x)+\cos(x)-1))+256*\cos(x)*\sin(x)*\arctan(2^{(1/2)}*(-(-1+\cos(x))/\sin(x))^{(1/2)}+1)+256*\cos(x)*\sin(x)*\arctan(2^{(1/2)}*(-(-1+\cos(x))/\sin(x))^{(1/2)}-1)-344*2^{(1/2)}*\cos(x)*\arctan((-(-1+\cos(x))/\sin(x))^{(1/2)})+688*2^{(1/2)}*\sin(x)*\arctan((-(-1+\cos(x))/\sin(x))^{(1/2)})-516*2^{(1/2)}*\cos(x)^2*\arctan((-(-1+\cos(x))/\sin(x))^{(1/2)})-19*2^{(1/2)}*\sin(x)*(-(-1+\cos(x))/\sin(x))^{(3/2)}+11*2^{(1/2)}*\cos(x)^2*(-(-1+\cos(x))/\sin(x))^{(1/2)}+688*2^{(1/2)}*\arctan(2^{(1/2)}*(-(-1+\cos(x))/\sin(x))^{(1/2)}) \end{aligned}$$

```

an((-(-1+cos(x))/sin(x))^(1/2))+96*cos(x)^2*ln(-(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)*sin(x)-sin(x)+cos(x)-1)/(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)*sin(x)+sin(x)-cos(x)+1))+96*cos(x)^2*ln(-(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)*sin(x)+sin(x)-cos(x)+1)/(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)*sin(x)-sin(x)+cos(x)-1))+384*cos(x)^2*arctan(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)+1)+384*cos(x)^2*arctan(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)-1)+64*cos(x)*ln(-(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)*sin(x)-sin(x)+cos(x)-1)/(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)*sin(x)+sin(x)-cos(x)+1))+64*cos(x)*ln(-(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)*sin(x)+sin(x)-cos(x)+1)/(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)*sin(x)-sin(x)+cos(x)-1))+256*cos(x)*arctan(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)+1)+256*cos(x)*arctan(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)-1)-128*sin(x)*ln(-(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)*sin(x)-sin(x)+cos(x)-1)/(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)*sin(x)+sin(x)-cos(x)+1))-128*sin(x)*ln(-(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)*sin(x)+sin(x)-cos(x)+1)/(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)*sin(x)-sin(x)+cos(x)-1))-512*sin(x)*arctan(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)+1)-512*sin(x)*arctan(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)-1)-19*2^(1/2)*(-(-1+cos(x))/sin(x))^(3/2)-32*cos(x)^3*ln(-(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)*sin(x)+sin(x)-cos(x)+1)/(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)*sin(x)-sin(x)+cos(x)-1))-128*cos(x)^3*arctan(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)-1)-32*cos(x)^3*ln(-(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)*sin(x)-sin(x)+cos(x)-1)/(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)*sin(x)+sin(x)-cos(x)+1))-128*cos(x)^3*arctan(2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2)+1)+19*(-(-1+cos(x))/sin(x))^(5/2)*2^(1/2)+11*(-(-1+cos(x))/sin(x))^(7/2)*2^(1/2)-11*2^(1/2)*(-(-1+cos(x))/sin(x))^(1/2))/(a*(1+sin(x))/sin(x))^(5/2)/sin(x)^5/(-(-1+cos(x))/sin(x))^(5/2)*2^(1/2)

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \csc(x) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*csc(x))^(5/2),x, algorithm="maxima")

[Out] integrate((a*csc(x) + a)^(-5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a + \frac{a}{\sin(x)}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + a/sin(x))^(5/2),x)

[Out] `int(1/(a + a/sin(x))^(5/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a \csc(x) + a)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+a*csc(x))**(5/2), x)`

[Out] `Integral((a*csc(x) + a)**(-5/2), x)`

3.19 $\int \sqrt{\csc(e + fx)} \sqrt{a + a \csc(e + fx)} dx$

Optimal. Leaf size=37

$$\frac{2\sqrt{a} \sinh^{-1}\left(\frac{\sqrt{a} \cot(e+fx)}{\sqrt{a \csc(e+fx)+a}}\right)}{f}$$

[Out] $-2*\operatorname{arcsinh}(\cot(f*x+e)*a^{(1/2)/(a+a*\csc(f*x+e))^{(1/2)}}*a^{(1/2)})/f$

Rubi [A] time = 0.06, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {3801, 215}

$$\frac{2\sqrt{a} \sinh^{-1}\left(\frac{\sqrt{a} \cot(e+fx)}{\sqrt{a \csc(e+fx)+a}}\right)}{f}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[Csc[e + f*x]]*Sqrt[a + a*Csc[e + f*x]],x]`

[Out] $(-2*\operatorname{Sqrt}[a]*\operatorname{ArcSinh}[(\operatorname{Sqrt}[a]*\operatorname{Cot}[e + f*x])/\operatorname{Sqrt}[a + a*\operatorname{Csc}[e + f*x]])]/f$

Rule 215

`Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]`

Rule 3801

`Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Dist[(-2*a*Sqrt[(a*d)/b])/(b*f), Subst[Int[1/Sqrt[1 + x^2/a], x], x, (b*Cot[e + f*x])/Sqrt[a + b*Csc[e + f*x]]], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[(a*d)/b, 0]`

Rubi steps

$$\int \sqrt{\csc(e+fx)} \sqrt{a+a \csc(e+fx)} dx = -\frac{2 \operatorname{Subst} \left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}} dx, x, \frac{a \cot(e+fx)}{\sqrt{a+a \csc(e+fx)}} \right)}{f}$$

$$= -\frac{2\sqrt{a} \sinh^{-1} \left(\frac{\sqrt{a} \cot(e+fx)}{\sqrt{a+a \csc(e+fx)}} \right)}{f}$$

Mathematica [B] time = 0.40, size = 108, normalized size = 2.92

$$\frac{2 \cot(e+fx) \sqrt{a(\csc(e+fx)+1)} \left(\log(\csc(e+fx)+1) - \log \left(\csc^{\frac{3}{2}}(e+fx) + \sqrt{\csc(e+fx)} + \sqrt{\cot^2(e+fx)} \right) \right)}{f \sqrt{\cot^2(e+fx)} \sqrt{\csc(e+fx)+1}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Csc[e + f*x]]*Sqrt[a + a*Csc[e + f*x]],x]

[Out] (2*Cot[e + f*x]*Sqrt[a*(1 + Csc[e + f*x])]*(Log[1 + Csc[e + f*x]] - Log[Sqrt[Csc[e + f*x]] + Csc[e + f*x]^(3/2) + Sqrt[Cot[e + f*x]^2]*Sqrt[1 + Csc[e + f*x]]]))/(f*Sqrt[Cot[e + f*x]^2]*Sqrt[1 + Csc[e + f*x]])

fricas [B] time = 0.80, size = 283, normalized size = 7.65

$$\sqrt{a} \log \left(\frac{a \cos^3(fx+e) - 7a \cos^2(fx+e) - 9a \cos(fx+e) + (a \cos^2(fx+e) + 8a \cos(fx+e) - a) \sin(fx+e) + \frac{4(\cos^3(fx+e) + 3 \cos^2(fx+e) - \cos(fx+e) - 2 \cos(fx+e) + 1) \sin(fx+e)}{\sqrt{a}}}{\cos^3(fx+e) + \cos^2(fx+e) + (\cos^2(fx+e) - 1) \sin(fx+e) - \cos(fx+e) - 1} \right)$$

$$2f$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)^(1/2)*(a+a*csc(f*x+e))^(1/2),x, algorithm="fricas")

[Out] [1/2*sqrt(a)*log((a*cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 - 9*a*cos(f*x + e) + (a*cos(f*x + e)^2 + 8*a*cos(f*x + e) - a)*sin(f*x + e) + 4*(cos(f*x + e)^3 - 7*cos(f*x + e)^2 - 9*cos(f*x + e) + 1)*sin(f*x + e)))]

heck sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)2*a^2*(a*atan((sqrt(a^3+a*(a*tan(1/2*(f*x+exp(1))))^2)-sqrt(a)*a*tan(1/2*(f*x+exp(1))))/sqrt(-a)/a)/sqrt(-a)/a-1/2*sqrt(a)*ln(abs(sqrt(a^3+a*(a*tan(1/2*(f*x+exp(1))))^2)-sqrt(a)*a*tan(1/2*(f*x+exp(1))))/a)*sign(sin(f*x+exp(1)))/a/f

maple [B] time = 2.35, size = 113, normalized size = 3.05

$$\frac{\sqrt{2} \sqrt{\frac{1}{\sin(fx+e)}} (-1 + \cos(fx + e)) \sqrt{\frac{a(1+\sin(fx+e))}{\sin(fx+e)}} \left(\operatorname{arcsinh}\left(\frac{-1+\cos(fx+e)}{\sin(fx+e)}\right) + \operatorname{arctanh}\left(\frac{\sqrt{2}}{2\sqrt{\frac{1}{1+\cos(fx+e)}}}\right) \right)}{f(1 - \cos(fx + e) + \sin(fx + e)) \sqrt{\frac{1}{1+\cos(fx+e)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(f*x+e)^(1/2)*(a+a*csc(f*x+e))^(1/2), x)

[Out] 1/f*2^(1/2)*(1/sin(f*x+e))^(1/2)*(-1+cos(f*x+e))*(a*(1+sin(f*x+e))/sin(f*x+e))^(1/2)*(arcsinh((-1+cos(f*x+e))/sin(f*x+e))+arctanh(1/2*2^(1/2)/(1/(1+cos(f*x+e))))^(1/2))/(1-cos(f*x+e)+sin(f*x+e))/(1/(1+cos(f*x+e)))^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \csc(fx + e) + a} \sqrt{\csc(fx + e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)^(1/2)*(a+a*csc(f*x+e))^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(a*csc(f*x + e) + a)*sqrt(csc(f*x + e)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{a + \frac{a}{\sin(e + fx)}} \sqrt{\frac{1}{\sin(e + fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/sin(e + f*x))^(1/2)*(1/sin(e + f*x))^(1/2), x)

[Out] int((a + a/sin(e + f*x))^(1/2)*(1/sin(e + f*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\csc(e + fx) + 1)} \sqrt{\csc(e + fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)**(1/2)*(a+a*csc(f*x+e))**(1/2), x)

[Out] Integral(sqrt(a*(csc(e + f*x) + 1))*sqrt(csc(e + f*x)), x)

3.20 $\int \sqrt{-\csc(e + fx)} \sqrt{a - a \csc(e + fx)} dx$

Optimal. Leaf size=38

$$-\frac{2\sqrt{a} \sinh^{-1}\left(\frac{\sqrt{a} \cot(e+fx)}{\sqrt{a-a \csc(e+fx)}}\right)}{f}$$

[Out] $-2*\operatorname{arcsinh}(\cot(f*x+e)*a^{(1/2)/(a-a*\csc(f*x+e))^{(1/2)}}*a^{(1/2)})/f$

Rubi [A] time = 0.07, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3801, 215}

$$-\frac{2\sqrt{a} \sinh^{-1}\left(\frac{\sqrt{a} \cot(e+fx)}{\sqrt{a-a \csc(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[-\text{Csc}[e + f*x]]*\text{Sqrt}[a - a*\text{Csc}[e + f*x]],x]$

[Out] $(-2*\text{Sqrt}[a]*\text{ArcSinh}[(\text{Sqrt}[a]*\text{Cot}[e + f*x])/\text{Sqrt}[a - a*\text{Csc}[e + f*x]]])/f$

Rule 215

$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^2], x_Symbol] \rightarrow \text{Simp}[\text{ArcSinh}[(\text{Rt}[b, 2]*x)/\text{Sqrt}[a]]/\text{Rt}[b, 2], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{GtQ}[a, 0] \ \&\& \ \text{PosQ}[b]$

Rule 3801

$\text{Int}[\text{Sqrt}[\csc[(e_) + (f_)*(x_)]*(d_)]*\text{Sqrt}[\csc[(e_) + (f_)*(x_)]*(b_) + (a_)], x_Symbol] \rightarrow \text{Dist}[(-2*a*\text{Sqrt}[(a*d)/b])/(b*f), \text{Subst}[\text{Int}[1/\text{Sqrt}[1 + x^2/a], x], x, (b*\text{Cot}[e + f*x])/\text{Sqrt}[a + b*\text{Csc}[e + f*x]]], x] /; \text{FreeQ}\{a, b, d, e, f\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{GtQ}[(a*d)/b, 0]$

Rubi steps

$$\int \sqrt{-\csc(e+fx)} \sqrt{a-a\csc(e+fx)} dx = \frac{2 \operatorname{Subst} \left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}} dx, x, -\frac{a \cot(e+fx)}{\sqrt{a-a\csc(e+fx)}} \right)}{f}$$

$$= -\frac{2\sqrt{a} \sinh^{-1} \left(\frac{\sqrt{a} \cot(e+fx)}{\sqrt{a-a\csc(e+fx)}} \right)}{f}$$

Mathematica [B] time = 0.79, size = 101, normalized size = 2.66

$$\frac{2 \tan \left(\frac{1}{2}(e+fx) \right) \sqrt{-\csc(e+fx)} \sqrt{a-a\csc(e+fx)} \left(\tanh^{-1} \left(\sqrt{\sec^2 \left(\frac{1}{2}(e+fx) \right)} \right) \right) + \sinh^{-1} \left(\tan \left(\frac{1}{2}(e+fx) \right) \right)}{f \left(\tan \left(\frac{1}{2}(e+fx) \right) - 1 \right) \sqrt{\sec^2 \left(\frac{1}{2}(e+fx) \right)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-Csc[e + f*x]]*Sqrt[a - a*Csc[e + f*x]],x]

[Out] (2*(ArcSinh[Tan[(e + f*x)/2]] + ArcTanh[Sqrt[Sec[(e + f*x)/2]^2]])*Sqrt[-Csc[e + f*x]]*Sqrt[a - a*Csc[e + f*x]]*Tan[(e + f*x)/2])/(f*Sqrt[Sec[(e + f*x)/2]^2]*(-1 + Tan[(e + f*x)/2]))

fricas [B] time = 1.08, size = 296, normalized size = 7.79

$$\left[\sqrt{a} \log \left(\frac{a \cos^3(fx+e) - 7a \cos^2(fx+e) - 4(\cos^3(fx+e) + 3 \cos^2(fx+e) + (\cos^2(fx+e) - 2 \cos(fx+e) - 3) \sin(fx+e) - \cos(fx+e) - 3) \sqrt{a} \sqrt{\frac{a \sin(fx+e) - a}{\sin(fx+e)}}}{\cos^3(fx+e) + \cos^2(fx+e) - (\cos^2(fx+e) - 1) \sin(fx+e) - \cos(fx+e)} \right) \right]$$

$$2f$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-csc(f*x+e))^(1/2)*(a-a*csc(f*x+e))^(1/2),x, algorithm="fricas")

[Out] [1/2*sqrt(a)*log((a*cos(f*x + e)^3 - 7*a*cos(f*x + e)^2 - 4*(cos(f*x + e)^3 + 3*cos(f*x + e)^2 + (cos(f*x + e)^2 - 2*cos(f*x + e) - 3)*sin(f*x + e) - cos(f*x + e) - 3)*sqrt(a)*sqrt((a*sin(f*x + e) - a)/sin(f*x + e))*sqrt(-1/s

$$\frac{(f*x + e) - 9*a*\cos(f*x + e) - (a*\cos(f*x + e)^2 + 8*a*\cos(f*x + e) - a)*\sin(f*x + e) - a}{(\cos(f*x + e)^3 + \cos(f*x + e)^2 - (\cos(f*x + e)^2 - 1)*\sin(f*x + e) - \cos(f*x + e) - 1)}/f, \frac{\sqrt{-a}*\arctan(-1/2*(\cos(f*x + e)^2 - 2*\sin(f*x + e) - 1)*\sqrt{-a}*\sqrt{(a*\sin(f*x + e) - a)/\sin(f*x + e)})*\sqrt{-1/\sin(f*x + e)}}{a*\cos(f*x + e)}/f]$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(f*x+e))^(1/2)*(a-a*csc(f*x+e))^(1/2),x, algorithm="giac")
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unable
to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
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maple [B] time = 2.11, size = 117, normalized size = 3.08

$$\frac{2\sqrt{-\frac{1}{\sin(fx+e)}}(-1+\cos(fx+e))\sqrt{\frac{a(\sin(fx+e)-1)}{\sin(fx+e)}}\left(\arctan\left(\frac{1}{\sqrt{-\frac{2}{1+\cos(fx+e)}}}\right)-\arctan\left(\frac{\sin(fx+e)\sqrt{-\frac{2}{1+\cos(fx+e)}}}{2}\right)\right)}{f(-1+\cos(fx+e)+\sin(fx+e))\sqrt{-\frac{2}{1+\cos(fx+e)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-csc(f*x+e))^(1/2)*(a-a*csc(f*x+e))^(1/2),x)

[Out] -2/f*(-1/sin(f*x+e))^(1/2)*(-1+cos(f*x+e))*(a*(sin(f*x+e)-1)/sin(f*x+e))^(1/2)*(arctan(1/(-2/(1+cos(f*x+e))))^(1/2))-arctan(1/2*sin(f*x+e)*(-2/(1+cos(f*x+e))))^(1/2))/(-1+cos(f*x+e)+sin(f*x+e))/(-2/(1+cos(f*x+e)))^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a \csc(fx+e)} + a \sqrt{-\csc(fx+e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-csc(f*x+e))^(1/2)*(a-a*csc(f*x+e))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-a*csc(f*x + e) + a)*sqrt(-csc(f*x + e)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{a - \frac{a}{\sin(e+fx)}} \sqrt{-\frac{1}{\sin(e+fx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a - a/sin(e + f*x))^(1/2)*(-1/sin(e + f*x))^(1/2),x)

[Out] int((a - a/sin(e + f*x))^(1/2)*(-1/sin(e + f*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\csc(e+fx)} \sqrt{-a(\csc(e+fx)-1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-csc(f*x+e))**(1/2)*(a-a*csc(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(-csc(e + f*x))*sqrt(-a*(csc(e + f*x) - 1)), x)
```

3.21 $\int \csc^{\frac{4}{3}}(c + dx) \sqrt{a + a \csc(c + dx)} dx$

Optimal. Leaf size=254

$$\frac{4 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} a^2 \cot(c + dx) \left(1 - \sqrt[3]{\csc(c + dx)}\right) \sqrt{\frac{\csc^{\frac{2}{3}}(c + dx) + \sqrt[3]{\csc(c + dx)} + 1}{(-\sqrt[3]{\csc(c + dx)} + \sqrt{3} + 1)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\csc(c + dx)} - \sqrt{3} + 1}{-\sqrt[3]{\csc(c + dx)} + \sqrt{3} + 1}\right)\right) - 7 - 4}{5d \sqrt{\frac{1 - \sqrt[3]{\csc(c + dx)}}{(-\sqrt[3]{\csc(c + dx)} + \sqrt{3} + 1)^2}} (a - a \csc(c + dx)) \sqrt{a \csc(c + dx) + a}}$$

[Out] $-6/5*a*\cos(d*x+c)*\csc(d*x+c)^{(4/3)}/d/(a+a*\csc(d*x+c))^{(1/2)}-4/5*3^{(3/4)}*a^2*\cot(d*x+c)*(1-\csc(d*x+c)^{(1/3)})*EllipticF((1-\csc(d*x+c)^{(1/3)}-3^{(1/2)})/(1-\csc(d*x+c)^{(1/3)}+3^{(1/2)}),I*3^{(1/2)}+2*I)*(1/2*6^{(1/2)}+1/2*2^{(1/2)})*((1+\csc(d*x+c)^{(1/3)}+\csc(d*x+c)^{(2/3)})/(1-\csc(d*x+c)^{(1/3)}+3^{(1/2)})^2)^{(1/2)}/d/(a-a*\csc(d*x+c))/(a+a*\csc(d*x+c))^{(1/2)}/((1-\csc(d*x+c)^{(1/3)})/(1-\csc(d*x+c)^{(1/3)}+3^{(1/2)})^2)^{(1/2)}$

Rubi [A] time = 0.28, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3806, 50, 63, 218}

$$\frac{4 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} a^2 \cot(c + dx) \left(1 - \sqrt[3]{\csc(c + dx)}\right) \sqrt{\frac{\csc^{\frac{2}{3}}(c + dx) + \sqrt[3]{\csc(c + dx)} + 1}{(-\sqrt[3]{\csc(c + dx)} + \sqrt{3} + 1)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\csc(c + dx)} - \sqrt{3} + 1}{-\sqrt[3]{\csc(c + dx)} + \sqrt{3} + 1}\right)\right) - 7 - 4}{5d \sqrt{\frac{1 - \sqrt[3]{\csc(c + dx)}}{(-\sqrt[3]{\csc(c + dx)} + \sqrt{3} + 1)^2}} (a - a \csc(c + dx)) \sqrt{a \csc(c + dx) + a}}$$

Antiderivative was successfully verified.

[In] `Int[Csc[c + d*x]^(4/3)*Sqrt[a + a*Csc[c + d*x]],x]`

[Out] $(-6*a*\cos[c + d*x]*\csc[c + d*x]^{(4/3)})/(5*d*\sqrt{a + a*\csc[c + d*x]}) - (4*3^{(3/4)}*\sqrt{2 + \sqrt{3}}*a^2*\cot[c + d*x]*(1 - \csc[c + d*x]^{(1/3)})*\sqrt{(1 + \csc[c + d*x]^{(1/3)} + \csc[c + d*x]^{(2/3)})/(1 + \sqrt{3} - \csc[c + d*x]^{(1/3)})^2}*EllipticF[ArcSin[(1 - \sqrt{3} - \csc[c + d*x]^{(1/3)})/(1 + \sqrt{3} - \csc[c + d*x]^{(1/3)})], -7 - 4*\sqrt{3}]))/(5*d*\sqrt{(1 - \csc[c + d*x]^{(1/3)})/(1 + \sqrt{3} - \csc[c + d*x]^{(1/3)})^2}*(a - a*\csc[c + d*x])*Sqrt[a + a*\csc[c + d*x]])$

Rule 50

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,`

$c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{GtQ}[n, 0] \&\& \text{NeQ}[m + n + 1, 0] \&\& \text{!(IGtQ}[m, 0] \&\& (\text{!IntegerQ}[n] \mid\mid (\text{GtQ}[m, 0] \&\& \text{LtQ}[m - n, 0]))) \&\& \text{!ILtQ}[m + n + 2, 0] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 63

$\text{Int}[(a_.) + (b_.)*(x_)^m]*((c_.) + (d_.)*(x_)^n), x_Symbol] \text{:> With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{p*(m+1)-1}*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{1/p}], x]] \text{/; FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 218

$\text{Int}[1/\text{Sqrt}[(a_) + (b_.)*(x_)^3], x_Symbol] \text{:> With}\{r = \text{Numer}[\text{Rt}[b/a, 3]], s = \text{Denom}[\text{Rt}[b/a, 3]]\}, \text{Simp}[(2*\text{Sqrt}[2 + \text{Sqrt}[3]]*(s + r*x)*\text{Sqrt}[(s^2 - r*s*x + r^2*x^2)/((1 + \text{Sqrt}[3])*s + r*x)^2]*\text{EllipticF}[\text{ArcSin}[(1 - \text{Sqrt}[3])*s + r*x]/((1 + \text{Sqrt}[3])*s + r*x)], -7 - 4*\text{Sqrt}[3]])/(3^{1/4}*r*\text{Sqrt}[a + b*x^3]*\text{Sqrt}[(s*(s + r*x))/((1 + \text{Sqrt}[3])*s + r*x)^2]), x]] \text{/; FreeQ}\{a, b\}, x] \& \& \text{PosQ}[a]$

Rule 3806

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_)])*(d_.))^n]*\text{Sqrt}[\text{csc}[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] \text{:> Dist}[(a^2*d*\text{Cot}[e + f*x])/(f*\text{Sqrt}[a + b*\text{Csc}[e + f*x]]*\text{Sqrt}[a - b*\text{Csc}[e + f*x]]), \text{Subst}[\text{Int}[(d*x)^{n-1}/\text{Sqrt}[a - b*x], x], x, \text{Csc}[e + f*x]], x] \text{/; FreeQ}\{a, b, d, e, f, n\}, x] \&\& \text{EqQ}[a^2 - b^2, 0]$

Rubi steps

$$\begin{aligned}
\int \csc^{\frac{4}{3}}(c+dx) \sqrt{a+a \csc(c+dx)} dx &= \frac{(a^2 \cot(c+dx)) \operatorname{Subst}\left(\int \frac{\sqrt[3]{x}}{\sqrt{a-ax}} dx, x, \csc(c+dx)\right)}{d\sqrt{a-a \csc(c+dx)} \sqrt{a+a \csc(c+dx)}} \\
&= -\frac{6a \cos(c+dx) \csc^{\frac{4}{3}}(c+dx)}{5d\sqrt{a+a \csc(c+dx)}} + \frac{(2a^2 \cot(c+dx)) \operatorname{Subst}\left(\int \frac{1}{x^{2/3} \sqrt{a-ax}} dx, x, \csc(c+dx)\right)}{5d\sqrt{a-a \csc(c+dx)} \sqrt{a+a \csc(c+dx)}} \\
&= -\frac{6a \cos(c+dx) \csc^{\frac{4}{3}}(c+dx)}{5d\sqrt{a+a \csc(c+dx)}} + \frac{(6a^2 \cot(c+dx)) \operatorname{Subst}\left(\int \frac{1}{\sqrt{a-ax^3}} dx, x, \csc(c+dx)\right)}{5d\sqrt{a-a \csc(c+dx)} \sqrt{a+a \csc(c+dx)}} \\
&= -\frac{6a \cos(c+dx) \csc^{\frac{4}{3}}(c+dx)}{5d\sqrt{a+a \csc(c+dx)}} - \frac{4 \cdot 3^{3/4} \sqrt{2+\sqrt{3}} a^2 \cot(c+dx) (1-\sqrt[3]{\csc(c+dx)})}{5d \sqrt{\frac{1-\sqrt[3]{\csc(c+dx)}}{(1+\sqrt{3}-\sqrt[3]{\csc(c+dx)})}}}
\end{aligned}$$

Mathematica [C] time = 0.38, size = 102, normalized size = 0.40

$$\frac{2\sqrt{a(\csc(c+dx)+1)} \left({}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{3}{2}; 1-\csc(c+dx)\right) + 3\sqrt[3]{\csc(c+dx)} \right) \left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right) \right)}{5d \left(\sin\left(\frac{1}{2}(c+dx)\right) + \cos\left(\frac{1}{2}(c+dx)\right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]^(4/3)*Sqrt[a + a*Csc[c + d*x]], x]

[Out] (-2*Sqrt[a*(1 + Csc[c + d*x])]*(3*Csc[c + d*x]^(1/3) + 2*Hypergeometric2F1[1/2, 2/3, 3/2, 1 - Csc[c + d*x]])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))/(5*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\sqrt{a \csc(dx+c)+a} \csc(dx+c)^{\frac{4}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^(4/3)*(a+a*csc(d*x+c))^(1/2), x, algorithm="fricas")

[Out] integral(sqrt(a*csc(d*x + c) + a)*csc(d*x + c)^(4/3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \csc(dx+c)+a} \csc(dx+c)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^(4/3)*(a+a*csc(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(a*csc(d*x + c) + a)*csc(d*x + c)^(4/3), x)

maple [F] time = 2.64, size = 0, normalized size = 0.00

$$\int \left(\csc^{\frac{4}{3}}(dx + c) \right) \sqrt{a + a \csc(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^(4/3)*(a+a*csc(d*x+c))^(1/2),x)

[Out] int(csc(d*x+c)^(4/3)*(a+a*csc(d*x+c))^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \csc(dx + c) + a} \csc(dx + c)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^(4/3)*(a+a*csc(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(a*csc(d*x + c) + a)*csc(d*x + c)^(4/3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a + \frac{a}{\sin(c + dx)}} \left(\frac{1}{\sin(c + dx)} \right)^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/sin(c + d*x))^(1/2)*(1/sin(c + d*x))^(4/3),x)

[Out] int((a + a/sin(c + d*x))^(1/2)*(1/sin(c + d*x))^(4/3), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**(4/3)*(a+a*csc(d*x+c))**(1/2),x)

[Out] Timed out

3.22 $\int \sqrt[3]{\csc(c+dx)} \sqrt{a+a \csc(c+dx)} dx$

Optimal. Leaf size=213

$$\frac{2 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} a^2 \cot(c+dx) \left(1 - \sqrt[3]{\csc(c+dx)}\right) \sqrt{\frac{\csc^2(c+dx) + \sqrt[3]{\csc(c+dx)} + 1}{(-\sqrt[3]{\csc(c+dx)} + \sqrt{3} + 1)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\csc(c+dx)} - \sqrt{3} + 1}{-\sqrt[3]{\csc(c+dx)} + \sqrt{3} + 1}\right)\right) - 7 - 4}{d \sqrt{\frac{1 - \sqrt[3]{\csc(c+dx)}}{(-\sqrt[3]{\csc(c+dx)} + \sqrt{3} + 1)^2}} (a - a \csc(c+dx)) \sqrt{a \csc(c+dx) + a}}$$

[Out] $-2 \cdot 3^{3/4} \cdot a^2 \cdot \cot(d \cdot x + c) \cdot (1 - \csc(d \cdot x + c)^{1/3}) \cdot \text{EllipticF}((1 - \csc(d \cdot x + c)^{1/3}) - 3^{1/2}) / (1 - \csc(d \cdot x + c)^{1/3} + 3^{1/2}), I \cdot 3^{1/2} + 2 \cdot I) \cdot (1/2 \cdot 6^{1/2} + 1/2 \cdot 2^{1/2}) \cdot ((1 + \csc(d \cdot x + c)^{1/3} + \csc(d \cdot x + c)^{2/3}) / (1 - \csc(d \cdot x + c)^{1/3} + 3^{1/2}))^2)^{1/2} / d / (a - a \cdot \csc(d \cdot x + c)) / (a + a \cdot \csc(d \cdot x + c))^{1/2} / ((1 - \csc(d \cdot x + c)^{1/3}) / (1 - \csc(d \cdot x + c)^{1/3} + 3^{1/2}))^2)^{1/2}$

Rubi [A] time = 0.13, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {3806, 63, 218}

$$\frac{2 \cdot 3^{3/4} \sqrt{2 + \sqrt{3}} a^2 \cot(c+dx) \left(1 - \sqrt[3]{\csc(c+dx)}\right) \sqrt{\frac{\csc^2(c+dx) + \sqrt[3]{\csc(c+dx)} + 1}{(-\sqrt[3]{\csc(c+dx)} + \sqrt{3} + 1)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\csc(c+dx)} - \sqrt{3} + 1}{-\sqrt[3]{\csc(c+dx)} + \sqrt{3} + 1}\right)\right) - 7 - 4}{d \sqrt{\frac{1 - \sqrt[3]{\csc(c+dx)}}{(-\sqrt[3]{\csc(c+dx)} + \sqrt{3} + 1)^2}} (a - a \csc(c+dx)) \sqrt{a \csc(c+dx) + a}}$$

Antiderivative was successfully verified.

[In] `Int[Csc[c + d*x]^(1/3)*Sqrt[a + a*Csc[c + d*x]],x]`

[Out] $(-2 \cdot 3^{3/4} \cdot \text{Sqrt}[2 + \text{Sqrt}[3]] \cdot a^2 \cdot \text{Cot}[c + d \cdot x] \cdot (1 - \text{Csc}[c + d \cdot x]^{1/3}) \cdot \text{Sqrt}[(1 + \text{Csc}[c + d \cdot x]^{1/3} + \text{Csc}[c + d \cdot x]^{2/3}) / (1 + \text{Sqrt}[3] - \text{Csc}[c + d \cdot x]^{1/3})]^2 \cdot \text{EllipticF}[\text{ArcSin}[(1 - \text{Sqrt}[3] - \text{Csc}[c + d \cdot x]^{1/3}) / (1 + \text{Sqrt}[3] - \text{Csc}[c + d \cdot x]^{1/3})], -7 - 4 \cdot \text{Sqrt}[3]]) / (d \cdot \text{Sqrt}[(1 - \text{Csc}[c + d \cdot x]^{1/3}) / (1 + \text{Sqrt}[3] - \text{Csc}[c + d \cdot x]^{1/3})]^2 \cdot (a - a \cdot \text{Csc}[c + d \cdot x]) \cdot \text{Sqrt}[a + a \cdot \text{Csc}[c + d \cdot x]])$

Rule 63

`Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]`

Rule 218

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]]]/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.)
+ (a_)], x_Symbol] := Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt[3]{\csc(c+dx)} \sqrt{a+a \csc(c+dx)} dx = \frac{(a^2 \cot(c+dx)) \operatorname{Subst}\left(\int \frac{1}{x^{2/3} \sqrt{a-ax}} dx, x, \csc(c+dx)\right)}{d \sqrt{a-a \csc(c+dx)} \sqrt{a+a \csc(c+dx)}}$$

$$= \frac{(3a^2 \cot(c+dx)) \operatorname{Subst}\left(\int \frac{1}{\sqrt{a-ax^3}} dx, x, \sqrt[3]{\csc(c+dx)}\right)}{d \sqrt{a-a \csc(c+dx)} \sqrt{a+a \csc(c+dx)}}$$

$$= \frac{2 \cdot 3^{3/4} \sqrt{2+\sqrt{3}} a^2 \cot(c+dx) \left(1 - \sqrt[3]{\csc(c+dx)}\right) \sqrt{\frac{1+\sqrt[3]{\csc(c+dx)}+\csc^2(c+dx)}{(1+\sqrt{3}-\sqrt[3]{\csc(c+dx)})^2}}}{d \sqrt{\frac{1-\sqrt[3]{\csc(c+dx)}}{(1+\sqrt{3}-\sqrt[3]{\csc(c+dx)})^2}} (a-a \csc(c+dx))}$$

Mathematica [C] time = 0.23, size = 46, normalized size = 0.22

$$\frac{2a \cot(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{3}{2}; 1 - \csc(c+dx)\right)}{d \sqrt{a(\csc(c+dx)+1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^(1/3)*Sqrt[a + a*Csc[c + d*x]], x]
```

```
[Out] (-2*a*Cot[c + d*x]*Hypergeometric2F1[1/2, 2/3, 3/2, 1 - Csc[c + d*x]])/(d*S
qrt[a*(1 + Csc[c + d*x]))]
```

fricas [F] time = 0.91, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{a \csc(dx+c) + a} \csc(dx+c)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(d*x+c)^(1/3)*(a+a*csc(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] `integral(sqrt(a*csc(d*x + c) + a)*csc(d*x + c)^(1/3), x)`

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \csc(dx+c) + a} \csc(dx+c)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(d*x+c)^(1/3)*(a+a*csc(d*x+c))^(1/2),x, algorithm="giac")`

[Out] `integrate(sqrt(a*csc(d*x + c) + a)*csc(d*x + c)^(1/3), x)`

maple [F] time = 4.17, size = 0, normalized size = 0.00

$$\int \left(\csc^{\frac{1}{3}}(dx+c)\right) \sqrt{a + a \csc(dx+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csc(d*x+c)^(1/3)*(a+a*csc(d*x+c))^(1/2),x)`

[Out] `int(csc(d*x+c)^(1/3)*(a+a*csc(d*x+c))^(1/2),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \csc(dx+c) + a} \csc(dx+c)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(d*x+c)^(1/3)*(a+a*csc(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(a*csc(d*x + c) + a)*csc(d*x + c)^(1/3), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a + \frac{a}{\sin(c+dx)}} \left(\frac{1}{\sin(c+dx)}\right)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + a/sin(c + d*x))^(1/2)*(1/sin(c + d*x))^(1/3), x)`

[Out] `int((a + a/sin(c + d*x))^(1/2)*(1/sin(c + d*x))^(1/3), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\csc(c + dx) + 1)} \sqrt[3]{\csc(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(d*x+c)**(1/3)*(a+a*csc(d*x+c))**(1/2), x)`

[Out] `Integral(sqrt(a*(csc(c + d*x) + 1))*csc(c + d*x)**(1/3), x)`

$$3.23 \quad \int \frac{\sqrt{a+a \csc(c+dx)}}{\csc^{\frac{2}{3}}(c+dx)} dx$$

Optimal. Leaf size=254

$$\frac{3^{3/4} \sqrt{2 + \sqrt{3}} a^2 \cot(c + dx) \left(1 - \sqrt[3]{\csc(c + dx)}\right) \sqrt{\frac{\csc^{\frac{2}{3}}(c+dx) + \sqrt[3]{\csc(c+dx)} + 1}{(-\sqrt[3]{\csc(c+dx)} + \sqrt{3} + 1)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\csc(c+dx)} - \sqrt{3} + 1}{-\sqrt[3]{\csc(c+dx)} + \sqrt{3} + 1}\right)\right) - 7 - 4\sqrt{3}}{2d \sqrt{\frac{1 - \sqrt[3]{\csc(c+dx)}}{(-\sqrt[3]{\csc(c+dx)} + \sqrt{3} + 1)^2}} (a - a \csc(c + dx)) \sqrt{a \csc(c + dx) + a}}$$

[Out] $-3/2*a*\cos(d*x+c)*\csc(d*x+c)^{(1/3)}/d/(a+a*\csc(d*x+c))^{(1/2)}-1/2*3^{(3/4)}*a^2*\cot(d*x+c)*(1-\csc(d*x+c)^{(1/3)})*EllipticF((1-\csc(d*x+c)^{(1/3)}-3^{(1/2)})/(1-\csc(d*x+c)^{(1/3)}+3^{(1/2)}), I*3^{(1/2)}+2*I)*(1/2*6^{(1/2)}+1/2*2^{(1/2)})*((1+\csc(d*x+c)^{(1/3)}+\csc(d*x+c)^{(2/3)})/(1-\csc(d*x+c)^{(1/3)}+3^{(1/2)})^2)^{(1/2)}/d/(a-a*\csc(d*x+c))/(a+a*\csc(d*x+c))^{(1/2)}/((1-\csc(d*x+c)^{(1/3)})/(1-\csc(d*x+c)^{(1/3)}+3^{(1/2)})^2)^{(1/2)}$

Rubi [A] time = 0.14, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3806, 51, 63, 218}

$$\frac{3^{3/4} \sqrt{2 + \sqrt{3}} a^2 \cot(c + dx) \left(1 - \sqrt[3]{\csc(c + dx)}\right) \sqrt{\frac{\csc^{\frac{2}{3}}(c+dx) + \sqrt[3]{\csc(c+dx)} + 1}{(-\sqrt[3]{\csc(c+dx)} + \sqrt{3} + 1)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\csc(c+dx)} - \sqrt{3} + 1}{-\sqrt[3]{\csc(c+dx)} + \sqrt{3} + 1}\right)\right) - 7 - 4\sqrt{3}}{2d \sqrt{\frac{1 - \sqrt[3]{\csc(c+dx)}}{(-\sqrt[3]{\csc(c+dx)} + \sqrt{3} + 1)^2}} (a - a \csc(c + dx)) \sqrt{a \csc(c + dx) + a}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Csc[c + d*x]]/Csc[c + d*x]^(2/3), x]

[Out] $(-3*a*\cos[c + d*x]*\csc[c + d*x]^{(1/3)})/(2*d*\sqrt{a + a*\csc[c + d*x]}) - (3^{(3/4)}*\sqrt{2 + \sqrt{3}}*a^2*\cot[c + d*x]*(1 - \csc[c + d*x]^{(1/3)})*\sqrt{(1 + \csc[c + d*x]^{(1/3)} + \csc[c + d*x]^{(2/3)})/(1 + \sqrt{3} - \csc[c + d*x]^{(1/3)})^2}*EllipticF[ArcSin[(1 - \sqrt{3} - \csc[c + d*x]^{(1/3)})/(1 + \sqrt{3} - \csc[c + d*x]^{(1/3)})], -7 - 4*\sqrt{3}])/(2*d*\sqrt{(1 - \csc[c + d*x]^{(1/3)})/(1 + \sqrt{3} - \csc[c + d*x]^{(1/3)})^2}*(a - a*\csc[c + d*x])*Sqrt[a + a*\csc[c + d*x]])$

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(


```

m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]

```

Rule 63

```

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

```

Rule 218

```

Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]]/(3^(1/4)*r*Sqrt[a + b*x^3
])*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]

```

Rule 3806

```

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] := Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a + a \csc(c + dx)}}{\csc^{\frac{2}{3}}(c + dx)} dx &= \frac{(a^2 \cot(c + dx)) \text{Subst}\left(\int \frac{1}{x^{5/3} \sqrt{a-ax}} dx, x, \csc(c + dx)\right)}{d\sqrt{a - a \csc(c + dx)} \sqrt{a + a \csc(c + dx)}} \\
&= -\frac{3a \cos(c + dx) \sqrt[3]{\csc(c + dx)}}{2d\sqrt{a + a \csc(c + dx)}} + \frac{(a^2 \cot(c + dx)) \text{Subst}\left(\int \frac{1}{x^{2/3} \sqrt{a-ax}} dx, x, \csc(c + dx)\right)}{4d\sqrt{a - a \csc(c + dx)} \sqrt{a + a \csc(c + dx)}} \\
&= -\frac{3a \cos(c + dx) \sqrt[3]{\csc(c + dx)}}{2d\sqrt{a + a \csc(c + dx)}} + \frac{(3a^2 \cot(c + dx)) \text{Subst}\left(\int \frac{1}{\sqrt{a-ax^3}} dx, x, \sqrt[3]{\csc(c + dx)}\right)}{4d\sqrt{a - a \csc(c + dx)} \sqrt{a + a \csc(c + dx)}} \\
&= -\frac{3a \cos(c + dx) \sqrt[3]{\csc(c + dx)}}{2d\sqrt{a + a \csc(c + dx)}} - \frac{3^{3/4} \sqrt{2 + \sqrt{3}} a^2 \cot(c + dx) (1 - \sqrt[3]{\csc(c + dx)})}{2d \sqrt{\frac{1 - \sqrt[3]{\csc(c + dx)}}{(1 + \sqrt{3} - \sqrt[3]{\csc(c + dx)})^2}}} \sqrt{\frac{1}{1 - \sqrt[3]{\csc(c + dx)}}}
\end{aligned}$$

Mathematica [C] time = 0.46, size = 110, normalized size = 0.43

$$\frac{\sqrt{a(\csc(c + dx) + 1)} \left(\csc^{\frac{2}{3}}(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{3}{2}; 1 - \csc(c + dx)\right) + 3 \right) \left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right) \right)}{2d \csc^{\frac{2}{3}}(c + dx) \left(\sin\left(\frac{1}{2}(c + dx)\right) + \cos\left(\frac{1}{2}(c + dx)\right) \right)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + a*Csc[c + d*x]]/Csc[c + d*x]^(2/3), x]

[Out] -1/2*(Sqrt[a*(1 + Csc[c + d*x])]*(3 + Csc[c + d*x]^(2/3)*Hypergeometric2F1[1/2, 2/3, 3/2, 1 - Csc[c + d*x]])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))/(d*Csc[c + d*x]^(2/3)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a \csc(dx + c) + a}}{\csc(dx + c)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*csc(d*x+c))^(1/2)/csc(d*x+c)^(2/3), x, algorithm="fricas")

[Out] integral(sqrt(a*csc(d*x + c) + a)/csc(d*x + c)^(2/3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a \csc(dx + c) + a}}{\csc(dx + c)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*csc(d*x+c))^(1/2)/csc(d*x+c)^(2/3),x, algorithm="giac")

[Out] integrate(sqrt(a*csc(d*x + c) + a)/csc(d*x + c)^(2/3), x)

maple [F] time = 2.98, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + a \csc(dx + c)}}{\csc(dx + c)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*csc(d*x+c))^(1/2)/csc(d*x+c)^(2/3),x)

[Out] int((a+a*csc(d*x+c))^(1/2)/csc(d*x+c)^(2/3),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a \csc(dx + c) + a}}{\csc(dx + c)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*csc(d*x+c))^(1/2)/csc(d*x+c)^(2/3),x, algorithm="maxima")

[Out] integrate(sqrt(a*csc(d*x + c) + a)/csc(d*x + c)^(2/3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a + \frac{a}{\sin(c+dx)}}}{\left(\frac{1}{\sin(c+dx)}\right)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/sin(c + d*x))^(1/2)/(1/sin(c + d*x))^(2/3),x)

[Out] int((a + a/sin(c + d*x))^(1/2)/(1/sin(c + d*x))^(2/3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a(\csc(c + dx) + 1)}}{\csc^{\frac{2}{3}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*csc(d*x+c))**(1/2)/csc(d*x+c)**(2/3), x)

[Out] Integral(sqrt(a*(csc(c + d*x) + 1))/csc(c + d*x)**(2/3), x)

3.24 $\int \csc^3(c + dx) \sqrt{a + a \csc(c + dx)} dx$

Optimal. Leaf size=514

$$\frac{8\sqrt{2}3^{3/4}a^2 \cot(c + dx) \left(1 - \sqrt[3]{\csc(c + dx)}\right) \sqrt{\frac{\csc^2(c+dx) + \sqrt[3]{\csc(c+dx)} + 1}{(-\sqrt[3]{\csc(c+dx)} + \sqrt{3} + 1)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\csc(c+dx)} - \sqrt{3} + 1}{-\sqrt[3]{\csc(c+dx)} + \sqrt{3} + 1}\right) \mid -7 - 4\sqrt{3}\right)}{7d \sqrt{\frac{1 - \sqrt[3]{\csc(c+dx)}}{(-\sqrt[3]{\csc(c+dx)} + \sqrt{3} + 1)^2}} (a - a \csc(c + dx)) \sqrt{a \csc(c + dx) + a}}$$

[Out] $-6/7*a*\cos(d*x+c)*\csc(d*x+c)^{(5/3)}/d/(a+a*\csc(d*x+c))^{(1/2)}+24/7*a*\cot(d*x+c)/d/(1-\csc(d*x+c)^{(1/3)}+3^{(1/2)})/(a+a*\csc(d*x+c))^{(1/2)}+8/7*3^{(3/4)}*a^2*\cot(d*x+c)*(1-\csc(d*x+c)^{(1/3)})*EllipticF((1-\csc(d*x+c)^{(1/3)}-3^{(1/2)})/(1-\csc(d*x+c)^{(1/3)}+3^{(1/2)}), I*3^{(1/2)}+2*I)*2^{(1/2)}*((1+\csc(d*x+c)^{(1/3)}+\csc(d*x+c)^{(2/3)})/(1-\csc(d*x+c)^{(1/3)}+3^{(1/2)})^2)^{(1/2)}/d/(a-a*\csc(d*x+c))/(a+a*\csc(d*x+c))^{(1/2)}/((1-\csc(d*x+c)^{(1/3)})/(1-\csc(d*x+c)^{(1/3)}+3^{(1/2)})^2)^{(1/2)}-12/7*3^{(1/4)}*a^2*\cot(d*x+c)*(1-\csc(d*x+c)^{(1/3)})*EllipticE((1-\csc(d*x+c)^{(1/3)}-3^{(1/2)})/(1-\csc(d*x+c)^{(1/3)}+3^{(1/2)}), I*3^{(1/2)}+2*I)*(1/2*6^{(1/2)}-1/2*2^{(1/2)})*((1+\csc(d*x+c)^{(1/3)}+\csc(d*x+c)^{(2/3)})/(1-\csc(d*x+c)^{(1/3)}+3^{(1/2)})^2)^{(1/2)}/d/(a-a*\csc(d*x+c))/(a+a*\csc(d*x+c))^{(1/2)}/((1-\csc(d*x+c)^{(1/3)})/(1-\csc(d*x+c)^{(1/3)}+3^{(1/2)})^2)^{(1/2)}$

Rubi [A] time = 0.30, antiderivative size = 514, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3806, 50, 63, 303, 218, 1877}

$$\frac{8\sqrt{2}3^{3/4}a^2 \cot(c + dx) \left(1 - \sqrt[3]{\csc(c + dx)}\right) \sqrt{\frac{\csc^2(c+dx) + \sqrt[3]{\csc(c+dx)} + 1}{(-\sqrt[3]{\csc(c+dx)} + \sqrt{3} + 1)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\csc(c+dx)} - \sqrt{3} + 1}{-\sqrt[3]{\csc(c+dx)} + \sqrt{3} + 1}\right) \mid -7 - 4\sqrt{3}\right)}{7d \sqrt{\frac{1 - \sqrt[3]{\csc(c+dx)}}{(-\sqrt[3]{\csc(c+dx)} + \sqrt{3} + 1)^2}} (a - a \csc(c + dx)) \sqrt{a \csc(c + dx) + a}}$$

Antiderivative was successfully verified.

[In] Int[Csc[c + d*x]^(5/3)*Sqrt[a + a*Csc[c + d*x]],x]

[Out] $(24*a*\cot[c + d*x])/(7*d*(1 + \text{Sqrt}[3] - \text{Csc}[c + d*x]^{(1/3)})*\text{Sqrt}[a + a*\text{Csc}[c + d*x]]) - (6*a*\cos[c + d*x]*\text{Csc}[c + d*x]^{(5/3)})/(7*d*\text{Sqrt}[a + a*\text{Csc}[c + d*x]]) - (12*3^{(1/4)}*\text{Sqrt}[2 - \text{Sqrt}[3]]*a^2*\cot[c + d*x]*(1 - \text{Csc}[c + d*x]^{(1/3)})*\text{Sqrt}[(1 + \text{Csc}[c + d*x]^{(1/3)} + \text{Csc}[c + d*x]^{(2/3)})/(1 + \text{Sqrt}[3] - \text{Csc}[c + d*x]^{(1/3)})^2]*\text{EllipticE}[\text{ArcSin}[(1 - \text{Sqrt}[3] - \text{Csc}[c + d*x]^{(1/3)})/(1 + \text{Sqrt}[3] - \text{Csc}[c + d*x]^{(1/3)})], -7 - 4*\text{Sqrt}[3]])/(7*d*\text{Sqrt}[(1 - \text{Csc}[c + d*x]^{(1/3)})/(1 + \text{Sqrt}[3] - \text{Csc}[c + d*x]^{(1/3)})^2]*(a - a*\text{Csc}[c + d*x])*Sqrt[$

$$a + a \operatorname{Csc}[c + d x]] + (8 \sqrt{2} \cdot 3^{3/4} \cdot a^2 \cdot \operatorname{Cot}[c + d x] \cdot (1 - \operatorname{Csc}[c + d x]^{1/3}) \cdot \sqrt{(1 + \operatorname{Csc}[c + d x]^{1/3} + \operatorname{Csc}[c + d x]^{2/3}) / (1 + \sqrt{3} - \operatorname{Csc}[c + d x]^{1/3})^2} \cdot \operatorname{EllipticF}[\operatorname{ArcSin}[(1 - \sqrt{3} - \operatorname{Csc}[c + d x]^{1/3}) / (1 + \sqrt{3} - \operatorname{Csc}[c + d x]^{1/3})], -7 - 4 \sqrt{3}]) / (7 d \sqrt{(1 - \operatorname{Csc}[c + d x]^{1/3}) / (1 + \sqrt{3} - \operatorname{Csc}[c + d x]^{1/3})^2} \cdot (a - a \operatorname{Csc}[c + d x]) \cdot \sqrt{a + a \operatorname{Csc}[c + d x]})$$

Rule 50

$$\operatorname{Int}[(a + b x)^m (c + d x)^n, x] \rightarrow \operatorname{Simp}[(a + b x)^{m+1} (c + d x)^n / (b(m+n+1)), x] + \operatorname{Dist}[(n(b c - a d)) / (b(m+n+1)), \operatorname{Int}[(a + b x)^m (c + d x)^{n-1}, x], x] /;$$

$$\text{FreeQ}\{a, b, c, d, x\} \ \&\& \ \text{NeQ}[b c - a d, 0] \ \&\& \ \text{GtQ}[n, 0] \ \&\& \ \text{NeQ}[m + n + 1, 0] \ \&\& \ !(\text{IGtQ}[m, 0] \ \&\& \ (!\text{IntegerQ}[n] \ || \ (\text{GtQ}[m, 0] \ \&\& \ \text{LtQ}[m - n, 0]))) \ \&\& \ !\text{ILtQ}[m + n + 2, 0] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$$

Rule 63

$$\operatorname{Int}[(a + b x)^m (c + d x)^n, x] \rightarrow \operatorname{With}\{p = \operatorname{Denominator}[m]\}, \operatorname{Dist}[p/b, \operatorname{Subst}[\operatorname{Int}[x^{p(m+1)-1} (c - (a d)/b + (d x^p)/b)^n, x], x, (a + b x)^{1/p}], x] /;$$

$$\text{FreeQ}\{a, b, c, d, x\} \ \&\& \ \text{NeQ}[b c - a d, 0] \ \&\& \ \text{LtQ}[-1, m, 0] \ \&\& \ \text{LeQ}[-1, n, 0] \ \&\& \ \text{LeQ}[\operatorname{Denominator}[n], \operatorname{Denominator}[m]] \ \&\& \ \text{IntLinearQ}[a, b, c, d, m, n, x]$$

Rule 218

$$\operatorname{Int}[1/\sqrt{(a + b x)^3}, x] \rightarrow \operatorname{With}\{r = \operatorname{Numer}[\operatorname{Rt}[b/a, 3]], s = \operatorname{Denom}[\operatorname{Rt}[b/a, 3]]\}, \operatorname{Simp}[(2 \sqrt{2 + \sqrt{3}}) (s + r x) \sqrt{(s^2 - r s x + r^2 x^2) / ((1 + \sqrt{3}) s + r x)^2} \cdot \operatorname{EllipticF}[\operatorname{ArcSin}[(1 - \sqrt{3}) s + r x] / ((1 + \sqrt{3}) s + r x)], -7 - 4 \sqrt{3})] / (3^{1/4} r \sqrt{a + b x^3}) \cdot \sqrt{(s(s + r x)) / ((1 + \sqrt{3}) s + r x)^2}, x] /;$$

$$\text{FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a]$$

Rule 303

$$\operatorname{Int}[x/\sqrt{(a + b x)^3}, x] \rightarrow \operatorname{With}\{r = \operatorname{Numer}[\operatorname{Rt}[b/a, 3]], s = \operatorname{Denom}[\operatorname{Rt}[b/a, 3]]\}, \operatorname{Dist}[(\sqrt{2} s) / (\sqrt{2 + \sqrt{3}} r), \operatorname{Int}[1/\sqrt{a + b x^3}, x], x] + \operatorname{Dist}[1/r, \operatorname{Int}[(1 - \sqrt{3}) s + r x] / \sqrt{a + b x^3}, x], x] /;$$

$$\text{FreeQ}\{a, b, x\} \ \&\& \ \text{PosQ}[a]$$

Rule 1877

$$\operatorname{Int}[(c + d x) / \sqrt{(a + b x)^3}, x] \rightarrow \operatorname{With}\{r = \operatorname{Numer}[\operatorname{Simplify}[(1 - \sqrt{3}) d / c]], s = \operatorname{Denom}[\operatorname{Simplify}[(1 - \sqrt{3}) d / c]]\}, \operatorname{Simp}[(2 d s^3 \sqrt{a + b x^3}) / (a r^2 ((1 + \sqrt{3}) s + r x)), x] - \operatorname{Simp}[(3^{1/4} \sqrt{2 - \sqrt{3}}) d s (s + r x) \sqrt{(s^2 - r s x + r^2 x^2) / ((1 + \sqrt{3}) s + r x)^2} \cdot \operatorname{EllipticF}[\operatorname{ArcSin}[(1 - \sqrt{3}) s + r x] / ((1 + \sqrt{3}) s + r x)], -7 - 4 \sqrt{3})] / (3^{1/4} r \sqrt{a + b x^3}) \cdot \sqrt{(s(s + r x)) / ((1 + \sqrt{3}) s + r x)^2}, x] /;$$

```
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]]/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/( (1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && EqQ[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]
```

Rule 3806

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)], x_Symbol] := Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x, Csc[e + f*x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned} \int \csc^{\frac{5}{3}}(c + dx) \sqrt{a + a \csc(c + dx)} dx &= \frac{(a^2 \cot(c + dx)) \operatorname{Subst}\left(\int \frac{x^{2/3}}{\sqrt{a-ax}} dx, x, \csc(c + dx)\right)}{d\sqrt{a - a \csc(c + dx)} \sqrt{a + a \csc(c + dx)}} \\ &= -\frac{6a \cos(c + dx) \csc^{\frac{5}{3}}(c + dx)}{7d\sqrt{a + a \csc(c + dx)}} + \frac{(4a^2 \cot(c + dx)) \operatorname{Subst}\left(\int \frac{1}{\sqrt[3]{x} \sqrt{a-ax}} dx, x, \csc(c + dx)\right)}{7d\sqrt{a - a \csc(c + dx)} \sqrt{a + a \csc(c + dx)}} \\ &= -\frac{6a \cos(c + dx) \csc^{\frac{5}{3}}(c + dx)}{7d\sqrt{a + a \csc(c + dx)}} + \frac{(12a^2 \cot(c + dx)) \operatorname{Subst}\left(\int \frac{x}{\sqrt{a-ax^3}} dx, x, \csc(c + dx)\right)}{7d\sqrt{a - a \csc(c + dx)} \sqrt{a + a \csc(c + dx)}} \\ &= -\frac{6a \cos(c + dx) \csc^{\frac{5}{3}}(c + dx)}{7d\sqrt{a + a \csc(c + dx)}} - \frac{(12a^2 \cot(c + dx)) \operatorname{Subst}\left(\int \frac{1-\sqrt{3}-x}{\sqrt{a-ax^3}} dx, x, \csc(c + dx)\right)}{7d\sqrt{a - a \csc(c + dx)} \sqrt{a + a \csc(c + dx)}} \\ &= \frac{24a \cot(c + dx)}{7d(1 + \sqrt{3} - \sqrt[3]{\csc(c + dx)}) \sqrt{a + a \csc(c + dx)}} - \frac{6a \cos(c + dx) \csc^{\frac{5}{3}}(c + dx)}{7d\sqrt{a + a \csc(c + dx)}} \end{aligned}$$

Mathematica [C] time = 1.18, size = 120, normalized size = 0.23

$$\frac{2\sqrt{a(\csc(c + dx) + 1)} \left(3(\csc(c + dx) + 4) - 8\sqrt[3]{\csc(c + dx)} {}_2F_1\left(\frac{1}{2}, \frac{4}{3}; \frac{3}{2}; 1 - \csc(c + dx)\right)\right) \left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right)}{7d\sqrt[3]{\csc(c + dx)} \left(\sin\left(\frac{1}{2}(c + dx)\right) + \cos\left(\frac{1}{2}(c + dx)\right)\right)}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]^(5/3)*Sqrt[a + a*Csc[c + d*x]],x]

[Out] (-2*Sqrt[a*(1 + Csc[c + d*x])]*(3*(4 + Csc[c + d*x]) - 8*Csc[c + d*x]^(1/3))*Hypergeometric2F1[1/2, 4/3, 3/2, 1 - Csc[c + d*x]])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))/(7*d*Csc[c + d*x]^(1/3)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))

fricas [F] time = 0.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{a \csc(dx + c) + a} \csc(dx + c)^{\frac{5}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^(5/3)*(a+a*csc(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(a*csc(d*x + c) + a)*csc(d*x + c)^(5/3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \csc(dx + c) + a} \csc(dx + c)^{\frac{5}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^(5/3)*(a+a*csc(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(a*csc(d*x + c) + a)*csc(d*x + c)^(5/3), x)

maple [F] time = 2.41, size = 0, normalized size = 0.00

$$\int \left(\csc^{\frac{5}{3}}(dx + c)\right) \sqrt{a + a \csc(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^(5/3)*(a+a*csc(d*x+c))^(1/2),x)

[Out] int(csc(d*x+c)^(5/3)*(a+a*csc(d*x+c))^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \csc(dx + c) + a} \csc(dx + c)^{\frac{5}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^(5/3)*(a+a*csc(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(a*csc(d*x + c) + a)*csc(d*x + c)^(5/3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a + \frac{a}{\sin(c + dx)}} \left(\frac{1}{\sin(c + dx)} \right)^{5/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + a/sin(c + d*x))^(1/2)*(1/sin(c + d*x))^(5/3), x)`

[Out] `int((a + a/sin(c + d*x))^(1/2)*(1/sin(c + d*x))^(5/3), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(d*x+c)**(5/3)*(a+a*csc(d*x+c))**(1/2), x)`

[Out] Timed out

3.25 $\int \csc^{\frac{2}{3}}(c + dx) \sqrt{a + a \csc(c + dx)} dx$

Optimal. Leaf size=470

$$\frac{2\sqrt{2} 3^{3/4} a^2 \cot(c + dx) \left(1 - \sqrt[3]{\csc(c + dx)}\right) \sqrt{\frac{\csc^{\frac{2}{3}}(c + dx) + \sqrt[3]{\csc(c + dx)} + 1}{(-\sqrt[3]{\csc(c + dx)} + \sqrt{3} + 1)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\csc(c + dx)} - \sqrt{3} + 1}{-\sqrt[3]{\csc(c + dx)} + \sqrt{3} + 1}\right) \middle| -7 - 4\sqrt{3}\right)}{d \sqrt{\frac{1 - \sqrt[3]{\csc(c + dx)}}{(-\sqrt[3]{\csc(c + dx)} + \sqrt{3} + 1)^2}} (a - a \csc(c + dx)) \sqrt{a \csc(c + dx) + a}}$$

[Out] $6*a*\cot(d*x+c)/d/(1-\csc(d*x+c)^{(1/3)}+3^{(1/2)})/(a+a*\csc(d*x+c))^{(1/2)}+2*3^{(3/4)}*a^2*\cot(d*x+c)*(1-\csc(d*x+c)^{(1/3)})*EllipticF((1-\csc(d*x+c)^{(1/3)}-3^{(1/2)})/(1-\csc(d*x+c)^{(1/3)}+3^{(1/2)}), I*3^{(1/2)}+2*I)*2^{(1/2)}*((1+\csc(d*x+c)^{(1/3)}+\csc(d*x+c)^{(2/3)})/(1-\csc(d*x+c)^{(1/3)}+3^{(1/2)})^2)^{(1/2)}/d/(a-a*\csc(d*x+c))^{(1/2)}/((1-\csc(d*x+c)^{(1/3)})/(1-\csc(d*x+c)^{(1/3)}+3^{(1/2)}))^2)^{(1/2)}-3*3^{(1/4)}*a^2*\cot(d*x+c)*(1-\csc(d*x+c)^{(1/3)})*EllipticE((1-\csc(d*x+c)^{(1/3)}-3^{(1/2)})/(1-\csc(d*x+c)^{(1/3)}+3^{(1/2)}), I*3^{(1/2)}+2*I)*(1/2*6^{(1/2)}-1/2*2^{(1/2)})*((1+\csc(d*x+c)^{(1/3)}+\csc(d*x+c)^{(2/3)})/(1-\csc(d*x+c)^{(1/3)}+3^{(1/2)})^2)^{(1/2)}/d/(a-a*\csc(d*x+c))/(a+a*\csc(d*x+c))^{(1/2)}/((1-\csc(d*x+c)^{(1/3)})/(1-\csc(d*x+c)^{(1/3)}+3^{(1/2)})^2)^{(1/2)}$

Rubi [A] time = 0.25, antiderivative size = 470, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3806, 63, 303, 218, 1877}

$$\frac{2\sqrt{2} 3^{3/4} a^2 \cot(c + dx) \left(1 - \sqrt[3]{\csc(c + dx)}\right) \sqrt{\frac{\csc^{\frac{2}{3}}(c + dx) + \sqrt[3]{\csc(c + dx)} + 1}{(-\sqrt[3]{\csc(c + dx)} + \sqrt{3} + 1)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\csc(c + dx)} - \sqrt{3} + 1}{-\sqrt[3]{\csc(c + dx)} + \sqrt{3} + 1}\right) \middle| -7 - 4\sqrt{3}\right)}{d \sqrt{\frac{1 - \sqrt[3]{\csc(c + dx)}}{(-\sqrt[3]{\csc(c + dx)} + \sqrt{3} + 1)^2}} (a - a \csc(c + dx)) \sqrt{a \csc(c + dx) + a}}$$

Antiderivative was successfully verified.

[In] Int[Csc[c + d*x]^(2/3)*Sqrt[a + a*Csc[c + d*x]], x]

[Out] $(6*a*\cot(c + d*x))/(d*(1 + \sqrt{3} - \csc(c + d*x)^{(1/3)})*\sqrt{a + a*\csc(c + d*x)}) - (3*3^{(1/4)}*\sqrt{2 - \sqrt{3}})*a^2*\cot(c + d*x)*(1 - \csc(c + d*x)^{(1/3)})*\sqrt{(1 + \csc(c + d*x)^{(1/3)} + \csc(c + d*x)^{(2/3)})/(1 + \sqrt{3} - \csc(c + d*x)^{(1/3}))^2}*EllipticE[ArcSin[(1 - \sqrt{3} - \csc(c + d*x)^{(1/3)})/(1 + \sqrt{3} - \csc(c + d*x)^{(1/3)})], -7 - 4*\sqrt{3}]/(d*\sqrt{(1 - \csc(c + d*x)^{(1/3)})/(1 + \sqrt{3} - \csc(c + d*x)^{(1/3}))^2}*(a - a*\csc(c + d*x))*\sqrt{a + a*\csc(c + d*x)}) + (2*\sqrt{2}*3^{(3/4)}*a^2*\cot(c + d*x)*(1 - \csc(c + d*x)^{(1/3)})*\sqrt{(1 + \csc(c + d*x)^{(1/3)} + \csc(c + d*x)^{(2/3)})/(1 + \sqrt{3} - \csc(c + d*x)^{(1/3}))^2})$

$c[c + d*x]^{(1/3)^2} * \text{EllipticF}[\text{ArcSin}[(1 - \sqrt{3} - \text{Csc}[c + d*x]^{(1/3)}) / (1 + \sqrt{3} - \text{Csc}[c + d*x]^{(1/3)})], -7 - 4*\sqrt{3}] / (d*\sqrt{(1 - \text{Csc}[c + d*x]^{(1/3)}) / (1 + \sqrt{3} - \text{Csc}[c + d*x]^{(1/3)})^2} * (a - a*\text{Csc}[c + d*x]) * \sqrt{a + a*\text{Csc}[c + d*x]})$

Rule 63

$\text{Int}[(a_. + (b_.)*(x_.))^{(m_.)} * ((c_.) + (d_.)*(x_.))^{(n_.)}, x_Symbol] := \text{With}[\{p = \text{Denominator}[m]\}, \text{Dist}[p/b, \text{Subst}[\text{Int}[x^{(p*(m+1)-1)} * (c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^{(1/p)}], x]] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{LtQ}[-1, m, 0] \&\& \text{LeQ}[-1, n, 0] \&\& \text{LeQ}[\text{Denominator}[n], \text{Denominator}[m]] \&\& \text{IntLinearQ}[a, b, c, d, m, n, x]$

Rule 218

$\text{Int}[1/\sqrt{(a_. + (b_.)*(x_.)^3}, x_Symbol] := \text{With}[\{r = \text{Numer}[\text{Rt}[b/a, 3]], s = \text{Denom}[\text{Rt}[b/a, 3]]\}, \text{Simp}[(2*\sqrt{2 + \sqrt{3}})*(s + r*x)*\sqrt{(s^2 - r*s*x + r^2*x^2)} / ((1 + \sqrt{3})*s + r*x)^2 * \text{EllipticF}[\text{ArcSin}[(1 - \sqrt{3})*s + r*x] / ((1 + \sqrt{3})*s + r*x)], -7 - 4*\sqrt{3}] / (3^{(1/4)}*r*\sqrt{a + b*x^3} * \sqrt{(s*(s + r*x)) / ((1 + \sqrt{3})*s + r*x)^2}), x]] /; \text{FreeQ}\{a, b\}, x] \& \& \text{PosQ}[a]$

Rule 303

$\text{Int}[(x_)/\sqrt{(a_. + (b_.)*(x_.)^3}, x_Symbol] := \text{With}[\{r = \text{Numer}[\text{Rt}[b/a, 3]], s = \text{Denom}[\text{Rt}[b/a, 3]]\}, \text{Dist}[(\sqrt{2}*s)/(\sqrt{2 + \sqrt{3}}*r), \text{Int}[1/\sqrt{a + b*x^3}, x], x] + \text{Dist}[1/r, \text{Int}[(1 - \sqrt{3})*s + r*x]/\sqrt{a + b*x^3}, x], x]] /; \text{FreeQ}\{a, b\}, x] \&\& \text{PosQ}[a]$

Rule 1877

$\text{Int}[(c_. + (d_.)*(x_.))/\sqrt{(a_. + (b_.)*(x_.)^3}, x_Symbol] := \text{With}[\{r = \text{Numer}[\text{Simplify}[(1 - \sqrt{3})*d]/c], s = \text{Denom}[\text{Simplify}[(1 - \sqrt{3})*d]/c]\}, \text{Simp}[(2*d*s^3*\sqrt{a + b*x^3}) / (a*r^2*((1 + \sqrt{3})*s + r*x)), x] - \text{Simp}[(3^{(1/4)}*\sqrt{2 - \sqrt{3}})*d*s*(s + r*x)*\sqrt{(s^2 - r*s*x + r^2*x^2)} / ((1 + \sqrt{3})*s + r*x)^2 * \text{EllipticE}[\text{ArcSin}[(1 - \sqrt{3})*s + r*x] / ((1 + \sqrt{3})*s + r*x)], -7 - 4*\sqrt{3}] / (r^2*\sqrt{a + b*x^3} * \sqrt{(s*(s + r*x)) / ((1 + \sqrt{3})*s + r*x)^2}), x]] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{PosQ}[a] \&\& \text{EqQ}[b*c^3 - 2*(5 - 3*\sqrt{3})*a*d^3, 0]$

Rule 3806

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^{(n_.)} * \sqrt{\text{csc}[(e_.) + (f_.)*(x_.)]*(b_. + (a_.))}, x_Symbol] := \text{Dist}[(a^2*d*\text{Cot}[e + f*x]) / (f*\sqrt{a + b*\text{Csc}[e + f*x]} * \sqrt{a - b*\text{Csc}[e + f*x]}), \text{Subst}[\text{Int}[(d*x)^{(n-1)} / \sqrt{a - b*x}, x], x,$

$\text{Csc}[e + f*x]], x] /; \text{FreeQ}[\{a, b, d, e, f, n\}, x] \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rubi steps

$$\begin{aligned} \int \text{csc}^{\frac{2}{3}}(c + dx) \sqrt{a + a \text{csc}(c + dx)} dx &= \frac{(a^2 \cot(c + dx)) \text{Subst}\left(\int \frac{1}{\sqrt[3]{x} \sqrt{a-ax}} dx, x, \text{csc}(c + dx)\right)}{d \sqrt{a - a \text{csc}(c + dx)} \sqrt{a + a \text{csc}(c + dx)}} \\ &= \frac{(3a^2 \cot(c + dx)) \text{Subst}\left(\int \frac{x}{\sqrt{a-ax^3}} dx, x, \sqrt[3]{\text{csc}(c + dx)}\right)}{d \sqrt{a - a \text{csc}(c + dx)} \sqrt{a + a \text{csc}(c + dx)}} \\ &= -\frac{(3a^2 \cot(c + dx)) \text{Subst}\left(\int \frac{1-\sqrt{3}-x}{\sqrt{a-ax^3}} dx, x, \sqrt[3]{\text{csc}(c + dx)}\right)}{d \sqrt{a - a \text{csc}(c + dx)} \sqrt{a + a \text{csc}(c + dx)}} - \frac{(3\sqrt{2}(2 - \sqrt{3})) \text{Subst}\left(\int \frac{1}{\sqrt{a-ax^3}} dx, x, \sqrt[3]{\text{csc}(c + dx)}\right)}{d \sqrt{a - a \text{csc}(c + dx)} \sqrt{a + a \text{csc}(c + dx)}} \\ &= \frac{6a \cot(c + dx)}{d(1 + \sqrt{3} - \sqrt[3]{\text{csc}(c + dx)}) \sqrt{a + a \text{csc}(c + dx)}} - \frac{3\sqrt[4]{3} \sqrt{2 - \sqrt{3}} a^2 \cot(c + dx)}{d \sqrt{a - a \text{csc}(c + dx)} \sqrt{a + a \text{csc}(c + dx)}} \end{aligned}$$

Mathematica [C] time = 0.97, size = 109, normalized size = 0.23

$$\frac{2\sqrt{a(\text{csc}(c + dx) + 1)} \left(2\sqrt[3]{\text{csc}(c + dx)} {}_2F_1\left(\frac{1}{2}, \frac{4}{3}; \frac{3}{2}; 1 - \text{csc}(c + dx)\right) - 3\right) \left(\cos\left(\frac{1}{2}(c + dx)\right) - \sin\left(\frac{1}{2}(c + dx)\right)\right)}{d\sqrt[3]{\text{csc}(c + dx)} \left(\sin\left(\frac{1}{2}(c + dx)\right) + \cos\left(\frac{1}{2}(c + dx)\right)\right)}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]^(2/3)*Sqrt[a + a*Csc[c + d*x]],x]

[Out] (2*Sqrt[a*(1 + Csc[c + d*x])]*(-3 + 2*Csc[c + d*x]^(1/3)*Hypergeometric2F1[1/2, 4/3, 3/2, 1 - Csc[c + d*x]])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))/(d*Csc[c + d*x]^(1/3)*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))

fricas [F] time = 1.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{a \text{csc}(dx + c) + a \text{csc}(dx + c)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^(2/3)*(a+a*csc(d*x+c))^(1/2),x, algorithm="fricas")

[Out] `integral(sqrt(a*csc(d*x + c) + a)*csc(d*x + c)^(2/3), x)`

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \csc(dx + c) + a} \csc(dx + c)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(d*x+c)^(2/3)*(a+a*csc(d*x+c))^(1/2),x, algorithm="giac")`

[Out] `integral(sqrt(a*csc(d*x + c) + a)*csc(d*x + c)^(2/3), x)`

maple [F] time = 2.18, size = 0, normalized size = 0.00

$$\int \left(\csc^{\frac{2}{3}}(dx + c) \right) \sqrt{a + a \csc(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csc(d*x+c)^(2/3)*(a+a*csc(d*x+c))^(1/2), x)`

[Out] `int(csc(d*x+c)^(2/3)*(a+a*csc(d*x+c))^(1/2), x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \csc(dx + c) + a} \csc(dx + c)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(d*x+c)^(2/3)*(a+a*csc(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] `integral(sqrt(a*csc(d*x + c) + a)*csc(d*x + c)^(2/3), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{a + \frac{a}{\sin(c + dx)}} \left(\frac{1}{\sin(c + dx)} \right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + a/sin(c + d*x))^(1/2)*(1/sin(c + d*x))^(2/3), x)`

[Out] `int((a + a/sin(c + d*x))^(1/2)*(1/sin(c + d*x))^(2/3), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\csc(c + dx) + 1)} \csc^{\frac{2}{3}}(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**(2/3)*(a+a*csc(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(csc(c + d*x) + 1))*csc(c + d*x)**(2/3), x)
```

$$3.26 \quad \int \frac{\sqrt{a+a \csc(c+dx)}}{\sqrt[3]{\csc(c+dx)}} dx$$

Optimal. Leaf size=508

$$\frac{\sqrt{2} 3^{3/4} a^2 \cot(c+dx) \left(1 - \sqrt[3]{\csc(c+dx)}\right) \sqrt{\frac{\csc^2(c+dx) + \sqrt[3]{\csc(c+dx)} + 1}{(-\sqrt[3]{\csc(c+dx)} + \sqrt{3} + 1)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\csc(c+dx)} - \sqrt{3} + 1}{-\sqrt[3]{\csc(c+dx)} + \sqrt{3} + 1}\right) \mid -7 - 4\sqrt{3}\right)}{d \sqrt{\frac{1 - \sqrt[3]{\csc(c+dx)}}{(-\sqrt[3]{\csc(c+dx)} + \sqrt{3} + 1)^2}} (a - a \csc(c+dx)) \sqrt{a \csc(c+dx) + a}}$$

[Out] $-3*a*\cos(d*x+c)*\csc(d*x+c)^{(2/3)}/d/(a+a*\csc(d*x+c))^{(1/2)}-3*a*\cot(d*x+c)/d/(1-\csc(d*x+c))^{(1/3)+3^{(1/2)}}/(a+a*\csc(d*x+c))^{(1/2)}-3^{(3/4)}*a^2*\cot(d*x+c)*(1-\csc(d*x+c))^{(1/3)}*EllipticF((1-\csc(d*x+c))^{(1/3)}-3^{(1/2)})/(1-\csc(d*x+c))^{(1/3)+3^{(1/2)}}), I*3^{(1/2)+2*I}*2^{(1/2)}*((1+\csc(d*x+c))^{(1/3)}+\csc(d*x+c)^{(2/3)})/(1-\csc(d*x+c))^{(1/3)+3^{(1/2)}})^2)^{(1/2)}/d/(a-a*\csc(d*x+c))/(a+a*\csc(d*x+c))^{(1/2)}/((1-\csc(d*x+c))^{(1/3)})/(1-\csc(d*x+c))^{(1/3)+3^{(1/2)}})^2)^{(1/2)}+3/2*3^{(1/4)}*a^2*\cot(d*x+c)*(1-\csc(d*x+c))^{(1/3)}*EllipticE((1-\csc(d*x+c))^{(1/3)}-3^{(1/2)})/(1-\csc(d*x+c))^{(1/3)+3^{(1/2)}}), I*3^{(1/2)+2*I}*(1/2*6^{(1/2)}-1/2*2^{(1/2)})*((1+\csc(d*x+c))^{(1/3)}+\csc(d*x+c)^{(2/3)})/(1-\csc(d*x+c))^{(1/3)+3^{(1/2)}})^2)^{(1/2)}/d/(a-a*\csc(d*x+c))/(a+a*\csc(d*x+c))^{(1/2)}/((1-\csc(d*x+c))^{(1/3)})/(1-\csc(d*x+c))^{(1/3)+3^{(1/2)}})^2)^{(1/2)}$

Rubi [A] time = 0.27, antiderivative size = 508, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3806, 51, 63, 303, 218, 1877}

$$\frac{\sqrt{2} 3^{3/4} a^2 \cot(c+dx) \left(1 - \sqrt[3]{\csc(c+dx)}\right) \sqrt{\frac{\csc^2(c+dx) + \sqrt[3]{\csc(c+dx)} + 1}{(-\sqrt[3]{\csc(c+dx)} + \sqrt{3} + 1)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\csc(c+dx)} - \sqrt{3} + 1}{-\sqrt[3]{\csc(c+dx)} + \sqrt{3} + 1}\right) \mid -7 - 4\sqrt{3}\right)}{d \sqrt{\frac{1 - \sqrt[3]{\csc(c+dx)}}{(-\sqrt[3]{\csc(c+dx)} + \sqrt{3} + 1)^2}} (a - a \csc(c+dx)) \sqrt{a \csc(c+dx) + a}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Csc[c + d*x]]/Csc[c + d*x]^(1/3), x]

[Out] $(-3*a*\cot(c+d*x))/(d*(1+\sqrt{3}-\csc(c+d*x))^{(1/3)}*\sqrt{a+a*\csc(c+d*x)}) - (3*a*\cos(c+d*x)*\csc(c+d*x)^{(2/3)})/(d*\sqrt{a+a*\csc(c+d*x)}) + (3*3^{(1/4)}*\sqrt{2-\sqrt{3}}*a^2*\cot(c+d*x)*(1-\csc(c+d*x))^{(1/3)})*\sqrt{((1+\csc(c+d*x))^{(1/3)}+\csc(c+d*x)^{(2/3)})/(1+\sqrt{3}-\csc(c+d*x))^{(1/3)}})^2}*EllipticE[ArcSin[(1-\sqrt{3}-\csc(c+d*x))^{(1/3)})/(1+\sqrt{3}-\csc(c+d*x))^{(1/3)}], -7-4*\sqrt{3}]/(2*d*\sqrt{((1-\csc(c+d*x))^{(1/3)})/(1+\sqrt{3}-\csc(c+d*x))^{(1/3)}})^2)*(a-a*\csc(c+d*x))*\sqrt{a+a*\csc(c+d*x)}$

*Csc[c + d*x])) - (Sqrt[2]*3^(3/4)*a^2*Cot[c + d*x]*(1 - Csc[c + d*x]^(1/3)))*Sqrt[(1 + Csc[c + d*x]^(1/3) + Csc[c + d*x]^(2/3))/(1 + Sqrt[3] - Csc[c + d*x]^(1/3))^2]*EllipticF[ArcSin[(1 - Sqrt[3] - Csc[c + d*x]^(1/3))/(1 + Sqrt[3] - Csc[c + d*x]^(1/3))], -7 - 4*Sqrt[3]]/(d*Sqrt[(1 - Csc[c + d*x]^(1/3))/(1 + Sqrt[3] - Csc[c + d*x]^(1/3))^2]*(a - a*Csc[c + d*x])*Sqrt[a + a*Csc[c + d*x]])

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 218

Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x] /; FreeQ[{a, b}, x] && PosQ[a]

Rule 303

Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sqrt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x] /; FreeQ[{a, b}, x] && PosQ[a]

Rule 1877

Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c]]}, Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - Simp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/

$(1 + \sqrt{3})s + rx)^2 * \text{EllipticE}[\text{ArcSin}[\frac{(1 - \sqrt{3})s + rx}{(1 + \sqrt{3})s + rx}], -7 - 4\sqrt{3}]/(r^2 \sqrt{a + bx^3} \sqrt{(s(s + rx))/(1 + \sqrt{3})s + rx}^2), x] /; \text{FreeQ}\{a, b, c, d, x\} \&\& \text{PosQ}\{a\} \&\& \text{EqQ}\{b^3c - 2(5 - 3\sqrt{3})ad^3, 0\}$

Rule 3806

$\text{Int}[(\text{csc}[e.] + (f.)(x.))(d.)^{(n.)} \sqrt{\text{csc}[e.] + (f.)(x.)} (b. + (a.))], x_Symbol] :> \text{Dist}[(a^2 d \cot[e + f x]) / (f \sqrt{a + b \text{Csc}[e + f x]} \sqrt{a - b \text{Csc}[e + f x]}), \text{Subst}[\text{Int}[(d x)^{(n-1)} / \sqrt{a - b x}], x], x, \text{Csc}[e + f x]], x] /; \text{FreeQ}\{a, b, d, e, f, n\}, x\} \&\& \text{EqQ}\{a^2 - b^2, 0\}$

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a + a \csc(c + dx)}}{\sqrt[3]{\csc(c + dx)}} dx &= \frac{(a^2 \cot(c + dx)) \text{Subst}\left(\int \frac{1}{x^{4/3} \sqrt{a-ax}} dx, x, \csc(c + dx)\right)}{d \sqrt{a - a \csc(c + dx)} \sqrt{a + a \csc(c + dx)}} \\ &= -\frac{3a \cos(c + dx) \csc^{2/3}(c + dx)}{d \sqrt{a + a \csc(c + dx)}} - \frac{(a^2 \cot(c + dx)) \text{Subst}\left(\int \frac{1}{\sqrt[3]{x} \sqrt{a-ax}} dx, x, \csc(c + dx)\right)}{2d \sqrt{a - a \csc(c + dx)} \sqrt{a + a \csc(c + dx)}} \\ &= -\frac{3a \cos(c + dx) \csc^{2/3}(c + dx)}{d \sqrt{a + a \csc(c + dx)}} - \frac{(3a^2 \cot(c + dx)) \text{Subst}\left(\int \frac{x}{\sqrt{a-ax^3}} dx, x, \sqrt[3]{\csc(c + dx)}\right)}{2d \sqrt{a - a \csc(c + dx)} \sqrt{a + a \csc(c + dx)}} \\ &= -\frac{3a \cos(c + dx) \csc^{2/3}(c + dx)}{d \sqrt{a + a \csc(c + dx)}} + \frac{(3a^2 \cot(c + dx)) \text{Subst}\left(\int \frac{1-\sqrt{3}-x}{\sqrt{a-ax^3}} dx, x, \sqrt[3]{\csc(c + dx)}\right)}{2d \sqrt{a - a \csc(c + dx)} \sqrt{a + a \csc(c + dx)}} \\ &= -\frac{3a \cot(c + dx)}{d(1 + \sqrt{3} - \sqrt[3]{\csc(c + dx)}) \sqrt{a + a \csc(c + dx)}} - \frac{3a \cos(c + dx) \csc^{2/3}(c + dx)}{d \sqrt{a + a \csc(c + dx)}} + \dots \end{aligned}$$

Mathematica [C] time = 0.76, size = 46, normalized size = 0.09

$$\frac{2a \cot(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{4}{3}; \frac{3}{2}; 1 - \csc(c + dx)\right)}{d \sqrt{a(\csc(c + dx) + 1)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + a*Csc[c + d*x]]/Csc[c + d*x]^(1/3),x]

[Out] $(-2*a*\cot[c + d*x]*\text{Hypergeometric2F1}[1/2, 4/3, 3/2, 1 - \text{Csc}[c + d*x]])/(d*\text{Sqrt}[a*(1 + \text{Csc}[c + d*x])])$

fricas [F] time = 1.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a \csc(dx + c) + a}}{\csc(dx + c)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*csc(d*x+c))^(1/2)/csc(d*x+c)^(1/3),x, algorithm="fricas")`

[Out] `integral(sqrt(a*csc(d*x + c) + a)/csc(d*x + c)^(1/3), x)`

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a \csc(dx + c) + a}}{\csc(dx + c)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*csc(d*x+c))^(1/2)/csc(d*x+c)^(1/3),x, algorithm="giac")`

[Out] `integrate(sqrt(a*csc(d*x + c) + a)/csc(d*x + c)^(1/3), x)`

maple [F] time = 2.68, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + a \csc(dx + c)}}{\csc(dx + c)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+a*csc(d*x+c))^(1/2)/csc(d*x+c)^(1/3),x)`

[Out] `int((a+a*csc(d*x+c))^(1/2)/csc(d*x+c)^(1/3),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a \csc(dx + c) + a}}{\csc(dx + c)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*csc(d*x+c))^(1/2)/csc(d*x+c)^(1/3),x, algorithm="maxima")`

[Out] `integrate(sqrt(a*csc(d*x + c) + a)/csc(d*x + c)^(1/3), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a + \frac{a}{\sin(c+dx)}}}{\left(\frac{1}{\sin(c+dx)}\right)^{1/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/sin(c + d*x))^(1/2)/(1/sin(c + d*x))^(1/3), x)

[Out] int((a + a/sin(c + d*x))^(1/2)/(1/sin(c + d*x))^(1/3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a(\csc(c+dx)+1)}}{\sqrt[3]{\csc(c+dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*csc(d*x+c))**(1/2)/csc(d*x+c)**(1/3), x)

[Out] Integral(sqrt(a*(csc(c + d*x) + 1))/csc(c + d*x)**(1/3), x)

$$3.27 \quad \int \frac{\sqrt{a+a \csc(c+dx)}}{4 \csc^3(c+dx)} dx$$

Optimal. Leaf size=552

$$\frac{5 \cdot 3^{3/4} a^2 \cot(c+dx) \left(1 - \sqrt[3]{\csc(c+dx)}\right) \sqrt{\frac{\csc^2(c+dx) + \sqrt[3]{\csc(c+dx)} + 1}{(-\sqrt[3]{\csc(c+dx)} + \sqrt{3} + 1)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\csc(c+dx)} - \sqrt{3} + 1}{-\sqrt[3]{\csc(c+dx)} + \sqrt{3} + 1}\right) \mid -7 - 4\sqrt{3}\right) + 15 \sqrt[4]{a} \sqrt{a \csc(c+dx)}}{4\sqrt{2} d \sqrt{\frac{1 - \sqrt[3]{\csc(c+dx)}}{(-\sqrt[3]{\csc(c+dx)} + \sqrt{3} + 1)^2}} (a - a \csc(c+dx)) \sqrt{a \csc(c+dx) + a}}$$

[Out] $-3/4*a*\cos(d*x+c)/d/\csc(d*x+c)^{(1/3)}/(a+a*\csc(d*x+c))^{(1/2)}-15/8*a*\cos(d*x+c)*\csc(d*x+c)^{(2/3)}/d/(a+a*\csc(d*x+c))^{(1/2)}-15/8*a*\cot(d*x+c)/d/(1-\csc(d*x+c))^{(1/3)+3^{(1/2)}}/(a+a*\csc(d*x+c))^{(1/2)}-5/8*3^{(3/4)}*a^2*\cot(d*x+c)*(1-\csc(d*x+c))^{(1/3)}*EllipticF((1-\csc(d*x+c))^{(1/3)}-3^{(1/2)})/(1-\csc(d*x+c))^{(1/3)+3^{(1/2)}}), I*3^{(1/2)+2*I}*2^{(1/2)}*((1+\csc(d*x+c))^{(1/3)}+\csc(d*x+c)^{(2/3)})/(1-\csc(d*x+c))^{(1/3)+3^{(1/2)}})^2)^{(1/2)}/d/(a-a*\csc(d*x+c))/(a+a*\csc(d*x+c))^{(1/2)}/((1-\csc(d*x+c))^{(1/3)})/(1-\csc(d*x+c))^{(1/3)+3^{(1/2)}})^2)^{(1/2)}+15/16*3^{(1/4)}*a^2*\cot(d*x+c)*(1-\csc(d*x+c))^{(1/3)}*EllipticE((1-\csc(d*x+c))^{(1/3)}-3^{(1/2)})/(1-\csc(d*x+c))^{(1/3)+3^{(1/2)}}), I*3^{(1/2)+2*I}*(1/2*6^{(1/2)}-1/2*2^{(1/2)})*((1+\csc(d*x+c))^{(1/3)}+\csc(d*x+c)^{(2/3)})/(1-\csc(d*x+c))^{(1/3)+3^{(1/2)}})^2)^{(1/2)}/d/(a-a*\csc(d*x+c))/(a+a*\csc(d*x+c))^{(1/2)}/((1-\csc(d*x+c))^{(1/3)})/(1-\csc(d*x+c))^{(1/3)+3^{(1/2)}})^2)^{(1/2)}$

Rubi [A] time = 0.31, antiderivative size = 552, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3806, 51, 63, 303, 218, 1877}

$$\frac{5 \cdot 3^{3/4} a^2 \cot(c+dx) \left(1 - \sqrt[3]{\csc(c+dx)}\right) \sqrt{\frac{\csc^2(c+dx) + \sqrt[3]{\csc(c+dx)} + 1}{(-\sqrt[3]{\csc(c+dx)} + \sqrt{3} + 1)^2}} F\left(\sin^{-1}\left(\frac{-\sqrt[3]{\csc(c+dx)} - \sqrt{3} + 1}{-\sqrt[3]{\csc(c+dx)} + \sqrt{3} + 1}\right) \mid -7 - 4\sqrt{3}\right) + 15 \sqrt[4]{a} \sqrt{a \csc(c+dx)}}{4\sqrt{2} d \sqrt{\frac{1 - \sqrt[3]{\csc(c+dx)}}{(-\sqrt[3]{\csc(c+dx)} + \sqrt{3} + 1)^2}} (a - a \csc(c+dx)) \sqrt{a \csc(c+dx) + a}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + a*Csc[c + d*x]]/Csc[c + d*x]^(4/3), x]

[Out] $(-15*a*\cot(c+d*x))/(8*d*(1+\sqrt{3})-\csc(c+d*x)^{(1/3)}*\sqrt{a+a*\csc(c+d*x)})-(3*a*\cos(c+d*x))/(4*d*\csc(c+d*x)^{(1/3)}*\sqrt{a+a*\csc(c+d*x)})-(15*a*\cos(c+d*x)*\csc(c+d*x)^{(2/3)})/(8*d*\sqrt{a+a*\csc(c+d*x)})+(15*3^{(1/4)}*\sqrt{2-\sqrt{3}}*a^2*\cot(c+d*x)*(1-\csc(c+d*x))^{(1/3)})*\sqrt{(1+\csc(c+d*x))^{(1/3)}+\csc(c+d*x)^{(2/3)}}/(1+\sqrt{3})-\csc(c+d*x)^{(1/3)})$

```

+ d*x]^(1/3))^2*EllipticE[ArcSin[(1 - Sqrt[3] - Csc[c + d*x]^(1/3))/(1 +
Sqrt[3] - Csc[c + d*x]^(1/3))], -7 - 4*Sqrt[3]]/(16*d*Sqrt[(1 - Csc[c + d*
x]^(1/3))/(1 + Sqrt[3] - Csc[c + d*x]^(1/3))^2]*(a - a*Csc[c + d*x])*Sqrt[a
+ a*Csc[c + d*x]]) - (5*3^(3/4)*a^2*Cot[c + d*x]*(1 - Csc[c + d*x]^(1/3))*
Sqrt[(1 + Csc[c + d*x]^(1/3) + Csc[c + d*x]^(2/3))/(1 + Sqrt[3] - Csc[c + d
*x]^(1/3))^2]*EllipticF[ArcSin[(1 - Sqrt[3] - Csc[c + d*x]^(1/3))/(1 + Sqrt
[3] - Csc[c + d*x]^(1/3))], -7 - 4*Sqrt[3]]/(4*Sqrt[2]*d*Sqrt[(1 - Csc[c +
d*x]^(1/3))/(1 + Sqrt[3] - Csc[c + d*x]^(1/3))^2]*(a - a*Csc[c + d*x])*Sqr
t[a + a*Csc[c + d*x]])

```

Rule 51

```

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]

```

Rule 63

```

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

```

Rule 218

```

Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[(2*Sqrt[2 + Sqrt[3]]*(s + r*x)*Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 + Sqrt[3])*s + r*x)^2]*EllipticF[ArcSin[((1 - Sqrt[3])*s
+ r*x)/((1 + Sqrt[3])*s + r*x)], -7 - 4*Sqrt[3]]/(3^(1/4)*r*Sqrt[a + b*x^3
]*Sqrt[(s*(s + r*x))/((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b}, x] &
& PosQ[a]

```

Rule 303

```

Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[2]*s)/(Sqrt[2 + Sqrt[3]]*r), Int[1/Sq
rt[a + b*x^3], x], x] + Dist[1/r, Int[((1 - Sqrt[3])*s + r*x)/Sqrt[a + b*x^
3], x], x]] /; FreeQ[{a, b}, x] && PosQ[a]

```

Rule 1877

```

Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] := With[{r = N

```

```

umer[Simplify[((1 - Sqrt[3])*d)/c]], s = Denom[Simplify[((1 - Sqrt[3])*d)/c
]], Simp[(2*d*s^3*Sqrt[a + b*x^3])/(a*r^2*((1 + Sqrt[3])*s + r*x)), x] - S
imp[(3^(1/4)*Sqrt[2 - Sqrt[3]]*d*s*(s + r*x)*Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 + Sqrt[3])*s + r*x)^2]*EllipticE[ArcSin[((1 - Sqrt[3])*s + r*x)/((1 + S
qrt[3])*s + r*x)], -7 - 4*Sqrt[3]])/(r^2*Sqrt[a + b*x^3]*Sqrt[(s*(s + r*x))/
((1 + Sqrt[3])*s + r*x)^2]), x]] /; FreeQ[{a, b, c, d}, x] && PosQ[a] && Eq
Q[b*c^3 - 2*(5 - 3*Sqrt[3])*a*d^3, 0]

```

Rule 3806

```

Int[(Csc[e_.] + (f_.)*(x_)]*(d_.)^(n_)*Sqrt[Csc[e_.] + (f_.)*(x_)]*(b_
+ (a_)), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]
]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,
Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]

```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a + a \csc(c + dx)}}{\csc^{\frac{4}{3}}(c + dx)} dx &= \frac{(a^2 \cot(c + dx)) \operatorname{Subst}\left(\int \frac{1}{x^{7/3} \sqrt{a-ax}} dx, x, \csc(c + dx)\right)}{d \sqrt{a - a \csc(c + dx)} \sqrt{a + a \csc(c + dx)}} \\
&= -\frac{3a \cos(c + dx)}{4d \sqrt[3]{\csc(c + dx)} \sqrt{a + a \csc(c + dx)}} + \frac{(5a^2 \cot(c + dx)) \operatorname{Subst}\left(\int \frac{1}{x^{4/3} \sqrt{a-ax}} dx, x, \csc(c + dx)\right)}{8d \sqrt{a - a \csc(c + dx)} \sqrt{a + a \csc(c + dx)}} \\
&= -\frac{3a \cos(c + dx)}{4d \sqrt[3]{\csc(c + dx)} \sqrt{a + a \csc(c + dx)}} - \frac{15a \cos(c + dx) \csc^{\frac{2}{3}}(c + dx)}{8d \sqrt{a + a \csc(c + dx)}} - \frac{(5a^2 \cot(c + dx)) \operatorname{Subst}\left(\int \frac{1}{x^{1/3} \sqrt{a-ax}} dx, x, \csc(c + dx)\right)}{16d \sqrt{a + a \csc(c + dx)}} \\
&= -\frac{3a \cos(c + dx)}{4d \sqrt[3]{\csc(c + dx)} \sqrt{a + a \csc(c + dx)}} - \frac{15a \cos(c + dx) \csc^{\frac{2}{3}}(c + dx)}{8d \sqrt{a + a \csc(c + dx)}} - \frac{(15a^2 \cot(c + dx)) \operatorname{Subst}\left(\int \frac{1}{x^{1/3} \sqrt{a-ax}} dx, x, \csc(c + dx)\right)}{16d \sqrt{a + a \csc(c + dx)}} \\
&= -\frac{3a \cos(c + dx)}{4d \sqrt[3]{\csc(c + dx)} \sqrt{a + a \csc(c + dx)}} - \frac{15a \cos(c + dx) \csc^{\frac{2}{3}}(c + dx)}{8d \sqrt{a + a \csc(c + dx)}} + \frac{(15a^2 \cot(c + dx)) \operatorname{Subst}\left(\int \frac{1}{x^{1/3} \sqrt{a-ax}} dx, x, \csc(c + dx)\right)}{16d \sqrt{a + a \csc(c + dx)}} \\
&= -\frac{15a \cot(c + dx)}{8d (1 + \sqrt{3} - \sqrt[3]{\csc(c + dx)}) \sqrt{a + a \csc(c + dx)}} - \frac{3a \cos(c + dx)}{4d \sqrt[3]{\csc(c + dx)} \sqrt{a + a \csc(c + dx)}}
\end{aligned}$$

Mathematica [C] time = 1.22, size = 72, normalized size = 0.13

$$\frac{a \cos(c + dx) \left(5 \csc^{\frac{4}{3}}(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{4}{3}; \frac{3}{2}; 1 - \csc(c + dx)\right) + 3 \right)}{4d \sqrt[3]{\csc(c + dx)} \sqrt{a(\csc(c + dx) + 1)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + a*Csc[c + d*x]]/Csc[c + d*x]^(4/3), x]

[Out] -1/4*(a*Cos[c + d*x]*(3 + 5*Csc[c + d*x]^(4/3)*Hypergeometric2F1[1/2, 4/3, 3/2, 1 - Csc[c + d*x]]))/(d*Csc[c + d*x]^(1/3)*Sqrt[a*(1 + Csc[c + d*x])])

fricas [F] time = 1.89, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a \csc(dx + c) + a}}{\csc(dx + c)^{\frac{4}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*csc(d*x+c))^(1/2)/csc(d*x+c)^(4/3), x, algorithm="fricas")

[Out] integral(sqrt(a*csc(d*x + c) + a)/csc(d*x + c)^(4/3), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a \csc(dx + c) + a}}{\csc(dx + c)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*csc(d*x+c))^(1/2)/csc(d*x+c)^(4/3), x, algorithm="giac")

[Out] integrate(sqrt(a*csc(d*x + c) + a)/csc(d*x + c)^(4/3), x)

maple [F] time = 1.80, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + a \csc(dx + c)}}{\csc(dx + c)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*csc(d*x+c))^(1/2)/csc(d*x+c)^(4/3), x)

[Out] int((a+a*csc(d*x+c))^(1/2)/csc(d*x+c)^(4/3), x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a \csc(dx + c) + a}}{\csc(dx + c)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*csc(d*x+c))^(1/2)/csc(d*x+c)^(4/3),x, algorithm="maxima")

[Out] integrate(sqrt(a*csc(d*x + c) + a)/csc(d*x + c)^(4/3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a + \frac{a}{\sin(c+dx)}}}{\left(\frac{1}{\sin(c+dx)}\right)^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/sin(c + d*x))^(1/2)/(1/sin(c + d*x))^(4/3),x)

[Out] int((a + a/sin(c + d*x))^(1/2)/(1/sin(c + d*x))^(4/3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a(\csc(c + dx) + 1)}}{\csc^{\frac{4}{3}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*csc(d*x+c))**(1/2)/csc(d*x+c)**(4/3),x)

[Out] Integral(sqrt(a*(csc(c + d*x) + 1))/csc(c + d*x)**(4/3), x)

3.28 $\int \csc^n(c + dx) \sqrt{a + a \csc(c + dx)} dx$

Optimal. Leaf size=48

$$\frac{2a \cot(c + dx) {}_2F_1\left(\frac{1}{2}, 1 - n; \frac{3}{2}; 1 - \csc(c + dx)\right)}{d \sqrt{a \csc(c + dx) + a}}$$

[Out] $-2*a*\cot(d*x+c)*\text{hypergeom}([1/2, 1-n], [3/2], 1-\csc(d*x+c))/d/(a+a*\csc(d*x+c))^{(1/2)}$

Rubi [A] time = 0.06, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3806, 65}

$$\frac{2a \cot(c + dx) {}_2F_1\left(\frac{1}{2}, 1 - n; \frac{3}{2}; 1 - \csc(c + dx)\right)}{d \sqrt{a \csc(c + dx) + a}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Csc}[c + d*x]^n * \text{Sqrt}[a + a*\text{Csc}[c + d*x]], x]$

[Out] $(-2*a*\text{Cot}[c + d*x]*\text{Hypergeometric2F1}[1/2, 1 - n, 3/2, 1 - \text{Csc}[c + d*x]])/(d * \text{Sqrt}[a + a*\text{Csc}[c + d*x]])$

Rule 65

$\text{Int}[(b_*)*(x_)^{(m_*)}*((c_*) + (d_*)*(x_))^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(c + d*x)^{(n + 1)} * \text{Hypergeometric2F1}[-m, n + 1, n + 2, 1 + (d*x)/c] / (d*(n + 1)*(-(d/(b*c)))^m), x] /;$ $\text{FreeQ}\{b, c, d, m, n\}, x \ \&\& \ !\text{IntegerQ}[n] \ \&\& \ (\text{IntegerQ}[m] \ || \ \text{GtQ}[-(d/(b*c)), 0])$

Rule 3806

$\text{Int}[(\csc[(e_*) + (f_*)*(x_)]*(d_*)^{(n_*)} * \text{Sqrt}[\csc[(e_*) + (f_*)*(x_)]*(b_*) + (a_*)], x_Symbol] \rightarrow \text{Dist}[(a^2*d*\text{Cot}[e + f*x]) / (f*\text{Sqrt}[a + b*\csc[e + f*x]] * \text{Sqrt}[a - b*\csc[e + f*x]]), \text{Subst}[\text{Int}[(d*x)^{(n - 1)} / \text{Sqrt}[a - b*x], x], x, \csc[e + f*x]], x] /;$ $\text{FreeQ}\{a, b, d, e, f, n\}, x \ \&\& \ \text{EqQ}[a^2 - b^2, 0]$

Rubi steps

$$\int \csc^n(c + dx) \sqrt{a + a \csc(c + dx)} dx = \frac{(a^2 \cot(c + dx)) \operatorname{Subst}\left(\int \frac{x^{-1+n}}{\sqrt{a-ax}} dx, x, \csc(c + dx)\right)}{d \sqrt{a - a \csc(c + dx)} \sqrt{a + a \csc(c + dx)}} \\ = -\frac{2a \cot(c + dx) {}_2F_1\left(\frac{1}{2}, 1 - n; \frac{3}{2}; 1 - \csc(c + dx)\right)}{d \sqrt{a + a \csc(c + dx)}}$$

Mathematica [A] time = 0.17, size = 48, normalized size = 1.00

$$-\frac{2a \cot(c + dx) {}_2F_1\left(\frac{1}{2}, 1 - n; \frac{3}{2}; 1 - \csc(c + dx)\right)}{d \sqrt{a(\csc(c + dx) + 1)}}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]^n*Sqrt[a + a*Csc[c + d*x]],x]

[Out] (-2*a*Cot[c + d*x]*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 - Csc[c + d*x]])/(d*Sqrt[a*(1 + Csc[c + d*x])])

fricas [F] time = 0.94, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\sqrt{a \csc(dx + c) + a} \csc(dx + c)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^n*(a+a*csc(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(a*csc(d*x + c) + a)*csc(d*x + c)^n, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \csc(dx + c) + a} \csc(dx + c)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^n*(a+a*csc(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(a*csc(d*x + c) + a)*csc(d*x + c)^n, x)

maple [F] time = 1.95, size = 0, normalized size = 0.00

$$\int (\csc^n(dx + c)) \sqrt{a + a \csc(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csc(d*x+c)^n*(a+a*csc(d*x+c))^(1/2),x)`

[Out] `int(csc(d*x+c)^n*(a+a*csc(d*x+c))^(1/2),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \csc(dx + c) + a} \csc(dx + c)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(d*x+c)^n*(a+a*csc(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(a*csc(d*x + c) + a)*csc(d*x + c)^n, x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{a + \frac{a}{\sin(c + dx)}} \left(\frac{1}{\sin(c + dx)} \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + a/sin(c + d*x))^(1/2)*(1/sin(c + d*x))^n,x)`

[Out] `int((a + a/sin(c + d*x))^(1/2)*(1/sin(c + d*x))^n, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a(\csc(c + dx) + 1)} \csc^n(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(d*x+c)**n*(a+a*csc(d*x+c))**(1/2),x)`

[Out] `Integral(sqrt(a*(csc(c + d*x) + 1))*csc(c + d*x)**n, x)`

3.29 $\int \csc^n(c + dx) \sqrt{a - a \csc(c + dx)} dx$

Optimal. Leaf size=69

$$\frac{2a \cos(c + dx) (-\csc(c + dx))^{-n} \csc^{n+1}(c + dx) {}_2F_1\left(\frac{1}{2}, 1 - n; \frac{3}{2}; \csc(c + dx) + 1\right)}{d \sqrt{a - a \csc(c + dx)}}$$

[Out] $-2*a*\cos(d*x+c)*\csc(d*x+c)^{(1+n)}*\text{hypergeom}([1/2, 1-n], [3/2], 1+\csc(d*x+c))/d / ((-\csc(d*x+c))^n)/(a-a*\csc(d*x+c))^{(1/2)}$

Rubi [A] time = 0.07, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3806, 67, 65}

$$\frac{2a \cos(c + dx) (-\csc(c + dx))^{-n} \csc^{n+1}(c + dx) {}_2F_1\left(\frac{1}{2}, 1 - n; \frac{3}{2}; \csc(c + dx) + 1\right)}{d \sqrt{a - a \csc(c + dx)}}$$

Antiderivative was successfully verified.

[In] `Int[Csc[c + d*x]^n*Sqrt[a - a*Csc[c + d*x]], x]`

[Out] $(-2*a*\text{Cos}[c + d*x]*\text{Csc}[c + d*x]^{(1 + n)}*\text{Hypergeometric2F1}[1/2, 1 - n, 3/2, 1 + \text{Csc}[c + d*x]])/(d*(-\text{Csc}[c + d*x])^n*\text{Sqrt}[a - a*\text{Csc}[c + d*x]])$

Rule 65

`Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((c + d*x)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m] || GtQ[-(d/(b*c)), 0])`

Rule 67

`Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] := Dist[(-(b*c)/d)^IntPart[m]*(b*x)^FracPart[m]/(-(d*x)/c)^FracPart[m], Int[(-(d*x)/c)^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0] && !GtQ[-(d/(b*c)), 0]`

Rule 3806

`Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)], x_Symbol] := Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[(d*x)^(n - 1)/Sqrt[a - b*x], x], x,`

Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned} \int \csc^n(c + dx) \sqrt{a - a \csc(c + dx)} dx &= \frac{(a^2 \cot(c + dx)) \operatorname{Subst}\left(\int \frac{x^{-1+n}}{\sqrt{a+ax}} dx, x, \csc(c + dx)\right)}{d\sqrt{a - a \csc(c + dx)} \sqrt{a + a \csc(c + dx)}} \\ &= -\frac{(a^2 \cos(c + dx)(-\csc(c + dx))^{-n} \csc^{1+n}(c + dx)) \operatorname{Subst}\left(\int \frac{(-x)^{-1+n}}{\sqrt{a+ax}} dx, \right)}{d\sqrt{a - a \csc(c + dx)} \sqrt{a + a \csc(c + dx)}} \\ &= -\frac{2a \cos(c + dx)(-\csc(c + dx))^{-n} \csc^{1+n}(c + dx) {}_2F_1\left(\frac{1}{2}, 1 - n; \frac{3}{2}; 1 + \csc(c + dx)\right)}{d\sqrt{a - a \csc(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.99, size = 73, normalized size = 1.06

$$\frac{2a \cos(c + dx) \csc^{2n+1}(c + dx) (-\csc^2(c + dx))^{-n} {}_2F_1\left(\frac{1}{2}, 1 - n; \frac{3}{2}; \csc(c + dx) + 1\right)}{d\sqrt{a - a \csc(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]^n*Sqrt[a - a*Csc[c + d*x]], x]

[Out] (-2*a*Cos[c + d*x]*Csc[c + d*x]^(1 + 2*n)*Hypergeometric2F1[1/2, 1 - n, 3/2, 1 + Csc[c + d*x]])/(d*(-Csc[c + d*x]^2)^n*Sqrt[a - a*Csc[c + d*x]])

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\sqrt{-a \csc(dx + c) + a} \csc(dx + c)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^n*(a-a*csc(d*x+c))^(1/2), x, algorithm="fricas")

[Out] integral(sqrt(-a*csc(d*x + c) + a)*csc(d*x + c)^n, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a \csc(dx + c) + a} \csc(dx + c)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^n*(a-a*csc(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(-a*csc(d*x + c) + a)*csc(d*x + c)^n, x)

maple [F] time = 2.45, size = 0, normalized size = 0.00

$$\int (\csc^n(dx + c)) \sqrt{a - a \csc(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(d*x+c)^n*(a-a*csc(d*x+c))^(1/2),x)

[Out] int(csc(d*x+c)^n*(a-a*csc(d*x+c))^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a \csc(dx + c) + a} \csc(dx + c)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^n*(a-a*csc(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-a*csc(d*x + c) + a)*csc(d*x + c)^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a - \frac{a}{\sin(c + dx)}} \left(\frac{1}{\sin(c + dx)} \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a - a/sin(c + d*x))^(1/2)*(1/sin(c + d*x))^n,x)

[Out] int((a - a/sin(c + d*x))^(1/2)*(1/sin(c + d*x))^n, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a(\csc(c + dx) - 1)} \csc^n(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)**n*(a-a*csc(d*x+c))**(1/2),x)

[Out] Integral(sqrt(-a*(csc(c + d*x) - 1))*csc(c + d*x)**n, x)

3.30 $\int \csc^3(e + fx)(a + a \csc(e + fx))^m dx$

Optimal. Leaf size=156

$$\frac{2^{m+\frac{1}{2}}(m^2 + m + 1) \cot(e + fx)(\csc(e + fx) + 1)^{-m-\frac{1}{2}}(a \csc(e + fx) + a)^m {}_2F_1\left(\frac{1}{2}, \frac{1}{2} - m; \frac{3}{2}; \frac{1}{2}(1 - \csc(e + fx))\right)}{f(m+1)(m+2)}$$

[Out] $\cot(f*x+e)*(a+a*\csc(f*x+e))^m/f/(m^2+3*m+2)-\cot(f*x+e)*(a+a*\csc(f*x+e))^{(1+m)}/a/f/(2+m)-2^{(1/2+m)}*(m^2+m+1)*\cot(f*x+e)*(1+\csc(f*x+e))^{(-1/2-m)}*(a+a*\csc(f*x+e))^m*\text{hypergeom}([1/2, 1/2-m], [3/2], 1/2-1/2*\csc(f*x+e))/f/(m^2+3*m+2)$

Rubi [A] time = 0.19, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3800, 4001, 3828, 3827, 69}

$$\frac{2^{m+\frac{1}{2}}(m^2 + m + 1) \cot(e + fx)(\csc(e + fx) + 1)^{-m-\frac{1}{2}}(a \csc(e + fx) + a)^m {}_2F_1\left(\frac{1}{2}, \frac{1}{2} - m; \frac{3}{2}; \frac{1}{2}(1 - \csc(e + fx))\right)}{f(m+1)(m+2)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Csc}[e + f*x]^3*(a + a*\text{Csc}[e + f*x])^m, x]$

[Out] $(\text{Cot}[e + f*x]*(a + a*\text{Csc}[e + f*x])^m)/(f*(2 + 3*m + m^2)) - (\text{Cot}[e + f*x]*(a + a*\text{Csc}[e + f*x])^{(1 + m)})/(a*f*(2 + m)) - (2^{(1/2 + m)}*(1 + m + m^2)*\text{Cot}[e + f*x]*(1 + \text{Csc}[e + f*x])^{(-1/2 - m)}*(a + a*\text{Csc}[e + f*x])^m*\text{Hypergeometric2F1}[1/2, 1/2 - m, 3/2, (1 - \text{Csc}[e + f*x])/2])/(f*(1 + m)*(2 + m))$

Rule 69

$\text{Int}[(a_) + (b_)*(x_)^{(m_)}*((c_) + (d_)*(x_)^{(n_)}, x_Symbol] := \text{Simp}[(a + b*x)^{(m+1)}*\text{Hypergeometric2F1}[-n, m+1, m+2, -((d*(a+b*x))/(b*c-a*d))]/(b*(m+1)*(b/(b*c-a*d))^n), x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& !\text{IntegerQ}[m] \&\& !\text{IntegerQ}[n] \&\& \text{GtQ}[b/(b*c - a*d), 0] \&\& (\text{RationalQ}[m] || !(\text{RationalQ}[n] \&\& \text{GtQ}[-(d/(b*c - a*d)), 0]))$

Rule 3800

$\text{Int}[\csc[(e_) + (f_)*(x_)]^3*(\csc[(e_) + (f_)*(x_)]*(b_) + (a_))^{(m_)}, x_Symbol] := -\text{Simp}[(\text{Cot}[e + f*x]*(a + b*\csc[e + f*x])^{(m+1)})/(b*f*(m+2)), x] + \text{Dist}[1/(b*(m+2)), \text{Int}[\text{Csc}[e + f*x]*(a + b*\csc[e + f*x])^m*(b*(m+1) - a*\csc[e + f*x]), x], x] /; \text{FreeQ}\{a, b, e, f, m\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& !\text{LtQ}[m, -2^{(-1)}]$

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.))^(m_.), x_Symbol] := Dist[(a^2*d*Cot[e + f*x])/(f*sqrt[a + b*Csc[e + f*
x]]*sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.))^(m_.), x_Symbol] := Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/((1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a]^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rule 4001

```
Int[csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.)*(cs
c[(e_.) + (f_.)*(x_.)]*(B_.) + (A_.)), x_Symbol] := -Simp[(B*Cot[e + f*x]*(a
+ b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m + 1
)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B, e
, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*(m
+ 1), 0] && !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}
\int \csc^3(e + fx)(a + a \csc(e + fx))^m dx &= -\frac{\cot(e + fx)(a + a \csc(e + fx))^{1+m}}{af(2 + m)} + \frac{\int \csc(e + fx)(a(1 + m) - a \csc(e + fx))^m dx}{a(2 + m)} \\
&= \frac{\cot(e + fx)(a + a \csc(e + fx))^m}{f(2 + 3m + m^2)} - \frac{\cot(e + fx)(a + a \csc(e + fx))^{1+m}}{af(2 + m)} + \\
&= \frac{\cot(e + fx)(a + a \csc(e + fx))^m}{f(2 + 3m + m^2)} - \frac{\cot(e + fx)(a + a \csc(e + fx))^{1+m}}{af(2 + m)} + \\
&= \frac{\cot(e + fx)(a + a \csc(e + fx))^m}{f(2 + 3m + m^2)} - \frac{\cot(e + fx)(a + a \csc(e + fx))^{1+m}}{af(2 + m)} + \\
&= \frac{\cot(e + fx)(a + a \csc(e + fx))^m}{f(2 + 3m + m^2)} - \frac{\cot(e + fx)(a + a \csc(e + fx))^{1+m}}{af(2 + m)} -
\end{aligned}$$

Mathematica [A] time = 4.61, size = 178, normalized size = 1.14

$$\tan^2\left(\frac{1}{2}(e+fx)\right)\left(\tan\left(\frac{1}{2}(e+fx)\right)+1\right)^{-2m}\left(a(\csc(e+fx)+1)\right)^m\left((m-2)m\cot^4\left(\frac{1}{2}(e+fx)\right)_2F_1\left(-m-2,-2m\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Csc[e + f*x]^3*(a + a*Csc[e + f*x])^m,x]

[Out] $-\frac{1}{4}((a(1 + \csc(e + fx)))^m((-2 + m)m\cot[(e + fx)/2]^4\text{Hypergeometric2F1}[-2 - m, -2m, -1 - m, -\tan[(e + fx)/2]] + (2 + m)(m\text{Hypergeometric2F1}[2 - m, -2m, 3 - m, -\tan[(e + fx)/2]] + 2(-2 + m)\cot[(e + fx)/2]^2\text{Hypergeometric2F1}[-2m, -m, 1 - m, -\tan[(e + fx)/2]]))\tan[(e + fx)/2]^2)/(f(-2 + m)m(2 + m)(1 + \tan[(e + fx)/2])^{(2m)})$

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a \csc(fx + e) + a\right)^m \csc(fx + e)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)^3*(a+a*csc(f*x+e))^m,x, algorithm="fricas")

[Out] integral((a*csc(f*x + e) + a)^m*csc(f*x + e)^3, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \csc(fx + e) + a)^m \csc(fx + e)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)^3*(a+a*csc(f*x+e))^m,x, algorithm="giac")

[Out] integrate((a*csc(f*x + e) + a)^m*csc(f*x + e)^3, x)

maple [F] time = 3.27, size = 0, normalized size = 0.00

$$\int (\csc^3(fx + e))(a + a \csc(fx + e))^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(f*x+e)^3*(a+a*csc(f*x+e))^m,x)

[Out] int(csc(f*x+e)^3*(a+a*csc(f*x+e))^m,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \csc(fx + e) + a)^m \csc(fx + e)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)^3*(a+a*csc(f*x+e))^m,x, algorithm="maxima")

[Out] integrate((a*csc(f*x + e) + a)^m*csc(f*x + e)^3, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{a}{\sin(e+fx)}\right)^m}{\sin(e+fx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/sin(e + f*x))^m/sin(e + f*x)^3,x)

[Out] int((a + a/sin(e + f*x))^m/sin(e + f*x)^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a (\csc(e + fx) + 1))^m \csc^3(e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)**3*(a+a*csc(f*x+e))**m,x)

[Out] Integral((a*(csc(e + f*x) + 1))**m*csc(e + f*x)**3, x)

3.31 $\int \csc^2(e + fx)(a + a \csc(e + fx))^m dx$

Optimal. Leaf size=109

$$\frac{2^{m+\frac{1}{2}} m \cot(e + fx) (\csc(e + fx) + 1)^{-m-\frac{1}{2}} (a \csc(e + fx) + a)^m {}_2F_1\left(\frac{1}{2}, \frac{1}{2} - m; \frac{3}{2}; \frac{1}{2}(1 - \csc(e + fx))\right)}{f(m+1)} \cot(e + fx)$$

[Out] $-\cot(f*x+e)*(a+a*\csc(f*x+e))^m/f/(1+m)-2^{(1/2+m)*m}*\cot(f*x+e)*(1+\csc(f*x+e))^{(-1/2-m)*(a+a*\csc(f*x+e))^m}*\text{hypergeom}([1/2, 1/2-m], [3/2], 1/2-1/2*\csc(f*x+e))/f/(1+m)$

Rubi [A] time = 0.10, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3798, 3828, 3827, 69}

$$\frac{2^{m+\frac{1}{2}} m \cot(e + fx) (\csc(e + fx) + 1)^{-m-\frac{1}{2}} (a \csc(e + fx) + a)^m {}_2F_1\left(\frac{1}{2}, \frac{1}{2} - m; \frac{3}{2}; \frac{1}{2}(1 - \csc(e + fx))\right)}{f(m+1)} \cot(e + fx)$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Csc}[e + f*x]^2*(a + a*\text{Csc}[e + f*x])^m, x]$

[Out] $-\left(\frac{\cot[e + f*x]*(a + a*\text{Csc}[e + f*x])^m}{f*(1 + m)}\right) - \left(2^{(1/2 + m)*m}*\cot[e + f*x]*(1 + \text{Csc}[e + f*x])^{(-1/2 - m)*(a + a*\text{Csc}[e + f*x])^m}*\text{Hypergeometric2F1}[1/2, 1/2 - m, 3/2, (1 - \text{Csc}[e + f*x])/2]\right)/f*(1 + m)$

Rule 69

$\text{Int}[(a_ + (b_)*(x_))^{(m_)}*((c_ + (d_)*(x_))^{(n_)}), x_Symbol] \rightarrow \text{Simp}[\left(\frac{(a + b*x)^{(m+1)}*\text{Hypergeometric2F1}[-n, m+1, m+2, -((d*(a + b*x))/(b*c - a*d))]}{b*(m+1)*(b/(b*c - a*d))^n}\right), x] /; \text{FreeQ}\{a, b, c, d, m, n\}, x \&\& \text{NeQ}\{b*c - a*d, 0\} \&\& \text{!IntegerQ}\{m\} \&\& \text{!IntegerQ}\{n\} \&\& \text{GtQ}\{b/(b*c - a*d), 0\} \&\& (\text{RationalQ}\{m\} \mid \mid \text{!(RationalQ}\{n\} \&\& \text{GtQ}\{-d/(b*c - a*d), 0\}))$

Rule 3798

$\text{Int}[\csc[(e_ + (f_)*(x_))]^2*(\csc[(e_ + (f_)*(x_))]*(b_ + (a_))^{(m_)}), x_Symbol] \rightarrow -\text{Simp}[\left(\frac{\cot[e + f*x]*(a + b*\csc[e + f*x])^m}{f*(m+1)}\right), x] + \text{Dist}[(a*m)/(b*(m+1)), \text{Int}[\text{Csc}[e + f*x]*(a + b*\csc[e + f*x])^m, x], x] /; \text{FreeQ}\{a, b, e, f, m\}, x \&\& \text{EqQ}\{a^2 - b^2, 0\} \&\& \text{!LtQ}\{m, -2^{(-1)}\}$

Rule 3827

$\text{Int}[(\csc[(e_ + (f_)*(x_))]*(d_))^{(n_)}*(\csc[(e_ + (f_)*(x_))]*(b_ + (a_))^{(m_)}), x_Symbol] \rightarrow \text{Dist}[(a^2*d*\cot[e + f*x])/f*\text{Sqrt}[a + b*\csc[e + f*x]]$

$x]]*\text{Sqrt}[a - b*\text{Csc}[e + f*x]], \text{Subst}[\text{Int}[\text{((d*x)}^{(n - 1)}*(a + b*x)^{(m - 1/2)})/\text{Sqrt}[a - b*x], x], x, \text{Csc}[e + f*x], x] /; \text{FreeQ}[\{a, b, d, e, f, m, n\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& !\text{IntegerQ}[m] \&\& \text{GtQ}[a, 0]$

Rule 3828

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^{(n_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^{(m_.)}, x_Symbol] :> \text{Dist}[(a^{\text{IntPart}[m]}*(a + b*\text{Csc}[e + f*x])^{\text{FracPart}[m]})/(1 + (b*\text{Csc}[e + f*x])/a)^{\text{FracPart}[m]}, \text{Int}[(1 + (b*\text{Csc}[e + f*x])/a)^m*(d*\text{Csc}[e + f*x])^n, x], x] /; \text{FreeQ}[\{a, b, d, e, f, m, n\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& !\text{IntegerQ}[m] \&\& !\text{GtQ}[a, 0]$

Rubi steps

$$\begin{aligned} \int \csc^2(e + fx)(a + a \csc(e + fx))^m dx &= -\frac{\cot(e + fx)(a + a \csc(e + fx))^m}{f(1 + m)} + \frac{m \int \csc(e + fx)(a + a \csc(e + fx))^{m-1} dx}{1 + m} \\ &= -\frac{\cot(e + fx)(a + a \csc(e + fx))^m}{f(1 + m)} + \frac{(m(1 + \csc(e + fx))^{-m}(a + a \csc(e + fx))^{m-1} \int \csc(e + fx) dx)}{1 + m} \\ &= -\frac{\cot(e + fx)(a + a \csc(e + fx))^m}{f(1 + m)} + \frac{(m \cot(e + fx)(1 + \csc(e + fx))^{-\frac{1}{2}}(a + a \csc(e + fx))^{m-1} \int \csc(e + fx) dx)}{1 + m} \\ &= -\frac{\cot(e + fx)(a + a \csc(e + fx))^m}{f(1 + m)} - \frac{2^{\frac{1}{2}+m} m \cot(e + fx)(1 + \csc(e + fx))^{m-1} \int \csc(e + fx) dx}{1 + m} \end{aligned}$$

Mathematica [A] time = 1.35, size = 126, normalized size = 1.16

$$\frac{\tan\left(\frac{1}{2}(e + fx)\right) \left(\tan\left(\frac{1}{2}(e + fx)\right) + 1\right)^{-2m} (a(\csc(e + fx) + 1))^m \left((m + 1) {}_2F_1\left(1 - m, -2m; 2 - m; -\tan\left(\frac{1}{2}(e + fx)\right)\right)\right)}{2f(m - 1)(m + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[e + f*x]^2*(a + a*Csc[e + f*x])^m,x]

[Out] $-1/2*((a*(1 + \text{Csc}[e + f*x]))^m*((-1 + m)*\text{Cot}[(e + f*x)/2]^2*\text{Hypergeometric2F1}[-1 - m, -2*m, -m, -\text{Tan}[(e + f*x)/2]] + (1 + m)*\text{Hypergeometric2F1}[1 - m, -2*m, 2 - m, -\text{Tan}[(e + f*x)/2]])*\text{Tan}[(e + f*x)/2])/(f*(-1 + m)*(1 + m)*(1 + \text{Tan}[(e + f*x)/2])^{(2*m)})$

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a \csc(fx + e) + a\right)^m \csc(fx + e)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)^2*(a+a*csc(f*x+e))^m,x, algorithm="fricas")

[Out] integral((a*csc(f*x + e) + a)^m*csc(f*x + e)^2, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \csc(fx + e) + a\right)^m \csc(fx + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)^2*(a+a*csc(f*x+e))^m,x, algorithm="giac")

[Out] integrate((a*csc(f*x + e) + a)^m*csc(f*x + e)^2, x)

maple [F] time = 2.00, size = 0, normalized size = 0.00

$$\int \left(\csc^2(fx + e)\right) \left(a + a \csc(fx + e)\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(f*x+e)^2*(a+a*csc(f*x+e))^m,x)

[Out] int(csc(f*x+e)^2*(a+a*csc(f*x+e))^m,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \csc(fx + e) + a\right)^m \csc(fx + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)^2*(a+a*csc(f*x+e))^m,x, algorithm="maxima")

[Out] integrate((a*csc(f*x + e) + a)^m*csc(f*x + e)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{a}{\sin(e+fx)}\right)^m}{\sin(e+fx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/sin(e + f*x))^m/sin(e + f*x)^2,x)
```

```
[Out] int((a + a/sin(e + f*x))^m/sin(e + f*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\csc(e + fx) + 1))^m \csc^2(e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+a*csc(f*x+e))**m,x)
```

```
[Out] Integral((a*(csc(e + f*x) + 1))**m*csc(e + f*x)**2, x)
```

3.32 $\int \csc(e + fx)(a + a \csc(e + fx))^m dx$

Optimal. Leaf size=74

$$\frac{2^{m+\frac{1}{2}} \cot(e + fx)(\csc(e + fx) + 1)^{-m-\frac{1}{2}}(a \csc(e + fx) + a)^m {}_2F_1\left(\frac{1}{2}, \frac{1}{2} - m; \frac{3}{2}; \frac{1}{2}(1 - \csc(e + fx))\right)}{f}$$

[Out] $-2^{(1/2+m)} \cot(f*x+e) (1+\csc(f*x+e))^{(-1/2-m)} (a+a*\csc(f*x+e))^m \text{hypergeom}($
 $[1/2, 1/2-m], [3/2], 1/2-1/2*\csc(f*x+e))/f$

Rubi [A] time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00,
 number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} =$
 0.158, Rules used = {3828, 3827, 69}

$$\frac{2^{m+\frac{1}{2}} \cot(e + fx)(\csc(e + fx) + 1)^{-m-\frac{1}{2}}(a \csc(e + fx) + a)^m {}_2F_1\left(\frac{1}{2}, \frac{1}{2} - m; \frac{3}{2}; \frac{1}{2}(1 - \csc(e + fx))\right)}{f}$$

Antiderivative was successfully verified.

[In] Int[Csc[e + f*x]*(a + a*Csc[e + f*x])^m,x]

[Out] $-((2^{(1/2 + m)} \text{Cot}[e + f*x] (1 + \text{Csc}[e + f*x])^{(-1/2 - m)} (a + a*\text{Csc}[e + f*x])^m \text{Hypergeometric2F1}[1/2, 1/2 - m, 3/2, (1 - \text{Csc}[e + f*x])/2])/f$

Rule 69

Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(((a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c - a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[m] && !IntegerQ[n] && GtQ[b/(b*c - a*d), 0] && (RationalQ[m] || !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))

Rule 3827

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2))/Sqrt[a - b*x], x], x, Csc[e + f*x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0]

Rule 3828

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m]

)]/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && !GtQ[a, 0]

Rubi steps

$$\begin{aligned} \int \csc(e + fx)(a + a \csc(e + fx))^m dx &= \left((1 + \csc(e + fx))^{-m} (a + a \csc(e + fx))^m \right) \int \csc(e + fx)(1 + \csc(e + fx)) \\ &= \frac{\left(\cot(e + fx)(1 + \csc(e + fx))^{-\frac{1}{2}-m} (a + a \csc(e + fx))^m \right) \text{Subst} \left(\int \frac{(1+x)^{-\frac{1}{2}}}{\sqrt{1-x}} \right)}{f \sqrt{1 - \csc(e + fx)}} \\ &= -\frac{2^{\frac{1}{2}+m} \cot(e + fx)(1 + \csc(e + fx))^{-\frac{1}{2}-m} (a + a \csc(e + fx))^m {}_2F_1 \left(\frac{1}{2}, \frac{1}{2} - \right)}{f} \end{aligned}$$

Mathematica [A] time = 0.20, size = 60, normalized size = 0.81

$$-\frac{\left(\tan \left(\frac{1}{2}(e + fx) \right) + 1 \right)^{-2m} (a(\csc(e + fx) + 1))^m {}_2F_1 \left(-2m, -m; 1 - m; -\tan \left(\frac{1}{2}(e + fx) \right) \right)}{fm}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[e + f*x]*(a + a*Csc[e + f*x])^m,x]

[Out] -(((a*(1 + Csc[e + f*x]))^m*Hypergeometric2F1[-2*m, -m, 1 - m, -Tan[(e + f*x)/2]])/(f*m*(1 + Tan[(e + f*x)/2])^(2*m)))

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral} \left((a \csc(fx + e) + a)^m \csc(fx + e), x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)*(a+a*csc(f*x+e))^m,x, algorithm="fricas")

[Out] integral((a*csc(f*x + e) + a)^m*csc(f*x + e), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \csc(fx + e) + a)^m \csc(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(f*x+e)*(a+a*csc(f*x+e))^m,x, algorithm="giac")`

[Out] `integrate((a*csc(f*x + e) + a)^m*csc(f*x + e), x)`

maple [F] time = 1.54, size = 0, normalized size = 0.00

$$\int \csc(fx + e) (a + a \csc(fx + e))^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csc(f*x+e)*(a+a*csc(f*x+e))^m,x)`

[Out] `int(csc(f*x+e)*(a+a*csc(f*x+e))^m,x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \csc(fx + e) + a)^m \csc(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(f*x+e)*(a+a*csc(f*x+e))^m,x, algorithm="maxima")`

[Out] `integrate((a*csc(f*x + e) + a)^m*csc(f*x + e), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{a}{\sin(e+fx)}\right)^m}{\sin(e+fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + a/sin(e + f*x))^m/sin(e + f*x),x)`

[Out] `int((a + a/sin(e + f*x))^m/sin(e + f*x), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a (\csc(e + fx) + 1))^m \csc(e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(f*x+e)*(a+a*csc(f*x+e))**m,x)`

[Out] `Integral((a*(csc(e + f*x) + 1))**m*csc(e + f*x), x)`

3.33 $\int (a + a \csc(e + fx))^m dx$

Optimal. Leaf size=84

$$\frac{\sqrt{2} \cot(e + fx)(a \csc(e + fx) + a)^m F_1\left(m + \frac{1}{2}; \frac{1}{2}, 1; m + \frac{3}{2}; \frac{1}{2}(\csc(e + fx) + 1), \csc(e + fx) + 1\right)}{f(2m + 1)\sqrt{1 - \csc(e + fx)}}$$

[Out] -AppellF1(1/2+m,1,1/2,3/2+m,1+csc(f*x+e),1/2+1/2*csc(f*x+e))*cot(f*x+e)*(a+a*csc(f*x+e))^m*2^(1/2)/f/(1+2*m)/(1-csc(f*x+e))^(1/2)

Rubi [A] time = 0.06, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3779, 3778, 136}

$$\frac{\sqrt{2} \cot(e + fx)(a \csc(e + fx) + a)^m F_1\left(m + \frac{1}{2}; \frac{1}{2}, 1; m + \frac{3}{2}; \frac{1}{2}(\csc(e + fx) + 1), \csc(e + fx) + 1\right)}{f(2m + 1)\sqrt{1 - \csc(e + fx)}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Csc[e + f*x])^m,x]

[Out] -((Sqrt[2]*AppellF1[1/2 + m, 1/2, 1, 3/2 + m, (1 + Csc[e + f*x])/2, 1 + Csc[e + f*x]]*Cot[e + f*x]*(a + a*Csc[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[1 - Csc[e + f*x]]))

Rule 136

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]/(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])

Rule 3778

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[(a^n*Cot[c + d*x])/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]]), Subst[Int[(1 + (b*x)/a)^(n - 1/2)/(x*Sqrt[1 - (b*x)/a]), x], x, Csc[c + d*x]], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]

Rule 3779

```
Int[(csc[c_.] + (d_.)*(x_.))*(b_.) + (a_.))^(n_), x_Symbol] := Dist[(a^IntPart[n]*(a + b*Csc[c + d*x])^FracPart[n])/(1 + (b*Csc[c + d*x])/a)^FracPart[n]], Int[(1 + (b*Csc[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned} \int (a + a \csc(e + fx))^m dx &= \left((1 + \csc(e + fx))^{-m} (a + a \csc(e + fx))^m \right) \int (1 + \csc(e + fx))^m dx \\ &= \frac{\left(\cot(e + fx) (1 + \csc(e + fx))^{-\frac{1}{2}-m} (a + a \csc(e + fx))^m \right) \text{Subst} \left(\int \frac{(1+x)^{-\frac{1}{2}+m}}{\sqrt{1-x}} dx, x, \csc(e + fx) \right)}{f \sqrt{1 - \csc(e + fx)}} \\ &= -\frac{\sqrt{2} F_1 \left(\frac{1}{2} + m; \frac{1}{2}, 1; \frac{3}{2} + m; \frac{1}{2} (1 + \csc(e + fx)), 1 + \csc(e + fx) \right) \cot(e + fx) (a + a \csc(e + fx))^m}{f (1 + 2m) \sqrt{1 - \csc(e + fx)}} \end{aligned}$$

Mathematica [F] time = 0.61, size = 0, normalized size = 0.00

$$\int (a + a \csc(e + fx))^m dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Csc[e + f*x])^m, x]
```

```
[Out] Integrate[(a + a*Csc[e + f*x])^m, x]
```

fricas [F] time = 0.83, size = 0, normalized size = 0.00

$$\text{integral} \left((a \csc(fx + e) + a)^m, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*csc(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((a*csc(f*x + e) + a)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \csc(fx + e) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*csc(f*x+e))^m,x, algorithm="giac")

[Out] integrate((a*csc(f*x + e) + a)^m, x)

maple [F] time = 1.20, size = 0, normalized size = 0.00

$$\int (a + a \csc(fx + e))^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*csc(f*x+e))^m,x)

[Out] int((a+a*csc(f*x+e))^m,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \csc(fx + e) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*csc(f*x+e))^m,x, algorithm="maxima")

[Out] integrate((a*csc(f*x + e) + a)^m, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a + \frac{a}{\sin(e + fx)} \right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/sin(e + f*x))^m,x)

[Out] int((a + a/sin(e + f*x))^m, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \csc(e + fx) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*csc(f*x+e))**m,x)

[Out] Integral((a*csc(e + f*x) + a)**m, x)

3.34 $\int (a + a \csc(e + fx))^m \sin(e + fx) dx$

Optimal. Leaf size=83

$$\frac{\sqrt{2} \cot(e + fx)(a \csc(e + fx) + a)^m F_1\left(m + \frac{1}{2}; \frac{1}{2}, 2; m + \frac{3}{2}; \frac{1}{2}(\csc(e + fx) + 1), \csc(e + fx) + 1\right)}{f(2m + 1)\sqrt{1 - \csc(e + fx)}}$$

[Out] AppellF1(1/2+m, 2, 1/2, 3/2+m, 1+csc(f*x+e), 1/2+1/2*csc(f*x+e))*cot(f*x+e)*(a+a*csc(f*x+e))^m*2^(1/2)/f/(1+2*m)/(1-csc(f*x+e))^(1/2)

Rubi [A] time = 0.09, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3828, 3827, 136}

$$\frac{\sqrt{2} \cot(e + fx)(a \csc(e + fx) + a)^m F_1\left(m + \frac{1}{2}; \frac{1}{2}, 2; m + \frac{3}{2}; \frac{1}{2}(\csc(e + fx) + 1), \csc(e + fx) + 1\right)}{f(2m + 1)\sqrt{1 - \csc(e + fx)}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Csc[e + f*x])^m*Sin[e + f*x], x]

[Out] (Sqrt[2]*AppellF1[1/2 + m, 1/2, 2, 3/2 + m, (1 + Csc[e + f*x])/2, 1 + Csc[e + f*x]]*Cot[e + f*x]*(a + a*Csc[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[1 - Csc[e + f*x]])

Rule 136

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]/(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !(GtQ[d/(d*a - c*b), 0] && SimplifierQ[c + d*x, a + b*x])

Rule 3827

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2))/Sqrt[a - b*x], x], x, Csc[e + f*x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0]

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/ (1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned} \int (a + a \csc(e + fx))^m \sin(e + fx) dx &= \left((1 + \csc(e + fx))^{-m} (a + a \csc(e + fx))^m \right) \int (1 + \csc(e + fx))^m \sin(e + fx) dx \\ &= \frac{\left(\cot(e + fx) (1 + \csc(e + fx))^{-\frac{1}{2}-m} (a + a \csc(e + fx))^m \right) \text{Subst} \left(\int \frac{(1+x)^{-\frac{1}{2}}}{\sqrt{1-x}} dx \right)}{f \sqrt{1 - \csc(e + fx)}} \\ &= \frac{\sqrt{2} F_1 \left(\frac{1}{2} + m; \frac{1}{2}, 2; \frac{3}{2} + m; \frac{1}{2} (1 + \csc(e + fx)), 1 + \csc(e + fx) \right) \cot(e + fx)}{f(1 + 2m) \sqrt{1 - \csc(e + fx)}} \end{aligned}$$

Mathematica [F] time = 3.77, size = 0, normalized size = 0.00

$$\int (a + a \csc(e + fx))^m \sin(e + fx) dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Csc[e + f*x])^m*Sin[e + f*x], x]
```

```
[Out] Integrate[(a + a*Csc[e + f*x])^m*Sin[e + f*x], x]
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral} \left((a \csc(fx + e) + a)^m \sin(fx + e), x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*csc(f*x+e))^m*sin(f*x+e), x, algorithm="fricas")
```

```
[Out] integral((a*csc(f*x + e) + a)^m*sin(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \csc(fx + e) + a)^m \sin(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*csc(f*x+e))^m*sin(f*x+e),x, algorithm="giac")

[Out] integrate((a*csc(f*x + e) + a)^m*sin(f*x + e), x)

maple [F] time = 2.26, size = 0, normalized size = 0.00

$$\int (a + a \csc(fx + e))^m \sin(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*csc(f*x+e))^m*sin(f*x+e),x)

[Out] int((a+a*csc(f*x+e))^m*sin(f*x+e),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \csc(fx + e) + a)^m \sin(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*csc(f*x+e))^m*sin(f*x+e),x, algorithm="maxima")

[Out] integrate((a*csc(f*x + e) + a)^m*sin(f*x + e), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(e + fx) \left(a + \frac{a}{\sin(e + fx)} \right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(e + f*x)*(a + a/sin(e + f*x))^m,x)

[Out] int(sin(e + f*x)*(a + a/sin(e + f*x))^m, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\csc(e + fx) + 1))^m \sin(e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*csc(f*x+e))^m*sin(f*x+e),x)

[Out] Integral((a*(csc(e + f*x) + 1))^m*sin(e + f*x), x)

3.35 $\int (a + a \csc(e + fx))^m \sin^2(e + fx) dx$

Optimal. Leaf size=84

$$\frac{\sqrt{2} \cot(e + fx)(a \csc(e + fx) + a)^m F_1\left(m + \frac{1}{2}; \frac{1}{2}, 3; m + \frac{3}{2}; \frac{1}{2}(\csc(e + fx) + 1), \csc(e + fx) + 1\right)}{f(2m + 1)\sqrt{1 - \csc(e + fx)}}$$

[Out] -AppellF1(1/2+m,3,1/2,3/2+m,1+csc(f*x+e),1/2+1/2*csc(f*x+e))*cot(f*x+e)*(a+a*csc(f*x+e))^m*2^(1/2)/f/(1+2*m)/(1-csc(f*x+e))^(1/2)

Rubi [A] time = 0.11, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3828, 3827, 136}

$$\frac{\sqrt{2} \cot(e + fx)(a \csc(e + fx) + a)^m F_1\left(m + \frac{1}{2}; \frac{1}{2}, 3; m + \frac{3}{2}; \frac{1}{2}(\csc(e + fx) + 1), \csc(e + fx) + 1\right)}{f(2m + 1)\sqrt{1 - \csc(e + fx)}}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Csc[e + f*x])^m*Sin[e + f*x]^2,x]

[Out] -((Sqrt[2]*AppellF1[1/2 + m, 1/2, 3, 3/2 + m, (1 + Csc[e + f*x])/2, 1 + Csc[e + f*x]]*Cot[e + f*x]*(a + a*Csc[e + f*x])^m)/(f*(1 + 2*m)*Sqrt[1 - Csc[e + f*x]]))

Rule 136

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[((b*e - a*f)^p*(a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]/(b^(p + 1)*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])

Rule 3827

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2))/Sqrt[a - b*x], x], x, Csc[e + f*x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0]

Rule 3828


```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) +
(a_.)^(m_.), x_Symbol] := Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/((1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a]^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned} \int (a + a \csc(e + fx))^m \sin^2(e + fx) dx &= \left((1 + \csc(e + fx))^{-m} (a + a \csc(e + fx))^m \right) \int (1 + \csc(e + fx))^m \sin^2(e + fx) dx \\ &= \frac{\left(\cot(e + fx) (1 + \csc(e + fx))^{-\frac{1}{2}-m} (a + a \csc(e + fx))^m \right) \text{Subst} \left(\int \frac{(1+x)}{\sqrt{1-x^2}} dx \right)}{f \sqrt{1 - \csc(e + fx)}} \\ &= -\frac{\sqrt{2} F_1 \left(\frac{1}{2} + m; \frac{1}{2}, 3; \frac{3}{2} + m; \frac{1}{2} (1 + \csc(e + fx)), 1 + \csc(e + fx) \right) \cot(e + fx)}{f (1 + 2m) \sqrt{1 - \csc(e + fx)}} \end{aligned}$$

Mathematica [F] time = 7.71, size = 0, normalized size = 0.00

$$\int (a + a \csc(e + fx))^m \sin^2(e + fx) dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + a*Csc[e + f*x])^m*Sin[e + f*x]^2,x]

[Out] Integrate[(a + a*Csc[e + f*x])^m*Sin[e + f*x]^2, x]

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral} \left(-\left(\cos(fx + e)^2 - 1 \right) (a \csc(fx + e) + a)^m, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*csc(f*x+e))^m*sin(f*x+e)^2,x, algorithm="fricas")

[Out] integral(-(cos(f*x + e)^2 - 1)*(a*csc(f*x + e) + a)^m, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \csc(fx + e) + a)^m \sin(fx + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*csc(f*x+e))^m*sin(f*x+e)^2,x, algorithm="giac")

[Out] integrate((a*csc(f*x + e) + a)^m*sin(f*x + e)^2, x)

maple [F] time = 4.69, size = 0, normalized size = 0.00

$$\int (a + a \csc(fx + e))^m (\sin^2(fx + e)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*csc(f*x+e))^m*sin(f*x+e)^2,x)

[Out] int((a+a*csc(f*x+e))^m*sin(f*x+e)^2,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \csc(fx + e) + a)^m \sin(fx + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*csc(f*x+e))^m*sin(f*x+e)^2,x, algorithm="maxima")

[Out] integrate((a*csc(f*x + e) + a)^m*sin(f*x + e)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(e + fx)^2 \left(a + \frac{a}{\sin(e + fx)} \right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(e + f*x)^2*(a + a/sin(e + f*x))^m,x)

[Out] int(sin(e + f*x)^2*(a + a/sin(e + f*x))^m, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a (\csc(e + fx) + 1))^m \sin^2(e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*csc(f*x+e))**m*sin(f*x+e)**2,x)

[Out] Integral((a*(csc(e + f*x) + 1))**m*sin(e + f*x)**2, x)

3.36 $\int (a + b \csc(c + dx))^4 dx$

Optimal. Leaf size=107

$$a^4x - \frac{b^2(17a^2 + 2b^2)\cot(c + dx)}{3d} - \frac{2ab(2a^2 + b^2)\tanh^{-1}(\cos(c + dx))}{d} - \frac{4ab^3\cot(c + dx)\csc(c + dx)}{3d} - \frac{b^2\cot(c + dx)}{3d}$$

[Out] $a^4x - 2ab(2a^2 + b^2)\operatorname{arctanh}(\cos(dx + c))/d - 1/3b^2(17a^2 + 2b^2)\cot(dx + c)/d - 4/3ab^3\cot(dx + c)\csc(dx + c)/d - 1/3b^2\cot(dx + c)(a + b\csc(dx + c))^2/d$

Rubi [A] time = 0.11, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3782, 4048, 3770, 3767, 8}

$$\frac{b^2(17a^2 + 2b^2)\cot(c + dx)}{3d} - \frac{2ab(2a^2 + b^2)\tanh^{-1}(\cos(c + dx))}{d} + a^4x - \frac{4ab^3\cot(c + dx)\csc(c + dx)}{3d} - \frac{b^2\cot(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b\text{Csc}[c + d*x])^4, x]$

[Out] $a^4x - (2ab(2a^2 + b^2)\text{ArcTanh}[\text{Cos}[c + d*x]])/d - (b^2(17a^2 + 2b^2)\text{Cot}[c + d*x])/(3d) - (4ab^3\text{Cot}[c + d*x]\text{Csc}[c + d*x])/(3d) - (b^2\text{Cot}[c + d*x](a + b\text{Csc}[c + d*x])^2)/(3d)$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 3767

$\text{Int}[\csc[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow -\text{Dist}[d^{(-1)}, \text{Subst}[\text{Int}[\text{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \text{Cot}[c + d*x]], x] /; \text{FreeQ}[\{c, d\}, x] \ \&\& \ \text{IGtQ}[n/2, 0]$

Rule 3770

$\text{Int}[\csc[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\text{Simp}[\text{ArcTanh}[\text{Cos}[c + d*x]]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 3782

$\text{Int}[(\csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.))^{(n_.)}, x_Symbol] \rightarrow -\text{Simp}[(b^2\text{Cot}[c + d*x]*(a + b\text{Csc}[c + d*x])^{(n - 2)})/(d*(n - 1)), x] + \text{Dist}[1/(n - 1), \text{Int}[(a + b\text{Csc}[c + d*x])^{(n - 3)}*\text{Simp}[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a^2$

$(n - 1)) * \text{Csc}[c + d*x] + (a*b^2*(3*n - 4)) * \text{Csc}[c + d*x]^2, x], x], x] /; \text{FreeQ}\{a, b, c, d, x\} \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{GtQ}[n, 2] \ \&\& \ \text{IntegerQ}[2*n]$

Rule 4048

$\text{Int}[(A + \text{csc}[e + f*x]) * (B + \text{csc}[e + f*x])^2 * (C + \text{csc}[e + f*x]) * \text{Cot}[e + f*x] / (2*f), x] + \text{Dist}[1/2, \text{Int}[\text{Simp}[2*A*a + (2*B*a + b*(2*A + C)) * \text{Csc}[e + f*x] + 2*(a*C + B*b) * \text{Csc}[e + f*x]^2, x], x] /; \text{FreeQ}\{a, b, e, f, A, B, C\}, x]$

Rubi steps

$$\begin{aligned} \int (a + b \csc(c + dx))^4 dx &= -\frac{b^2 \cot(c + dx)(a + b \csc(c + dx))^2}{3d} + \frac{1}{3} \int (a + b \csc(c + dx)) (3a^3 + b(9a^2 + 2b^2) \csc(c + dx)) dx \\ &= -\frac{4ab^3 \cot(c + dx) \csc(c + dx)}{3d} - \frac{b^2 \cot(c + dx)(a + b \csc(c + dx))^2}{3d} + \frac{1}{6} \int (6a^4 + 12ab(2a^2 + b^2) \csc(c + dx)) dx \\ &= a^4 x - \frac{4ab^3 \cot(c + dx) \csc(c + dx)}{3d} - \frac{b^2 \cot(c + dx)(a + b \csc(c + dx))^2}{3d} + (2ab(2a^2 + b^2) \csc(c + dx)) \\ &= a^4 x - \frac{2ab(2a^2 + b^2) \tanh^{-1}(\cos(c + dx))}{d} - \frac{4ab^3 \cot(c + dx) \csc(c + dx)}{3d} - \frac{b^2 \cot(c + dx)(a + b \csc(c + dx))^2}{3d} \\ &= a^4 x - \frac{2ab(2a^2 + b^2) \tanh^{-1}(\cos(c + dx))}{d} - \frac{b^2(17a^2 + 2b^2) \cot(c + dx)}{3d} - \frac{4ab^3 \cot(c + dx) \csc(c + dx)}{3d} \end{aligned}$$

Mathematica [B] time = 6.25, size = 568, normalized size = 5.31

$$\frac{a^4(c + dx) \sin^4(c + dx)(a + b \csc(c + dx))^4}{d(a \sin(c + dx) + b)^4} + \frac{2(2a^3b + ab^3) \sin^4(c + dx) \log\left(\sin\left(\frac{1}{2}(c + dx)\right)\right)(a + b \csc(c + dx))^4}{d(a \sin(c + dx) + b)^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Csc[c + d*x])^4, x]

[Out] $(a^4*(c + d*x)*(a + b*\text{Csc}[c + d*x])^4*\text{Sin}[c + d*x]^4)/(d*(b + a*\text{Sin}[c + d*x])^4) + ((-9*a^2*b^2*\text{Cos}[(c + d*x)/2] - b^4*\text{Cos}[(c + d*x)/2])* \text{Csc}[(c + d*x)/2]*(a + b*\text{Csc}[c + d*x])^4*\text{Sin}[c + d*x]^4)/(3*d*(b + a*\text{Sin}[c + d*x])^4) - (a*b^3*\text{Csc}[(c + d*x)/2]^2*(a + b*\text{Csc}[c + d*x])^4*\text{Sin}[c + d*x]^4)/(2*d*(b + a*\text{Sin}[c + d*x])^4) - (b^4*\text{Cot}[(c + d*x)/2]* \text{Csc}[(c + d*x)/2]^2*(a + b*\text{Csc}[c + d*x])^4*\text{Sin}[c + d*x]^4)/(24*d*(b + a*\text{Sin}[c + d*x])^4) - (2*(2*a^3*b + a*b^3)*(a + b*\text{Csc}[c + d*x])^4*\text{Log}[\text{Cos}[(c + d*x)/2]]*\text{Sin}[c + d*x]^4)/(d*(b + a*\text{Sin}[c + d*x])^4)$

$$\begin{aligned} & \text{in}[c + d*x])^4) + (2*(2*a^3*b + a*b^3)*(a + b*\text{Csc}[c + d*x])^4*\text{Log}[\text{Sin}[(c + \\ & d*x)/2]]*\text{Sin}[c + d*x]^4)/(d*(b + a*\text{Sin}[c + d*x])^4) + (a*b^3*(a + b*\text{Csc}[c + \\ & d*x])^4*\text{Sec}[(c + d*x)/2]^2*\text{Sin}[c + d*x]^4)/(2*d*(b + a*\text{Sin}[c + d*x])^4) + \\ & ((a + b*\text{Csc}[c + d*x])^4*\text{Sec}[(c + d*x)/2]*(9*a^2*b^2*\text{Sin}[(c + d*x)/2] + b^4* \\ & \text{Sin}[(c + d*x)/2])* \text{Sin}[c + d*x]^4)/(3*d*(b + a*\text{Sin}[c + d*x])^4) + (b^4*(a + \\ & b*\text{Csc}[c + d*x])^4*\text{Sec}[(c + d*x)/2]^2*\text{Sin}[c + d*x]^4*\text{Tan}[(c + d*x)/2])/(24*d \\ & *(b + a*\text{Sin}[c + d*x])^4) \end{aligned}$$

fricas [B] time = 0.78, size = 217, normalized size = 2.03

$$2(9a^2b^2 + b^4)\cos(dx + c)^3 - 3(2a^3b + ab^3 - (2a^3b + ab^3)\cos(dx + c)^2)\log\left(\frac{1}{2}\cos(dx + c) + \frac{1}{2}\right)\sin(dx + c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*csc(d*x+c))^4,x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/3*(2*(9*a^2*b^2 + b^4)*\cos(d*x + c)^3 - 3*(2*a^3*b + a*b^3 - (2*a^3*b + \\ & a*b^3)*\cos(d*x + c)^2)*\log(1/2*\cos(d*x + c) + 1/2)*\sin(d*x + c) + 3*(2*a^3*b \\ & b + a*b^3 - (2*a^3*b + a*b^3)*\cos(d*x + c)^2)*\log(-1/2*\cos(d*x + c) + 1/2)* \\ & \sin(d*x + c) - 3*(6*a^2*b^2 + b^4)*\cos(d*x + c) - 3*(a^4*d*x*\cos(d*x + c)^2 \\ & - a^4*d*x + 2*a*b^3*\cos(d*x + c))*\sin(d*x + c))/((d*\cos(d*x + c)^2 - d)*\sin \\ & (d*x + c)) \end{aligned}$$

giac [B] time = 0.78, size = 205, normalized size = 1.92

$$b^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^3 + 12 ab^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 24(dx + c)a^4 + 72 a^2 b^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 9 b^4 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*csc(d*x+c))^4,x, algorithm="giac")

[Out]
$$\begin{aligned} & 1/24*(b^4*\tan(1/2*d*x + 1/2*c)^3 + 12*a*b^3*\tan(1/2*d*x + 1/2*c)^2 + 24*(d* \\ & x + c)*a^4 + 72*a^2*b^2*\tan(1/2*d*x + 1/2*c) + 9*b^4*\tan(1/2*d*x + 1/2*c) + \\ & 48*(2*a^3*b + a*b^3)*\log(\text{abs}(\tan(1/2*d*x + 1/2*c))) - (176*a^3*b*\tan(1/2*d \\ & *x + 1/2*c)^3 + 88*a*b^3*\tan(1/2*d*x + 1/2*c)^3 + 72*a^2*b^2*\tan(1/2*d*x + \\ & 1/2*c)^2 + 9*b^4*\tan(1/2*d*x + 1/2*c)^2 + 12*a*b^3*\tan(1/2*d*x + 1/2*c) + b \\ & ^4)/\tan(1/2*d*x + 1/2*c)^3)/d \end{aligned}$$

maple [A] time = 1.39, size = 139, normalized size = 1.30

$$a^4x + \frac{a^4c}{d} + \frac{4a^3b \ln(\csc(dx + c) - \cot(dx + c))}{d} - \frac{6a^2b^2 \cot(dx + c)}{d} - \frac{2ab^3 \cot(dx + c) \csc(dx + c)}{d} + \frac{2ab^3 \ln(\csc(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*csc(d*x+c))^4,x)`

[Out] $a^4x + 1/d*a^4*c + 4/d*a^3*b*\ln(\csc(d*x+c) - \cot(d*x+c)) - 6/d*a^2*b^2*\cot(d*x+c) - 2*a*b^3*\cot(d*x+c)*\csc(d*x+c)/d + 2/d*a*b^3*\ln(\csc(d*x+c) - \cot(d*x+c)) - 2/3/d*b^4*\cot(d*x+c) - 1/3/d*b^4*\cot(d*x+c)*\csc(d*x+c)^2$

maxima [A] time = 0.34, size = 125, normalized size = 1.17

$$a^4x + \frac{ab^3 \left(\frac{2 \cos(dx+c)}{\cos(dx+c)^2-1} - \log(\cos(dx+c)+1) + \log(\cos(dx+c)-1) \right)}{d} - \frac{4a^3b \log(\cot(dx+c) + \csc(dx+c))}{d} - \frac{2a^2b^2 \cot(dx+c)}{d} - \frac{2ab^3 \cot(dx+c) \csc(dx+c)}{d} - \frac{2b^4 \cot(dx+c)}{3d} - \frac{2b^4 \cot(dx+c) \csc(dx+c)^2}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*csc(d*x+c))^4,x, algorithm="maxima")`

[Out] $a^4x + a*b^3*(2*\cos(d*x + c)/(\cos(d*x + c)^2 - 1) - \log(\cos(d*x + c) + 1) + \log(\cos(d*x + c) - 1))/d - 4*a^3*b*\log(\cot(d*x + c) + \csc(d*x + c))/d - 6*a^2*b^2/(d*\tan(d*x + c)) - 1/3*(3*\tan(d*x + c)^2 + 1)*b^4/(d*\tan(d*x + c)^3)$

mupad [B] time = 0.62, size = 314, normalized size = 2.93

$$\frac{b^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{24d} - \frac{b^4 \cot\left(\frac{c}{2} + \frac{dx}{2}\right)^3}{24d} - \frac{3b^4 \cot\left(\frac{c}{2} + \frac{dx}{2}\right)}{8d} + \frac{3b^4 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{8d} + \frac{2a^4 \operatorname{atan}\left(\frac{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)a^3 + 4 \sin\left(\frac{c}{2} + \frac{dx}{2}\right)a^2 b}{-\sin\left(\frac{c}{2} + \frac{dx}{2}\right)a^3 + 4 \cos\left(\frac{c}{2} + \frac{dx}{2}\right)a^2 b}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/sin(c + d*x))^4,x)`

[Out] $(b^4*\tan(c/2 + (d*x)/2)^3)/(24*d) - (b^4*\cot(c/2 + (d*x)/2)^3)/(24*d) - (3*b^4*\cot(c/2 + (d*x)/2))/(8*d) + (3*b^4*\tan(c/2 + (d*x)/2))/(8*d) + (2*a^4*a*\tan((a^3*\cos(c/2 + (d*x)/2) + 2*b^3*\sin(c/2 + (d*x)/2) + 4*a^2*b*\sin(c/2 + (d*x)/2))/(2*b^3*\cos(c/2 + (d*x)/2) - a^3*\sin(c/2 + (d*x)/2) + 4*a^2*b*\cos(c/2 + (d*x)/2)))/d + (2*a*b^3*\log(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)))/d + (4*a^3*b*\log(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)))/d - (3*a^2*b^2*\cot(c/2 + (d*x)/2))/d - (a*b^3*\cot(c/2 + (d*x)/2)^2)/(2*d) + (3*a^2*b^2*\tan(c/2 + (d*x)/2))/d + (a*b^3*\tan(c/2 + (d*x)/2)^2)/(2*d)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \csc(c + dx))^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csc(d*x+c))**4,x)
```

```
[Out] Integral((a + b*csc(c + d*x))**4, x)
```

3.37 $\int (a + b \csc(c + dx))^3 dx$

Optimal. Leaf size=73

$$a^3x - \frac{b(6a^2 + b^2) \tanh^{-1}(\cos(c + dx))}{2d} - \frac{5ab^2 \cot(c + dx)}{2d} - \frac{b^2 \cot(c + dx)(a + b \csc(c + dx))}{2d}$$

[Out] $a^3x - 1/2*b*(6*a^2+b^2)*\operatorname{arctanh}(\cos(d*x+c))/d - 5/2*a*b^2*\cot(d*x+c)/d - 1/2*b^2*\cot(d*x+c)*(a+b*\csc(d*x+c))/d$

Rubi [A] time = 0.05, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3782, 3770, 3767, 8}

$$-\frac{b(6a^2 + b^2) \tanh^{-1}(\cos(c + dx))}{2d} + a^3x - \frac{5ab^2 \cot(c + dx)}{2d} - \frac{b^2 \cot(c + dx)(a + b \csc(c + dx))}{2d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Csc[c + d*x])^3, x]

[Out] $a^3x - (b*(6*a^2 + b^2)*\operatorname{ArcTanh}[\operatorname{Cos}[c + d*x]])/(2*d) - (5*a*b^2*\operatorname{Cot}[c + d*x])/(2*d) - (b^2*\operatorname{Cot}[c + d*x]*(a + b*\operatorname{Csc}[c + d*x]))/(2*d)$

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x], Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3782

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.))^(n_), x_Symbol] := -Simp[(b^2*Cot[c + d*x]*(a + b*Csc[c + d*x])^(n - 2))/(d*(n - 1)), x] + Dist[1/(n - 1), Int[(a + b*Csc[c + d*x])^(n - 3)*Simp[a^3*(n - 1) + (b*(b^2*(n - 2) + 3*a^2*(n - 1)))*Csc[c + d*x] + (a*b^2*(3*n - 4))*Csc[c + d*x]^2, x], x], x] /; F

$\text{reeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{GtQ}[n, 2] \ \&\& \ \text{IntegerQ}[2*n]$

Rubi steps

$$\begin{aligned} \int (a + b \csc(c + dx))^3 dx &= -\frac{b^2 \cot(c + dx)(a + b \csc(c + dx))}{2d} + \frac{1}{2} \int (2a^3 + b(6a^2 + b^2) \csc(c + dx) + 5ab^2 \csc^3(c + dx)) dx \\ &= a^3 x - \frac{b^2 \cot(c + dx)(a + b \csc(c + dx))}{2d} + \frac{1}{2} (5ab^2) \int \csc^2(c + dx) dx + \frac{1}{2} (b(6a^2 + b^2)) \int \csc(c + dx) dx \\ &= a^3 x - \frac{b(6a^2 + b^2) \tanh^{-1}(\cos(c + dx))}{2d} - \frac{b^2 \cot(c + dx)(a + b \csc(c + dx))}{2d} - \frac{(5ab^2) \log(\sin(\frac{1}{2}(c + dx)))}{2d} \\ &= a^3 x - \frac{b(6a^2 + b^2) \tanh^{-1}(\cos(c + dx))}{2d} - \frac{5ab^2 \cot(c + dx)}{2d} - \frac{b^2 \cot(c + dx)(a + b \csc(c + dx))}{2d} \end{aligned}$$

Mathematica [B] time = 0.63, size = 152, normalized size = 2.08

$$\frac{8a^3 c + 8a^3 dx + 24a^2 b \log\left(\sin\left(\frac{1}{2}(c + dx)\right)\right) - 24a^2 b \log\left(\cos\left(\frac{1}{2}(c + dx)\right)\right) + 12ab^2 \tan\left(\frac{1}{2}(c + dx)\right) - 12ab^2 \cot\left(\frac{1}{2}(c + dx)\right)}{8d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Csc[c + d*x])^3, x]

[Out] (8*a^3*c + 8*a^3*d*x - 12*a*b^2*Cot[(c + d*x)/2] - b^3*Csc[(c + d*x)/2]^2 - 24*a^2*b*Log[Cos[(c + d*x)/2]] - 4*b^3*Log[Cos[(c + d*x)/2]] + 24*a^2*b*Log[Sin[(c + d*x)/2]] + 4*b^3*Log[Sin[(c + d*x)/2]] + b^3*Sec[(c + d*x)/2]^2 + 12*a*b^2*Tan[(c + d*x)/2])/(8*d)

fricas [B] time = 0.55, size = 155, normalized size = 2.12

$$\frac{4a^3 dx \cos(dx + c)^2 - 4a^3 dx + 12ab^2 \cos(dx + c) \sin(dx + c) + 2b^3 \cos(dx + c) + (6a^2 b + b^3 - (6a^2 b + b^3) \cos(dx + c)) \log\left(\frac{1}{2} \cos(dx + c) + \frac{1}{2}\right) - (6a^2 b + b^3 - (6a^2 b + b^3) \cos(dx + c))^2 \log\left(-\frac{1}{2} \cos(dx + c) + \frac{1}{2}\right)}{4(d \cos(dx + c) - d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*csc(d*x+c))^3,x, algorithm="fricas")

[Out] 1/4*(4*a^3*d*x*cos(d*x + c)^2 - 4*a^3*d*x + 12*a*b^2*cos(d*x + c)*sin(d*x + c) + 2*b^3*cos(d*x + c) + (6*a^2*b + b^3 - (6*a^2*b + b^3)*cos(d*x + c))^2*log(1/2*cos(d*x + c) + 1/2) - (6*a^2*b + b^3 - (6*a^2*b + b^3)*cos(d*x + c))^2*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^2 - d)

giac [A] time = 0.47, size = 134, normalized size = 1.84

$$\frac{b^3 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^2 + 8(dx+c)a^3 + 12ab^2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 4(6a^2b + b^3) \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)\right|\right) - \frac{36a^2b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{8d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*csc(d*x+c))^3,x, algorithm="giac")

[Out] $\frac{1}{8}*(b^3*\tan(1/2*d*x + 1/2*c)^2 + 8*(d*x + c)*a^3 + 12*a*b^2*\tan(1/2*d*x + 1/2*c) + 4*(6*a^2*b + b^3)*\log(\text{abs}(\tan(1/2*d*x + 1/2*c))) - (36*a^2*b*\tan(1/2*d*x + 1/2*c)^2 + 6*b^3*\tan(1/2*d*x + 1/2*c)^2 + 12*a*b^2*\tan(1/2*d*x + 1/2*c) + b^3)/\tan(1/2*d*x + 1/2*c)^2)/d$

maple [A] time = 1.37, size = 99, normalized size = 1.36

$$a^3x + \frac{a^3c}{d} + \frac{3a^2b \ln(\csc(dx+c) - \cot(dx+c))}{d} - \frac{3ab^2 \cot(dx+c)}{d} - \frac{b^3 \csc(dx+c) \cot(dx+c)}{2d} + \frac{b^3 \ln(\csc(dx+c))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*csc(d*x+c))^3,x)

[Out] $a^3x + 1/d*a^3*c + 3/d*a^2*b*\ln(\csc(d*x+c) - \cot(d*x+c)) - 3*a*b^2*\cot(d*x+c)/d - 1/2/d*b^3*\csc(d*x+c)*\cot(d*x+c) + 1/2/d*b^3*\ln(\csc(d*x+c) - \cot(d*x+c))$

maxima [A] time = 0.34, size = 95, normalized size = 1.30

$$a^3x + \frac{b^3 \left(\frac{2 \cos(dx+c)}{\cos(dx+c)^2 - 1} - \log(\cos(dx+c) + 1) + \log(\cos(dx+c) - 1) \right)}{4d} - \frac{3a^2b \log(\cot(dx+c) + \csc(dx+c))}{d} - \frac{3ab^2 \cot(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*csc(d*x+c))^3,x, algorithm="maxima")

[Out] $a^3x + 1/4*b^3*(2*\cos(d*x + c)/(\cos(d*x + c)^2 - 1) - \log(\cos(d*x + c) + 1) + \log(\cos(d*x + c) - 1))/d - 3*a^2*b*\log(\cot(d*x + c) + \csc(d*x + c))/d - 3*a*b^2/(d*\tan(d*x + c))$

mupad [B] time = 0.42, size = 234, normalized size = 3.21

$$\frac{b^3 \tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2}{8d} - \frac{b^3 \cot\left(\frac{c}{2} + \frac{dx}{2}\right)^2}{8d} + \frac{b^3 \ln\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{2d} + \frac{2a^3 \operatorname{atan}\left(\frac{2 \cos\left(\frac{c}{2} + \frac{dx}{2}\right) a^3 + 6 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) a^2 b + \sin\left(\frac{c}{2} + \frac{dx}{2}\right) b^3}{-2 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) a^3 + 6 \cos\left(\frac{c}{2} + \frac{dx}{2}\right) a^2 b + \cos\left(\frac{c}{2} + \frac{dx}{2}\right) b^3}\right)}{d} + \frac{3ab^2 \cot\left(\frac{c}{2} + \frac{dx}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/sin(c + d*x))^3,x)`

[Out] $(b^3 \tan(c/2 + (d*x)/2)^2)/(8*d) - (b^3 \cot(c/2 + (d*x)/2)^2)/(8*d) + (b^3 \log(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)))/(2*d) + (2*a^3 \operatorname{atan}((2*a^3 \cos(c/2 + (d*x)/2) + b^3 \sin(c/2 + (d*x)/2) + 6*a^2*b \sin(c/2 + (d*x)/2)))/(b^3 \cos(c/2 + (d*x)/2) - 2*a^3 \sin(c/2 + (d*x)/2) + 6*a^2*b \cos(c/2 + (d*x)/2)))/d + (3*a^2*b \log(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)))/d - (3*a*b^2 \cot(c/2 + (d*x)/2))/(2*d) + (3*a*b^2 \tan(c/2 + (d*x)/2))/(2*d)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \csc(c + dx))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*csc(d*x+c))**3,x)`

[Out] `Integral((a + b*csc(c + d*x))**3, x)`

3.38 $\int (a + b \csc(c + dx))^2 dx$

Optimal. Leaf size=34

$$a^2x - \frac{2ab \tanh^{-1}(\cos(c + dx))}{d} - \frac{b^2 \cot(c + dx)}{d}$$

[Out] $a^2x - 2a*b*\operatorname{arctanh}(\cos(d*x+c))/d - b^2*\cot(d*x+c)/d$

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3773, 3770, 3767, 8}

$$a^2x - \frac{2ab \tanh^{-1}(\cos(c + dx))}{d} - \frac{b^2 \cot(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + b*\operatorname{Csc}[c + d*x])^2, x]$

[Out] $a^2*x - (2*a*b*\operatorname{ArcTanh}[\operatorname{Cos}[c + d*x]])/d - (b^2*\operatorname{Cot}[c + d*x])/d$

Rule 8

$\operatorname{Int}[a_, x_Symbol] \rightarrow \operatorname{Simp}[a*x, x] /; \operatorname{FreeQ}[a, x]$

Rule 3767

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \rightarrow -\operatorname{Dist}[d^{(-1)}, \operatorname{Subst}[\operatorname{Int}[\operatorname{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \operatorname{Cot}[c + d*x]], x] /; \operatorname{FreeQ}[\{c, d\}, x] \ \&\& \ \operatorname{IGtQ}[n/2, 0]$

Rule 3770

$\operatorname{Int}[\operatorname{csc}[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow -\operatorname{Simp}[\operatorname{ArcTanh}[\operatorname{Cos}[c + d*x]]/d, x] /; \operatorname{FreeQ}[\{c, d\}, x]$

Rule 3773

$\operatorname{Int}[(\operatorname{csc}[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.))^2, x_Symbol] \rightarrow \operatorname{Simp}[a^2*x, x] + (\operatorname{Dist}[2*a*b, \operatorname{Int}[\operatorname{Csc}[c + d*x], x], x] + \operatorname{Dist}[b^2, \operatorname{Int}[\operatorname{Csc}[c + d*x]^2, x], x]) /; \operatorname{FreeQ}[\{a, b, c, d\}, x]$

Rubi steps

$$\begin{aligned}
\int (a + b \csc(c + dx))^2 dx &= a^2 x + (2ab) \int \csc(c + dx) dx + b^2 \int \csc^2(c + dx) dx \\
&= a^2 x - \frac{2ab \tanh^{-1}(\cos(c + dx))}{d} - \frac{b^2 \operatorname{Subst}(\int 1 dx, x, \cot(c + dx))}{d} \\
&= a^2 x - \frac{2ab \tanh^{-1}(\cos(c + dx))}{d} - \frac{b^2 \cot(c + dx)}{d}
\end{aligned}$$

Mathematica [B] time = 0.18, size = 76, normalized size = 2.24

$$\frac{2a \left(ac + adx + 2b \log \left(\sin \left(\frac{1}{2}(c + dx) \right) \right) - 2b \log \left(\cos \left(\frac{1}{2}(c + dx) \right) \right) \right) + b^2 \tan \left(\frac{1}{2}(c + dx) \right) + b^2 \left(-\cot \left(\frac{1}{2}(c + dx) \right) \right)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Csc[c + d*x])^2, x]

[Out] $(-(b^2 \cot[(c + d*x)/2]) + 2*a*(a*c + a*d*x - 2*b*\log[\cos[(c + d*x)/2]] + 2*b*\log[\sin[(c + d*x)/2]]) + b^2*\tan[(c + d*x)/2])/(2*d)$

fricas [B] time = 0.68, size = 77, normalized size = 2.26

$$\frac{a^2 dx \sin(dx + c) - ab \log \left(\frac{1}{2} \cos(dx + c) + \frac{1}{2} \right) \sin(dx + c) + ab \log \left(-\frac{1}{2} \cos(dx + c) + \frac{1}{2} \right) \sin(dx + c) - b^2 \cos(dx + c)}{d \sin(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*csc(d*x+c))^2,x, algorithm="fricas")

[Out] $(a^2*d*x*\sin(d*x + c) - a*b*\log(1/2*\cos(d*x + c) + 1/2)*\sin(d*x + c) + a*b*\log(-1/2*\cos(d*x + c) + 1/2)*\sin(d*x + c) - b^2*\cos(d*x + c))/(d*\sin(d*x + c))$

giac [B] time = 0.86, size = 74, normalized size = 2.18

$$\frac{2(dx + c)a^2 + 4ab \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) \right| \right) + b^2 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) - \frac{4ab \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + b^2}{\tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*csc(d*x+c))^2,x, algorithm="giac")

[Out] $\frac{1}{2}*(2*(d*x + c)*a^2 + 4*a*b*\log(\text{abs}(\tan(1/2*d*x + 1/2*c)))) + b^2*\tan(1/2*d*x + 1/2*c) - (4*a*b*\tan(1/2*d*x + 1/2*c) + b^2)/\tan(1/2*d*x + 1/2*c))/d$

maple [A] time = 0.83, size = 52, normalized size = 1.53

$$a^2x - \frac{b^2 \cot(dx + c)}{d} + \frac{2ab \ln(\csc(dx + c) - \cot(dx + c))}{d} + \frac{a^2c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*csc(d*x+c))^2,x)`

[Out] $a^2*x - 1/d*b^2*\cot(d*x+c) + 2/d*a*b*\ln(\csc(d*x+c) - \cot(d*x+c)) + 1/d*a^2*c$

maxima [A] time = 0.36, size = 43, normalized size = 1.26

$$a^2x - \frac{2ab \log(\cot(dx + c) + \csc(dx + c))}{d} - \frac{b^2}{d \tan(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*csc(d*x+c))^2,x, algorithm="maxima")`

[Out] $a^2*x - 2*a*b*\log(\cot(d*x + c) + \csc(d*x + c))/d - b^2/(d*\tan(d*x + c))$

mupad [B] time = 0.33, size = 105, normalized size = 3.09

$$\frac{2a^2 \operatorname{atan}\left(\frac{a \cos\left(\frac{c}{2} + \frac{dx}{2}\right) + 2b \sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{2b \cos\left(\frac{c}{2} + \frac{dx}{2}\right) - a \sin\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d} - \frac{b^2 \cot(c + dx)}{d} + \frac{2ab \ln\left(\frac{\sin\left(\frac{c}{2} + \frac{dx}{2}\right)}{\cos\left(\frac{c}{2} + \frac{dx}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/sin(c + d*x))^2,x)`

[Out] $(2*a^2*\operatorname{atan}((a*\cos(c/2 + (d*x)/2) + 2*b*\sin(c/2 + (d*x)/2))/(2*b*\cos(c/2 + (d*x)/2) - a*\sin(c/2 + (d*x)/2)))/d - (b^2*\cot(c + d*x))/d + (2*a*b*\log(\sin(c/2 + (d*x)/2)/\cos(c/2 + (d*x)/2)))/d$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \csc(c + dx))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*csc(d*x+c))**2,x)`

[Out] `Integral((a + b*csc(c + d*x))**2, x)`

$$3.39 \quad \int \frac{\csc^5(x)}{a+b \csc(x)} dx$$

Optimal. Leaf size=112

$$\frac{a(2a^2 + b^2) \tanh^{-1}(\cos(x))}{2b^4} - \frac{(3a^2 + 2b^2) \cot(x)}{3b^3} - \frac{2a^4 \tanh^{-1}\left(\frac{a+b \tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^4 \sqrt{a^2-b^2}} + \frac{a \cot(x) \csc(x)}{2b^2} - \frac{\cot(x) \csc^2(x)}{3b}$$

[Out] $1/2*a*(2*a^2+b^2)*\operatorname{arctanh}(\cos(x))/b^4-1/3*(3*a^2+2*b^2)*\cot(x)/b^3+1/2*a*\cot(x)*\csc(x)/b^2-1/3*\cot(x)*\csc(x)^2/b-2*a^4*\operatorname{arctanh}\left(\frac{a+b*\tan(1/2*x)}{a^2-b^2}\right)/b^4/(a^2-b^2)^{(1/2)}/b^4/(a^2-b^2)^{(1/2)}$

Rubi [A] time = 0.41, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {3851, 4092, 4082, 3998, 3770, 3831, 2660, 618, 206}

$$-\frac{(3a^2 + 2b^2) \cot(x)}{3b^3} + \frac{a(2a^2 + b^2) \tanh^{-1}(\cos(x))}{2b^4} - \frac{2a^4 \tanh^{-1}\left(\frac{a+b \tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^4 \sqrt{a^2-b^2}} + \frac{a \cot(x) \csc(x)}{2b^2} - \frac{\cot(x) \csc^2(x)}{3b}$$

Antiderivative was successfully verified.

[In] Int[Csc[x]^5/(a + b*Csc[x]),x]

[Out] $(a*(2*a^2 + b^2)*\operatorname{ArcTanh}[\operatorname{Cos}[x]])/(2*b^4) - (2*a^4*\operatorname{ArcTanh}[(a + b*\operatorname{Tan}[x/2])/\operatorname{Sqrt}[a^2 - b^2]])/(b^4*\operatorname{Sqrt}[a^2 - b^2]) - ((3*a^2 + 2*b^2)*\operatorname{Cot}[x])/(3*b^3) + (a*\operatorname{Cot}[x]*\operatorname{Csc}[x])/(2*b^2) - (\operatorname{Cot}[x]*\operatorname{Csc}[x]^2)/(3*b)$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 2660

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*

e^{2*x^2}), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3831

Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] := Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3851

Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] := -Simp[(d^3*Cot[e + f*x]*(d*Csc[e + f*x])^(n - 3))/(b*f*(n - 2)), x] + Dist[d^3/(b*(n - 2)), Int[((d*Csc[e + f*x])^(n - 3)*Simp[a*(n - 3) + b*(n - 3)*Csc[e + f*x] - a*(n - 2)*Csc[e + f*x]^2, x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 3]

Rule 3998

Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] := Dist[B/b, Int[Csc[e + f*x], x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]

Rule 4082

Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] := -Simp[(C*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 4092

Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] := -Simp[(C*Csc[e + f*x]*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e + f


```
*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(m + 3))*Csc[e + f*x]^2, x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && N
eQ[a^2 - b^2, 0] && !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^5(x)}{a + b \csc(x)} dx &= -\frac{\cot(x) \csc^2(x)}{3b} + \frac{\int \frac{\csc^2(x)(2a+2b \csc(x)-3a \csc^2(x))}{a+b \csc(x)} dx}{3b} \\
&= \frac{a \cot(x) \csc(x)}{2b^2} - \frac{\cot(x) \csc^2(x)}{3b} + \frac{\int \frac{\csc(x)(-3a^2+ab \csc(x)+2(3a^2+2b^2) \csc^2(x))}{a+b \csc(x)} dx}{6b^2} \\
&= -\frac{(3a^2 + 2b^2) \cot(x)}{3b^3} + \frac{a \cot(x) \csc(x)}{2b^2} - \frac{\cot(x) \csc^2(x)}{3b} + \frac{\int \frac{\csc(x)(-3a^2b-3a(2a^2+b^2) \csc(x))}{a+b \csc(x)} dx}{6b^3} \\
&= -\frac{(3a^2 + 2b^2) \cot(x)}{3b^3} + \frac{a \cot(x) \csc(x)}{2b^2} - \frac{\cot(x) \csc^2(x)}{3b} + \frac{a^4 \int \frac{\csc(x)}{a+b \csc(x)} dx}{b^4} - \frac{a(2a^2 + b^2)}{2b^4} \\
&= \frac{a(2a^2 + b^2) \tanh^{-1}(\cos(x))}{2b^4} - \frac{(3a^2 + 2b^2) \cot(x)}{3b^3} + \frac{a \cot(x) \csc(x)}{2b^2} - \frac{\cot(x) \csc^2(x)}{3b} + \frac{a^4}{2b^4} \\
&= \frac{a(2a^2 + b^2) \tanh^{-1}(\cos(x))}{2b^4} - \frac{(3a^2 + 2b^2) \cot(x)}{3b^3} + \frac{a \cot(x) \csc(x)}{2b^2} - \frac{\cot(x) \csc^2(x)}{3b} + \frac{(2a^4)}{2b^4} \\
&= \frac{a(2a^2 + b^2) \tanh^{-1}(\cos(x))}{2b^4} - \frac{(3a^2 + 2b^2) \cot(x)}{3b^3} + \frac{a \cot(x) \csc(x)}{2b^2} - \frac{\cot(x) \csc^2(x)}{3b} - \frac{(4a^4)}{2b^4} \\
&= \frac{a(2a^2 + b^2) \tanh^{-1}(\cos(x))}{2b^4} - \frac{2a^4 \tanh^{-1}\left(\frac{b\left(\frac{a}{b} + \tan\left(\frac{x}{2}\right)\right)}{\sqrt{a^2 - b^2}}\right)}{b^4 \sqrt{a^2 - b^2}} - \frac{(3a^2 + 2b^2) \cot(x)}{3b^3} + \frac{a \cot(x) \csc(x)}{2b^2}
\end{aligned}$$

Mathematica [A] time = 1.73, size = 125, normalized size = 1.12

$$\frac{b(3a^2 + 2b^2) \cos(3x) \csc^3(x) - 3b \cot(x) \csc(x) \left((a^2 + 2b^2) \csc(x) - 2ab \right) + 6a(2a^2 + b^2) \left(\log\left(\cos\left(\frac{x}{2}\right)\right) - \log\left(\cos(x)\right) \right)}{12b^4}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[x]^5/(a + b*Csc[x]),x]

[Out] $((24a^4 \operatorname{ArcTan}[(a + b \tan(x/2))/\sqrt{-a^2 + b^2}])/\sqrt{-a^2 + b^2} + b(3a^2 + 2b^2) \cos(3x) \operatorname{Csc}[x]^3 - 3b \operatorname{Cot}[x] \operatorname{Csc}[x](-2ab + (a^2 + 2b^2) \operatorname{Csc}[x]) + 6a(2a^2 + b^2)(\operatorname{Log}[\cos(x/2)] - \operatorname{Log}[\sin(x/2)]))/ (12b^4)$

fricas [B] time = 0.74, size = 607, normalized size = 5.42

$$\frac{4(3a^4b - a^2b^3 - 2b^5) \cos(x)^3 - 6(a^4 \cos(x)^2 - a^4) \sqrt{a^2 - b^2} \log\left(-\frac{(a^2 - 2b^2) \cos(x)^2 + 2ab \sin(x) + a^2 + b^2 - 2(b \cos(x) \sin(x) + a \cos(x)) \sqrt{a^2 - b^2}}{a^2 \cos(x)^2 - 2ab \sin(x) - a^2 - b^2}\right)}{12b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^5/(a+b*csc(x)),x, algorithm="fricas")

[Out] $[1/12*(4*(3a^4*b - a^2*b^3 - 2*b^5)*\cos(x)^3 - 6*(a^4*\cos(x)^2 - a^4)*\sqrt{a^2 - b^2}*\log(-((a^2 - 2*b^2)*\cos(x)^2 + 2*a*b*\sin(x) + a^2 + b^2 - 2*(b*\cos(x)*\sin(x) + a*\cos(x))*\sqrt{a^2 - b^2}))/ (a^2*\cos(x)^2 - 2*a*b*\sin(x) - a^2 - b^2))*\sin(x) + 6*(a^3*b^2 - a*b^4)*\cos(x)*\sin(x) + 3*(2*a^5 - a^3*b^2 - a*b^4 - (2*a^5 - a^3*b^2 - a*b^4)*\cos(x)^2)*\log(1/2*\cos(x) + 1/2)*\sin(x) - 3*(2*a^5 - a^3*b^2 - a*b^4 - (2*a^5 - a^3*b^2 - a*b^4)*\cos(x)^2)*\log(-1/2*\cos(x) + 1/2)*\sin(x) - 12*(a^4*b - b^5)*\cos(x))/((a^2*b^4 - b^6 - (a^2*b^4 - b^6)*\cos(x)^2)*\sin(x)), 1/12*(4*(3a^4*b - a^2*b^3 - 2*b^5)*\cos(x)^3 + 12*(a^4*\cos(x)^2 - a^4)*\sqrt{-a^2 + b^2}*\arctan(-\sqrt{-a^2 + b^2}*(b*\sin(x) + a))/((a^2 - b^2)*\cos(x))*\sin(x) + 6*(a^3*b^2 - a*b^4)*\cos(x)*\sin(x) + 3*(2*a^5 - a^3*b^2 - a*b^4 - (2*a^5 - a^3*b^2 - a*b^4)*\cos(x)^2)*\log(1/2*\cos(x) + 1/2)*\sin(x) - 3*(2*a^5 - a^3*b^2 - a*b^4 - (2*a^5 - a^3*b^2 - a*b^4)*\cos(x)^2)*\log(-1/2*\cos(x) + 1/2)*\sin(x) - 12*(a^4*b - b^5)*\cos(x))/((a^2*b^4 - b^6 - (a^2*b^4 - b^6)*\cos(x)^2)*\sin(x))]$

giac [A] time = 0.66, size = 194, normalized size = 1.73

$$\frac{2\left(\pi\left[\frac{x}{2\pi} + \frac{1}{2}\right] \operatorname{sgn}(b) + \arctan\left(\frac{b \tan\left(\frac{1}{2}x\right) + a}{\sqrt{-a^2 + b^2}}\right)\right) a^4}{\sqrt{-a^2 + b^2} b^4} + \frac{b^2 \tan\left(\frac{1}{2}x\right)^3 - 3ab \tan\left(\frac{1}{2}x\right)^2 + 12a^2 \tan\left(\frac{1}{2}x\right) + 9b^2 \tan\left(\frac{1}{2}x\right)}{24b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^5/(a+b*csc(x)),x, algorithm="giac")

[Out] $2*(\pi*\operatorname{floor}(1/2*x/\pi + 1/2)*\operatorname{sgn}(b) + \arctan((b*\tan(1/2*x) + a)/\sqrt{-a^2 + b^2}))*a^4/(\sqrt{-a^2 + b^2}*b^4) + 1/24*(b^2*\tan(1/2*x)^3 - 3*a*b*\tan(1/2*x)^2 + 12*a^2*\tan(1/2*x) + 9*b^2*\tan(1/2*x))/b^3 - 1/2*(2*a^3 + a*b^2)*\log(\dots)$

$\text{abs}(\tan(1/2*x))/b^4 + 1/24*(44*a^3*\tan(1/2*x)^3 + 22*a*b^2*\tan(1/2*x)^3 - 12*a^2*b*\tan(1/2*x)^2 - 9*b^3*\tan(1/2*x)^2 + 3*a*b^2*\tan(1/2*x) - b^3)/(b^4*\tan(1/2*x)^3)$

maple [A] time = 0.27, size = 162, normalized size = 1.45

$$\frac{\tan^3\left(\frac{x}{2}\right)}{24b} - \frac{a\left(\tan^2\left(\frac{x}{2}\right)\right)}{8b^2} + \frac{a^2\tan\left(\frac{x}{2}\right)}{2b^3} + \frac{3\tan\left(\frac{x}{2}\right)}{8b} + \frac{2a^4\arctan\left(\frac{2\tan\left(\frac{x}{2}\right)b+2a}{2\sqrt{-a^2+b^2}}\right)}{b^4\sqrt{-a^2+b^2}} - \frac{1}{24b\tan\left(\frac{x}{2}\right)^3} - \frac{a^2}{2b^3\tan\left(\frac{x}{2}\right)} - \frac{3}{8b\tan\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csc(x)^5/(a+b*csc(x)),x)`

[Out] $1/24/b*\tan(1/2*x)^3 - 1/8/b^2*a*\tan(1/2*x)^2 + 1/2/b^3*a^2*\tan(1/2*x) + 3/8/b*\tan(1/2*x) + 2/b^4*a^4/(-a^2+b^2)^{(1/2)}*\arctan(1/2*(2*\tan(1/2*x)*b+2*a)/(-a^2+b^2)^{(1/2)}) - 1/24/b/\tan(1/2*x)^3 - 1/2/b^3/\tan(1/2*x)*a^2 - 3/8/b/\tan(1/2*x) + 1/8*a/b^2/\tan(1/2*x)^2 - 1/b^4*a^3*\ln(\tan(1/2*x)) - 1/2/b^2*a*\ln(\tan(1/2*x))$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(x)^5/(a+b*csc(x)),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details)Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 0.79, size = 588, normalized size = 5.25

$$b^2 \left(\frac{3a \sin(2x) \sqrt{a^2-b^2}}{4} + \frac{3a \sin(3x) \ln\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right) \sqrt{a^2-b^2}}{8} - \frac{9a \ln\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right) \sin(x) \sqrt{a^2-b^2}}{8} \right) + b^3 \left(\frac{\cos(3x) \sqrt{a^2-b^2}}{2} - \frac{3 \cos(x) \sqrt{a^2-b^2}}{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(sin(x)^5*(a + b/sin(x))),x)`

[Out] $-(b^2*((3*a*\sin(2*x)*(a^2 - b^2)^{(1/2)})/4 + (3*a*\sin(3*x)*\log(\sin(x/2)/\cos(x/2))*(a^2 - b^2)^{(1/2)})/8 - (9*a*\log(\sin(x/2)/\cos(x/2))*\sin(x)*(a^2 - b^2))$

```

^(1/2))/8) + b^3*((cos(3*x)*(a^2 - b^2)^(1/2))/2 - (3*cos(x)*(a^2 - b^2)^(1/2))/2)/2) - b*((3*a^2*cos(x)*(a^2 - b^2)^(1/2))/4 - (3*a^2*cos(3*x)*(a^2 - b^2)^(1/2))/4) + (a^4*atan((a^4*sin(x/2)*(a^2 - b^2)^(1/2)*8i - b^4*sin(x/2)*(a^2 - b^2)^(1/2)*1i + a*b^3*cos(x/2)*(a^2 - b^2)^(1/2)*1i + a^3*b*cos(x/2)*(a^2 - b^2)^(1/2)*4i)/(b^5*cos(x/2) - 8*a^5*sin(x/2) + a^2*b^3*cos(x/2) + 4*a^3*b^2*sin(x/2) - 4*a^4*b*cos(x/2) + 2*a*b^4*sin(x/2)))*sin(x)*9i)/2 - (a^4*atan((a^4*sin(x/2)*(a^2 - b^2)^(1/2)*8i - b^4*sin(x/2)*(a^2 - b^2)^(1/2)*1i + a*b^3*cos(x/2)*(a^2 - b^2)^(1/2)*1i + a^3*b*cos(x/2)*(a^2 - b^2)^(1/2)*4i)/(b^5*cos(x/2) - 8*a^5*sin(x/2) + a^2*b^3*cos(x/2) + 4*a^3*b^2*sin(x/2) - 4*a^4*b*cos(x/2) + 2*a*b^4*sin(x/2)))*sin(3*x)*3i)/2 - (9*a^3*log(sin(x/2)/cos(x/2))*sin(x)*(a^2 - b^2)^(1/2))/4 + (3*a^3*sin(3*x)*log(sin(x/2)/cos(x/2))*(a^2 - b^2)^(1/2))/4)/((3*b^4*sin(3*x)*(a^2 - b^2)^(1/2))/4 - (9*b^4*sin(x)*(a^2 - b^2)^(1/2))/4)

```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^5(x)}{a + b \csc(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)**5/(a+b*csc(x)),x)

[Out] Integral(csc(x)**5/(a + b*csc(x)), x)

$$3.40 \quad \int \frac{\csc^4(x)}{a+b \csc(x)} dx$$

Optimal. Leaf size=84

$$-\frac{(2a^2 + b^2) \tanh^{-1}(\cos(x))}{2b^3} + \frac{2a^3 \tanh^{-1}\left(\frac{a+b \tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^3 \sqrt{a^2-b^2}} + \frac{a \cot(x)}{b^2} - \frac{\cot(x) \csc(x)}{2b}$$

[Out] $-1/2*(2*a^2+b^2)*\operatorname{arctanh}(\cos(x))/b^3+a*\cot(x)/b^2-1/2*\cot(x)*\csc(x)/b+2*a^3*\operatorname{arctanh}\left(\frac{a+b*\tan(1/2*x)}{\sqrt{a^2-b^2}}\right)/b^3/\sqrt{a^2-b^2}$

Rubi [A] time = 0.25, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {3851, 4082, 3998, 3770, 3831, 2660, 618, 206}

$$-\frac{(2a^2 + b^2) \tanh^{-1}(\cos(x))}{2b^3} + \frac{2a^3 \tanh^{-1}\left(\frac{a+b \tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^3 \sqrt{a^2-b^2}} + \frac{a \cot(x)}{b^2} - \frac{\cot(x) \csc(x)}{2b}$$

Antiderivative was successfully verified.

[In] Int[Csc[x]^4/(a + b*Csc[x]), x]

[Out] $-((2*a^2 + b^2)*\operatorname{ArcTanh}[\cos(x)])/(2*b^3) + (2*a^3*\operatorname{ArcTanh}[(a + b*\tan[x/2])/Sqrt[a^2 - b^2]])/(b^3*Sqrt[a^2 - b^2]) + (a*\cot[x])/b^2 - (\cot[x]*\csc[x])/(2*b)$

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 2660

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[

$a^2 - b^2, 0]$

Rule 3770

$\text{Int}[\text{csc}[(c_.) + (d_.)(x_.)], x_Symbol] \text{ :> } -\text{Simp}[\text{ArcTanh}[\text{Cos}[c + d*x]]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 3831

$\text{Int}[\text{csc}[(e_.) + (f_.)(x_.)]/(\text{csc}[(e_.) + (f_.)(x_.)]*(b_.) + (a_.)), x_Symbol] \text{ :> } \text{Dist}[1/b, \text{Int}[1/(1 + (a*\text{Sin}[e + f*x])/b), x], x] /; \text{FreeQ}[\{a, b, e, f\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0]$

Rule 3851

$\text{Int}[(\text{csc}[(e_.) + (f_.)(x_.)]*(d_.))^{(n_.)}/(\text{csc}[(e_.) + (f_.)(x_.)]*(b_.) + (a_.)), x_Symbol] \text{ :> } -\text{Simp}[(d^3*\text{Cot}[e + f*x]*(d*\text{Csc}[e + f*x])^{(n-3)})/(b*f*(n-2)), x] + \text{Dist}[d^3/(b*(n-2)), \text{Int}[(d*\text{Csc}[e + f*x])^{(n-3)}*\text{Simp}[a*(n-3) + b*(n-3)*\text{Csc}[e + f*x] - a*(n-2)*\text{Csc}[e + f*x]^2, x]]/(a + b*\text{Csc}[e + f*x]), x], x] /; \text{FreeQ}[\{a, b, d, e, f\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0] \ \&\& \ \text{GtQ}[n, 3]$

Rule 3998

$\text{Int}[(\text{csc}[(e_.) + (f_.)(x_.)]*(\text{csc}[(e_.) + (f_.)(x_.)]*(B_.) + (A_.)))/(\text{csc}[(e_.) + (f_.)(x_.)]*(b_.) + (a_.)), x_Symbol] \text{ :> } \text{Dist}[B/b, \text{Int}[\text{Csc}[e + f*x], x], x] + \text{Dist}[(A*b - a*B)/b, \text{Int}[\text{Csc}[e + f*x]/(a + b*\text{Csc}[e + f*x]), x], x] /; \text{FreeQ}[\{a, b, e, f, A, B\}, x] \ \&\& \ \text{NeQ}[A*b - a*B, 0]$

Rule 4082

$\text{Int}[\text{csc}[(e_.) + (f_.)(x_.)]*((A_.) + \text{csc}[(e_.) + (f_.)(x_.)]*(B_.) + \text{csc}[(e_.) + (f_.)(x_.)]^2*(C_.))*(\text{csc}[(e_.) + (f_.)(x_.)]*(b_.) + (a_.))^{(m_.)}, x_Symbol] \text{ :> } -\text{Simp}[(C*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^{(m+1)})/(b*f*(m+2)), x] + \text{Dist}[1/(b*(m+2)), \text{Int}[\text{Csc}[e + f*x]*(a + b*\text{Csc}[e + f*x])^m*\text{Simp}[b*A*(m+2) + b*C*(m+1) + (b*B*(m+2) - a*C)*\text{Csc}[e + f*x], x], x], x] /; \text{FreeQ}[\{a, b, e, f, A, B, C, m\}, x] \ \&\& \ !\text{LtQ}[m, -1]$

Rubi steps

$$\begin{aligned}
\int \frac{\csc^4(x)}{a + b \csc(x)} dx &= -\frac{\cot(x) \csc(x)}{2b} + \frac{\int \frac{\csc(x)(a+b \csc(x)-2a \csc^2(x))}{a+b \csc(x)} dx}{2b} \\
&= \frac{a \cot(x)}{b^2} - \frac{\cot(x) \csc(x)}{2b} + \frac{\int \frac{\csc(x)(ab+(2a^2+b^2) \csc(x))}{a+b \csc(x)} dx}{2b^2} \\
&= \frac{a \cot(x)}{b^2} - \frac{\cot(x) \csc(x)}{2b} - \frac{a^3 \int \frac{\csc(x)}{a+b \csc(x)} dx}{b^3} + \frac{(2a^2 + b^2) \int \csc(x) dx}{2b^3} \\
&= -\frac{(2a^2 + b^2) \tanh^{-1}(\cos(x))}{2b^3} + \frac{a \cot(x)}{b^2} - \frac{\cot(x) \csc(x)}{2b} - \frac{a^3 \int \frac{1}{1+\frac{a \sin(x)}{b}} dx}{b^4} \\
&= -\frac{(2a^2 + b^2) \tanh^{-1}(\cos(x))}{2b^3} + \frac{a \cot(x)}{b^2} - \frac{\cot(x) \csc(x)}{2b} - \frac{(2a^3) \text{Subst} \left(\int \frac{1}{1+\frac{2ax}{b}+x^2} dx, x, \tan \frac{x}{2} \right)}{b^4} \\
&= -\frac{(2a^2 + b^2) \tanh^{-1}(\cos(x))}{2b^3} + \frac{a \cot(x)}{b^2} - \frac{\cot(x) \csc(x)}{2b} + \frac{(4a^3) \text{Subst} \left(\int \frac{1}{-4\left(1-\frac{a^2}{b^2}\right)-x^2} dx, x, \tan \frac{x}{2} \right)}{b^4} \\
&= -\frac{(2a^2 + b^2) \tanh^{-1}(\cos(x))}{2b^3} + \frac{2a^3 \tanh^{-1} \left(\frac{b\left(\frac{a}{b} + \tan\left(\frac{x}{2}\right)\right)}{\sqrt{a^2-b^2}} \right)}{b^3 \sqrt{a^2-b^2}} + \frac{a \cot(x)}{b^2} - \frac{\cot(x) \csc(x)}{2b}
\end{aligned}$$

Mathematica [A] time = 0.50, size = 144, normalized size = 1.71

$$\frac{8a^2 \log\left(\sin\left(\frac{x}{2}\right)\right) - 8a^2 \log\left(\cos\left(\frac{x}{2}\right)\right) - \frac{16a^3 \tan^{-1}\left(\frac{a+b \tan\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} - 4ab \tan\left(\frac{x}{2}\right) + 4ab \cot\left(\frac{x}{2}\right) - b^2 \csc^2\left(\frac{x}{2}\right) + b^2 \sec^2\left(\frac{x}{2}\right)}{8b^3}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[x]^4/(a + b*Csc[x]), x]

[Out] ((-16*a^3*ArcTan[(a + b*Tan[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + 4*a*b*Cot[x/2] - b^2*Csc[x/2]^2 - 8*a^2*Log[Cos[x/2]] - 4*b^2*Log[Cos[x/2]] + 8*a^2*Log[Sin[x/2]] + 4*b^2*Log[Sin[x/2]] + b^2*Sec[x/2]^2 - 4*a*b*Tan[x/2])/(8*b^3)

fricas [B] time = 0.87, size = 524, normalized size = 6.24

$$\left[\frac{4(a^3b - ab^3)\cos(x)\sin(x) - 2(a^3\cos(x)^2 - a^3)\sqrt{a^2 - b^2} \log\left(\frac{(a^2 - 2b^2)\cos(x)^2 + 2ab\sin(x) + a^2 + b^2 + 2(b\cos(x)\sin(x) + a\cos(x))}{a^2\cos(x)^2 - 2ab\sin(x) - a^2 - b^2}\right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^4/(a+b*csc(x)),x, algorithm="fricas")

[Out] [1/4*(4*(a^3*b - a*b^3)*cos(x)*sin(x) - 2*(a^3*cos(x)^2 - a^3)*sqrt(a^2 - b^2)*log(((a^2 - 2*b^2)*cos(x)^2 + 2*a*b*sin(x) + a^2 + b^2 + 2*(b*cos(x)*sin(x) + a*cos(x))*sqrt(a^2 - b^2))/(a^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) - 2*(a^2*b^2 - b^4)*cos(x) - (2*a^4 - a^2*b^2 - b^4 - (2*a^4 - a^2*b^2 - b^4)*cos(x)^2)*log(1/2*cos(x) + 1/2) + (2*a^4 - a^2*b^2 - b^4 - (2*a^4 - a^2*b^2 - b^4)*cos(x)^2)*log(-1/2*cos(x) + 1/2))/(a^2*b^3 - b^5 - (a^2*b^3 - b^5)*cos(x)^2), 1/4*(4*(a^3*b - a*b^3)*cos(x)*sin(x) - 4*(a^3*cos(x)^2 - a^3)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*sin(x) + a)/((a^2 - b^2)*cos(x))) - 2*(a^2*b^2 - b^4)*cos(x) - (2*a^4 - a^2*b^2 - b^4 - (2*a^4 - a^2*b^2 - b^4)*cos(x)^2)*log(1/2*cos(x) + 1/2) + (2*a^4 - a^2*b^2 - b^4 - (2*a^4 - a^2*b^2 - b^4)*cos(x)^2)*log(-1/2*cos(x) + 1/2))/(a^2*b^3 - b^5 - (a^2*b^3 - b^5)*cos(x)^2)]

giac [A] time = 0.53, size = 141, normalized size = 1.68

$$\frac{2\left(\pi\left[\frac{x}{2\pi} + \frac{1}{2}\right]\operatorname{sgn}(b) + \arctan\left(\frac{b\tan\left(\frac{1}{2}x\right) + a}{\sqrt{-a^2 + b^2}}\right)\right)a^3}{\sqrt{-a^2 + b^2}b^3} + \frac{b\tan\left(\frac{1}{2}x\right)^2 - 4a\tan\left(\frac{1}{2}x\right)}{8b^2} + \frac{(2a^2 + b^2)\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{2b^3} - \frac{12a}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^4/(a+b*csc(x)),x, algorithm="giac")

[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(b) + arctan((b*tan(1/2*x) + a)/sqrt(-a^2 + b^2)))*a^3/(sqrt(-a^2 + b^2)*b^3) + 1/8*(b*tan(1/2*x)^2 - 4*a*tan(1/2*x))/b^2 + 1/2*(2*a^2 + b^2)*log(abs(tan(1/2*x)))/b^3 - 1/8*(12*a^2*tan(1/2*x)^2 + 6*b^2*tan(1/2*x)^2 - 4*a*b*tan(1/2*x) + b^2)/(b^3*tan(1/2*x)^2)

maple [A] time = 0.23, size = 112, normalized size = 1.33

$$\frac{\tan^2\left(\frac{x}{2}\right)}{8b} - \frac{a\tan\left(\frac{x}{2}\right)}{2b^2} - \frac{2a^3\arctan\left(\frac{2\tan\left(\frac{x}{2}\right)b + 2a}{2\sqrt{-a^2 + b^2}}\right)}{b^3\sqrt{-a^2 + b^2}} - \frac{1}{8b\tan\left(\frac{x}{2}\right)^2} + \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)a^2}{b^3} + \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{2b} + \frac{a}{2b^2\tan\left(\frac{x}{2}\right)}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^4(x)}{a + b \csc(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)**4/(a+b*csc(x)),x)

[Out] Integral(csc(x)**4/(a + b*csc(x)), x)

$$3.41 \quad \int \frac{\csc^3(x)}{a+b \csc(x)} dx$$

Optimal. Leaf size=62

$$-\frac{2a^2 \tanh^{-1}\left(\frac{a+b \tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^2 \sqrt{a^2-b^2}} + \frac{a \tanh^{-1}(\cos(x))}{b^2} - \frac{\cot(x)}{b}$$

[Out] a*arctanh(cos(x))/b^2-cot(x)/b-2*a^2*arctanh((a+b*tan(1/2*x))/(a^2-b^2)^(1/2))/b^2/(a^2-b^2)^(1/2)

Rubi [A] time = 0.15, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3790, 3789, 3770, 3831, 2660, 618, 206}

$$-\frac{2a^2 \tanh^{-1}\left(\frac{a+b \tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b^2 \sqrt{a^2-b^2}} + \frac{a \tanh^{-1}(\cos(x))}{b^2} - \frac{\cot(x)}{b}$$

Antiderivative was successfully verified.

[In] Int[Csc[x]^3/(a + b*Csc[x]),x]

[Out] (a*ArcTanh[Cos[x]])/b^2 - (2*a^2*ArcTanh[(a + b*Tan[x/2])/Sqrt[a^2 - b^2]])/(b^2*Sqrt[a^2 - b^2]) - Cot[x]/b

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 2660

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x]
  /; FreeQ[{c, d}, x]
```

Rule 3789

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] := Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3790

```
Int[csc[(e_.) + (f_.)*(x_)]^3/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] := -Simp[Cot[e + f*x]/(b*f), x] - Dist[a/b, Int[Csc[e + f*x]^2/(a + b*
Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] := Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^3(x)}{a + b \csc(x)} dx &= -\frac{\cot(x)}{b} - \frac{a \int \frac{\csc^2(x)}{a+b \csc(x)} dx}{b} \\
&= -\frac{\cot(x)}{b} - \frac{a \int \csc(x) dx}{b^2} + \frac{a^2 \int \frac{\csc(x)}{a+b \csc(x)} dx}{b^2} \\
&= \frac{a \tanh^{-1}(\cos(x))}{b^2} - \frac{\cot(x)}{b} + \frac{a^2 \int \frac{1}{1 + \frac{a \sin(x)}{b}} dx}{b^3} \\
&= \frac{a \tanh^{-1}(\cos(x))}{b^2} - \frac{\cot(x)}{b} + \frac{(2a^2) \text{Subst} \left(\int \frac{1}{1 + \frac{2ax}{b} + x^2} dx, x, \tan\left(\frac{x}{2}\right) \right)}{b^3} \\
&= \frac{a \tanh^{-1}(\cos(x))}{b^2} - \frac{\cot(x)}{b} - \frac{(4a^2) \text{Subst} \left(\int \frac{1}{-4\left(1 - \frac{a^2}{b^2}\right) - x^2} dx, x, \frac{2a}{b} + 2 \tan\left(\frac{x}{2}\right) \right)}{b^3} \\
&= \frac{a \tanh^{-1}(\cos(x))}{b^2} - \frac{2a^2 \tanh^{-1} \left(\frac{b\left(\frac{a}{b} + \tan\left(\frac{x}{2}\right)\right)}{\sqrt{a^2 - b^2}} \right)}{b^2 \sqrt{a^2 - b^2}} - \frac{\cot(x)}{b}
\end{aligned}$$

Mathematica [A] time = 0.20, size = 106, normalized size = 1.71

$$\frac{\csc\left(\frac{x}{2}\right) \sec\left(\frac{x}{2}\right) \left(2a^2 \sin(x) \tan^{-1} \left(\frac{a+b \tan\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}} \right) + \sqrt{b^2-a^2} \left(a \sin(x) \left(\log\left(\cos\left(\frac{x}{2}\right)\right) - \log\left(\sin\left(\frac{x}{2}\right)\right) \right) - b \cos(x) \right) \right)}{2b^2 \sqrt{b^2-a^2}}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[x]^3/(a + b*Csc[x]), x]

[Out] (Csc[x/2]*Sec[x/2]*(2*a^2*ArcTan[(a + b*Tan[x/2])/Sqrt[-a^2 + b^2]]*Sin[x] + Sqrt[-a^2 + b^2]*(-(b*Cos[x]) + a*(Log[Cos[x/2]] - Log[Sin[x/2]]))*Sin[x]))/(2*b^2*Sqrt[-a^2 + b^2])

fricas [B] time = 0.67, size = 308, normalized size = 4.97

$$\left[\frac{\sqrt{a^2 - b^2} a^2 \log \left(-\frac{(a^2 - 2b^2) \cos(x)^2 + 2ab \sin(x) + a^2 + b^2 - 2(b \cos(x) \sin(x) + a \cos(x)) \sqrt{a^2 - b^2}}{a^2 \cos(x)^2 - 2ab \sin(x) - a^2 - b^2} \right) \sin(x) + (a^3 - ab^2) \log \left(\frac{1}{2} \cos(x) + \right)}{2(a^2 b^2 - b^4) \sin(x)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^3/(a+b*csc(x)),x, algorithm="fricas")

[Out] [1/2*(sqrt(a^2 - b^2)*a^2*log(-((a^2 - 2*b^2)*cos(x)^2 + 2*a*b*sin(x) + a^2 + b^2 - 2*(b*cos(x)*sin(x) + a*cos(x))*sqrt(a^2 - b^2)))/(a^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2))*sin(x) + (a^3 - a*b^2)*log(1/2*cos(x) + 1/2)*sin(x) - (a^3 - a*b^2)*log(-1/2*cos(x) + 1/2)*sin(x) - 2*(a^2*b - b^3)*cos(x))/((a^2*b^2 - b^4)*sin(x)), -1/2*(2*sqrt(-a^2 + b^2)*a^2*arctan(-sqrt(-a^2 + b^2)*(b*sin(x) + a)/((a^2 - b^2)*cos(x)))*sin(x) - (a^3 - a*b^2)*log(1/2*cos(x) + 1/2)*sin(x) + (a^3 - a*b^2)*log(-1/2*cos(x) + 1/2)*sin(x) + 2*(a^2*b - b^3)*cos(x))/((a^2*b^2 - b^4)*sin(x))]

giac [A] time = 0.35, size = 98, normalized size = 1.58

$$\frac{2 \left(\pi \left\lfloor \frac{x}{2\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(b) + \arctan \left(\frac{b \tan\left(\frac{1}{2}x\right) + a}{\sqrt{-a^2 + b^2}} \right) \right) a^2}{\sqrt{-a^2 + b^2} b^2} - \frac{a \log \left(\left| \tan \left(\frac{1}{2}x \right) \right| \right)}{b^2} + \frac{\tan \left(\frac{1}{2}x \right)}{2b} + \frac{2a \tan \left(\frac{1}{2}x \right) - b}{2b^2 \tan \left(\frac{1}{2}x \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^3/(a+b*csc(x)),x, algorithm="giac")

[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(b) + arctan((b*tan(1/2*x) + a)/sqrt(-a^2 + b^2)))*a^2/(sqrt(-a^2 + b^2)*b^2) - a*log(abs(tan(1/2*x)))/b^2 + 1/2*tan(1/2*x)/b + 1/2*(2*a*tan(1/2*x) - b)/(b^2*tan(1/2*x))

maple [A] time = 0.21, size = 77, normalized size = 1.24

$$\frac{\tan\left(\frac{x}{2}\right)}{2b} + \frac{2a^2 \arctan\left(\frac{2 \tan\left(\frac{x}{2}\right)b + 2a}{2\sqrt{-a^2 + b^2}}\right)}{b^2 \sqrt{-a^2 + b^2}} - \frac{1}{2b \tan\left(\frac{x}{2}\right)} - \frac{a \ln\left(\tan\left(\frac{x}{2}\right)\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(x)^3/(a+b*csc(x)),x)

[Out] 1/2/b*tan(1/2*x)+2/b^2*a^2/(-a^2+b^2)^(1/2)*arctan(1/2*(2*tan(1/2*x)*b+2*a)/(-a^2+b^2)^(1/2))-1/2/b/tan(1/2*x)-1/b^2*a*ln(tan(1/2*x))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^3/(a+b*csc(x)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details) Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 0.49, size = 135, normalized size = 2.18

$$-\frac{1}{b \tan(x)} - \frac{a \ln\left(\tan\left(\frac{x}{2}\right)\right)}{b^2} - \frac{a^2 \operatorname{atan}\left(\frac{a^2 \tan\left(\frac{x}{2}\right) \sqrt{a^2-b^2} - 4i - b^2 \tan\left(\frac{x}{2}\right) \sqrt{a^2-b^2} - 1i + a b \sqrt{a^2-b^2} - 2i}{4 \tan\left(\frac{x}{2}\right) a^3 + 2 a^2 b - 3 \tan\left(\frac{x}{2}\right) a b^2 - b^3}\right)}{b^2 \sqrt{a^2-b^2}} 2i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(sin(x)^3*(a + b/sin(x))),x)

[Out] - 1/(b*tan(x)) - (a*log(tan(x/2)))/b^2 - (a^2*atan((a^2*tan(x/2)*(a^2 - b^2)^(1/2)*4i - b^2*tan(x/2)*(a^2 - b^2)^(1/2)*1i + a*b*(a^2 - b^2)^(1/2)*2i)/(4*a^3*tan(x/2) + 2*a^2*b - b^3 - 3*a*b^2*tan(x/2)))*2i)/(b^2*(a^2 - b^2)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(x)}{a + b \csc(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)**3/(a+b*csc(x)),x)

[Out] Integral(csc(x)**3/(a + b*csc(x)), x)

$$3.42 \quad \int \frac{\csc^2(x)}{a+b \csc(x)} dx$$

Optimal. Leaf size=53

$$\frac{2a \tanh^{-1}\left(\frac{a+b \tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}} - \frac{\tanh^{-1}(\cos(x))}{b}$$

[Out] $-\operatorname{arctanh}(\cos(x))/b+2*a*\operatorname{arctanh}((a+b*\tan(1/2*x))/(a^2-b^2)^{(1/2)})/b/(a^2-b^2)^{(1/2)}$

Rubi [A] time = 0.11, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3789, 3770, 3831, 2660, 618, 206}

$$\frac{2a \tanh^{-1}\left(\frac{a+b \tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{b\sqrt{a^2-b^2}} - \frac{\tanh^{-1}(\cos(x))}{b}$$

Antiderivative was successfully verified.

[In] `Int[Csc[x]^2/(a + b*Csc[x]), x]`

[Out] $-(\operatorname{ArcTanh}[\operatorname{Cos}[x]]/b) + (2*a*\operatorname{ArcTanh}[(a + b*\operatorname{Tan}[x/2])/Sqrt[a^2 - b^2]])/(b*Sqrt[a^2 - b^2])$

Rule 206

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rule 618

`Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]`

Rule 2660

`Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]`

Rule 3770

Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3789

Int[csc[(e_.) + (f_.)*(x_.)]^2/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]

Rule 3831

Int[csc[(e_.) + (f_.)*(x_.)]/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{\csc^2(x)}{a + b \csc(x)} dx &= \frac{\int \csc(x) dx}{b} - \frac{a \int \frac{\csc(x)}{a + b \csc(x)} dx}{b} \\
 &= -\frac{\tanh^{-1}(\cos(x))}{b} - \frac{a \int \frac{1}{1 + \frac{a \sin(x)}{b}} dx}{b^2} \\
 &= -\frac{\tanh^{-1}(\cos(x))}{b} - \frac{(2a) \text{Subst} \left(\int \frac{1}{1 + \frac{2ax}{b} + x^2} dx, x, \tan\left(\frac{x}{2}\right) \right)}{b^2} \\
 &= -\frac{\tanh^{-1}(\cos(x))}{b} + \frac{(4a) \text{Subst} \left(\int \frac{1}{-4\left(1 - \frac{a^2}{b^2}\right) - x^2} dx, x, \frac{2a}{b} + 2 \tan\left(\frac{x}{2}\right) \right)}{b^2} \\
 &= -\frac{\tanh^{-1}(\cos(x))}{b} + \frac{2a \tanh^{-1} \left(\frac{b\left(\frac{a}{b} + \tan\left(\frac{x}{2}\right)\right)}{\sqrt{a^2 - b^2}} \right)}{b\sqrt{a^2 - b^2}}
 \end{aligned}$$

Mathematica [A] time = 0.06, size = 62, normalized size = 1.17

$$\frac{-\frac{2a \tan^{-1} \left(\frac{a + b \tan\left(\frac{x}{2}\right)}{\sqrt{b^2 - a^2}} \right)}{\sqrt{b^2 - a^2}} + \log \left(\sin\left(\frac{x}{2}\right) \right) - \log \left(\cos\left(\frac{x}{2}\right) \right)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[x]^2/(a + b*Csc[x]),x]

[Out] $\left(\frac{-2*a*ArcTan[(a + b*Tan[x/2])/Sqrt[-a^2 + b^2]]}{Sqrt[-a^2 + b^2]} - \text{Log}[Cos[x/2]] + \text{Log}[Sin[x/2]]\right)/b$

fricas [B] time = 0.66, size = 245, normalized size = 4.62

$$\frac{\sqrt{a^2 - b^2} a \log\left(\frac{(a^2 - 2b^2) \cos(x)^2 + 2ab \sin(x) + a^2 + b^2 + 2(b \cos(x) \sin(x) + a \cos(x)) \sqrt{a^2 - b^2}}{a^2 \cos(x)^2 - 2ab \sin(x) - a^2 - b^2}\right) - (a^2 - b^2) \log\left(\frac{1}{2} \cos(x) + \frac{1}{2}\right) + (a^2 - b^2) \log\left(-\frac{1}{2} \cos(x) + \frac{1}{2}\right)}{2(a^2 b - b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^2/(a+b*csc(x)),x, algorithm="fricas")

[Out] $\left[\frac{1}{2} * (\sqrt{a^2 - b^2}) * a * \log\left(\frac{(a^2 - 2b^2) \cos(x)^2 + 2ab \sin(x) + a^2 + b^2 + 2(b \cos(x) \sin(x) + a \cos(x)) \sqrt{a^2 - b^2}}{a^2 \cos(x)^2 - 2ab \sin(x) - a^2 - b^2}\right) - (a^2 - b^2) * \log\left(\frac{1}{2} \cos(x) + \frac{1}{2}\right) + (a^2 - b^2) * \log\left(-\frac{1}{2} \cos(x) + \frac{1}{2}\right)\right] / (a^2 b - b^3) + \frac{1}{2} * (2 * \sqrt{-a^2 + b^2}) * a * \arctan\left(\frac{-\sqrt{-a^2 + b^2} * (b \sin(x) + a)}{(a^2 - b^2) \cos(x)}\right) - (a^2 - b^2) * \log\left(\frac{1}{2} \cos(x) + \frac{1}{2}\right) + (a^2 - b^2) * \log\left(-\frac{1}{2} \cos(x) + \frac{1}{2}\right) / (a^2 b - b^3)$

giac [A] time = 0.53, size = 63, normalized size = 1.19

$$-\frac{2 \left(\pi \left\lfloor \frac{x}{2\pi} + \frac{1}{2} \right\rfloor \text{sgn}(b) + \arctan\left(\frac{b \tan\left(\frac{1}{2}x\right) + a}{\sqrt{-a^2 + b^2}}\right) \right) a}{\sqrt{-a^2 + b^2} b} + \frac{\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)^2/(a+b*csc(x)),x, algorithm="giac")

[Out] $-2 * (\pi * \text{floor}(1/2 * x / \pi + 1/2) * \text{sgn}(b) + \arctan((b * \tan(1/2 * x) + a) / \sqrt{-a^2 + b^2})) * a / (\sqrt{-a^2 + b^2} * b) + \log(\text{abs}(\tan(1/2 * x))) / b$

maple [A] time = 0.22, size = 53, normalized size = 1.00

$$-\frac{2a \arctan\left(\frac{2 \tan\left(\frac{x}{2}\right) b + 2a}{2 \sqrt{-a^2 + b^2}}\right)}{b \sqrt{-a^2 + b^2}} + \frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(csc(x)^2/(a+b*csc(x)),x)`

[Out] $-2/b*a/(-a^2+b^2)^{(1/2)}*\arctan(1/2*(2*\tan(1/2*x)*b+2*a)/(-a^2+b^2)^{(1/2)})+1/b*\ln(\tan(1/2*x))$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(x)^2/(a+b*csc(x)),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details)Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 0.43, size = 129, normalized size = 2.43

$$\frac{\ln\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{b} - \frac{2a \operatorname{atanh}\left(\frac{\sqrt{a^2-b^2}\left(4i\sin\left(\frac{x}{2}\right)a^2+2i\cos\left(\frac{x}{2}\right)ab-1i\sin\left(\frac{x}{2}\right)b^2\right)}{a^3\sin\left(\frac{x}{2}\right)4i+b\cos\left(\frac{x}{2}\right)(a^2-b^2)1i+a^2b\cos\left(\frac{x}{2}\right)1i-ab^2\sin\left(\frac{x}{2}\right)3i}\right)}{b\sqrt{a^2-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(sin(x)^2*(a + b/sin(x))),x)`

[Out] $\log(\sin(x/2)/\cos(x/2))/b - (2*a*\operatorname{atanh}(((a^2 - b^2)^{(1/2)}*(a^2*\sin(x/2)*4i - b^2*\sin(x/2)*1i + a*b*\cos(x/2)*2i))/(a^3*\sin(x/2)*4i + b*\cos(x/2)*(a^2 - b^2)*1i + a^2*b*\cos(x/2)*1i - a*b^2*\sin(x/2)*3i)))/(b*(a^2 - b^2)^{(1/2)})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(x)}{a + b \csc(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(x)**2/(a+b*csc(x)),x)`

[Out] `Integral(csc(x)**2/(a + b*csc(x)), x)`

$$3.43 \quad \int \frac{\csc(x)}{a+b \csc(x)} dx$$

Optimal. Leaf size=40

$$-\frac{2 \tanh^{-1}\left(\frac{a+b \tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

[Out] $-2*\operatorname{arctanh}((a+b*\tan(1/2*x))/(a^2-b^2)^{(1/2)})/(a^2-b^2)^{(1/2)}$

Rubi [A] time = 0.07, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3831, 2660, 618, 206}

$$-\frac{2 \tanh^{-1}\left(\frac{a+b \tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

[In] `Int[Csc[x]/(a + b*Csc[x]),x]`

[Out] `(-2*ArcTanh[(a + b*Tan[x/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2]`

Rule 206

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rule 618

`Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]`

Rule 2660

`Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]`

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_.)), x_Symbol]
  := Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f}, x]
  && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\csc(x)}{a + b \csc(x)} dx &= \frac{\int \frac{1}{1 + \frac{a \sin(x)}{b}} dx}{b} \\ &= \frac{2 \operatorname{Subst}\left(\int \frac{1}{1 + \frac{2ax}{b} + x^2} dx, x, \tan\left(\frac{x}{2}\right)\right)}{b} \\ &= -\frac{4 \operatorname{Subst}\left(\int \frac{1}{-4\left(1 - \frac{a^2}{b^2}\right) - x^2} dx, x, \frac{2a}{b} + 2 \tan\left(\frac{x}{2}\right)\right)}{b} \\ &= -\frac{2 \tanh^{-1}\left(\frac{b\left(\frac{a}{b} + \tan\left(\frac{x}{2}\right)\right)}{\sqrt{a^2 - b^2}}\right)}{\sqrt{a^2 - b^2}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 1.00

$$\frac{2 \tan^{-1}\left(\frac{a + b \tan\left(\frac{x}{2}\right)}{\sqrt{b^2 - a^2}}\right)}{\sqrt{b^2 - a^2}}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[x]/(a + b*Csc[x]), x]

[Out] (2*ArcTan[(a + b*Tan[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2]

fricas [A] time = 0.53, size = 154, normalized size = 3.85

$$\left[\frac{\log\left(-\frac{(a^2 - 2b^2) \cos(x)^2 + 2ab \sin(x) + a^2 + b^2 - 2(b \cos(x) \sin(x) + a \cos(x)) \sqrt{a^2 - b^2}}{a^2 \cos(x)^2 - 2ab \sin(x) - a^2 - b^2}\right)}{2 \sqrt{a^2 - b^2}}, -\frac{\sqrt{-a^2 + b^2} \arctan\left(-\frac{\sqrt{-a^2 + b^2} (b \sin(x) + a)}{(a^2 - b^2) \cos(x)}\right)}{a^2 - b^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)/(a+b*csc(x)),x, algorithm="fricas")

[Out] [1/2*log(-((a^2 - 2*b^2)*cos(x)^2 + 2*a*b*sin(x) + a^2 + b^2 - 2*(b*cos(x))*sin(x) + a*cos(x))*sqrt(a^2 - b^2))/(a^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2))/sqrt(a^2 - b^2), -sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*sin(x) + a)/((a^2 - b^2)*cos(x)))/(a^2 - b^2)]

giac [A] time = 0.45, size = 48, normalized size = 1.20

$$\frac{2 \left(\pi \left[\frac{x}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(b) + \arctan \left(\frac{b \tan\left(\frac{1}{2}x\right) + a}{\sqrt{-a^2 + b^2}} \right) \right)}{\sqrt{-a^2 + b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)/(a+b*csc(x)),x, algorithm="giac")

[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(b) + arctan((b*tan(1/2*x) + a)/sqrt(-a^2 + b^2)))/sqrt(-a^2 + b^2)

maple [A] time = 0.18, size = 39, normalized size = 0.98

$$\frac{2 \arctan \left(\frac{2 \tan\left(\frac{x}{2}\right)b + 2a}{2\sqrt{-a^2 + b^2}} \right)}{\sqrt{-a^2 + b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(x)/(a+b*csc(x)),x)

[Out] 2/(-a^2+b^2)^(1/2)*arctan(1/2*(2*tan(1/2*x)*b+2*a)/(-a^2+b^2)^(1/2))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(x)/(a+b*csc(x)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details)Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 0.29, size = 36, normalized size = 0.90

$$\frac{2 \operatorname{atanh}\left(\frac{a+b \tan\left(\frac{x}{2}\right)}{\sqrt{a+b} \sqrt{a-b}}\right)}{\sqrt{a+b} \sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(sin(x)*(a + b/sin(x))),x)`

[Out] `-(2*atanh((a + b*tan(x/2))/((a + b)^(1/2)*(a - b)^(1/2))))/((a + b)^(1/2)*(a - b)^(1/2))`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(x)}{a + b \csc(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(x)/(a+b*csc(x)),x)`

[Out] `Integral(csc(x)/(a + b*csc(x)), x)`

$$3.44 \quad \int \frac{1}{a+b \csc(c+dx)} dx$$

Optimal. Leaf size=57

$$\frac{2b \tanh^{-1}\left(\frac{a+b \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{ad\sqrt{a^2-b^2}} + \frac{x}{a}$$

[Out] x/a+2*b*arctanh((a+b*tan(1/2*d*x+1/2*c))/(a^2-b^2)^(1/2))/a/d/(a^2-b^2)^(1/2)

Rubi [A] time = 0.06, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3783, 2660, 618, 206}

$$\frac{2b \tanh^{-1}\left(\frac{a+b \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{ad\sqrt{a^2-b^2}} + \frac{x}{a}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Csc[c + d*x])^(-1), x]

[Out] x/a + (2*b*ArcTanh[(a + b*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a*Sqrt[a^2 - b^2]*d)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 2660

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3783

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.))^(−1), x_Symbol] := Simp[x/a, x]
− Dist[1/a, Int[1/(1 + (a*Sin[c + d*x])/b), x], x] /; FreeQ[{a, b, c, d}, x]
] && NeQ[a^2 − b^2, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{a + b \csc(c + dx)} dx &= \frac{x}{a} - \frac{\int \frac{1}{1 + \frac{a \sin(c+dx)}{b}} dx}{a} \\ &= \frac{x}{a} - \frac{2 \operatorname{Subst} \left(\int \frac{1}{1 + \frac{2ax}{b} + x^2} dx, x, \tan \left(\frac{1}{2}(c + dx) \right) \right)}{ad} \\ &= \frac{x}{a} + \frac{4 \operatorname{Subst} \left(\int \frac{1}{-4 \left(1 - \frac{a^2}{b^2} \right) - x^2} dx, x, \frac{2a}{b} + 2 \tan \left(\frac{1}{2}(c + dx) \right) \right)}{ad} \\ &= \frac{x}{a} + \frac{2b \tanh^{-1} \left(\frac{b \left(\frac{a}{b} + \tan \left(\frac{1}{2}(c+dx) \right) \right)}{\sqrt{a^2 - b^2}} \right)}{a \sqrt{a^2 - b^2} d} \end{aligned}$$

Mathematica [A] time = 0.11, size = 59, normalized size = 1.04

$$\frac{-\frac{2b \tan^{-1} \left(\frac{a+b \tan \left(\frac{1}{2}(c+dx) \right)}{\sqrt{b^2-a^2}} \right)}{d \sqrt{b^2-a^2}} + \frac{c}{d} + x}{a}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Csc[c + d*x])^(−1), x]

[Out] (c/d + x − (2*b*ArcTan[(a + b*Tan[(c + d*x)/2])/Sqrt[−a^2 + b^2]])/(Sqrt[−a^2 + b^2]*d)/a

fricas [A] time = 0.61, size = 238, normalized size = 4.18

$$\left[\frac{2(a^2 - b^2)dx + \sqrt{a^2 - b^2} b \log \left(\frac{(a^2 - 2b^2) \cos(dx+c)^2 + 2ab \sin(dx+c) + a^2 + b^2 + 2(b \cos(dx+c) \sin(dx+c) + a \cos(dx+c)) \sqrt{a^2 - b^2}}{a^2 \cos(dx+c)^2 - 2ab \sin(dx+c) - a^2 - b^2} \right)}{2(a^3 - ab^2)d} \right],$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*csc(d*x+c)),x, algorithm="fricas")

[Out] $\left[\frac{1}{2} \cdot (2 \cdot (a^2 - b^2) \cdot d \cdot x + \sqrt{a^2 - b^2}) \cdot b \cdot \log\left(\frac{(a^2 - 2 \cdot b^2) \cdot \cos(d \cdot x + c)^2 + 2 \cdot a \cdot b \cdot \sin(d \cdot x + c) + a^2 + b^2 + 2 \cdot (b \cdot \cos(d \cdot x + c) \cdot \sin(d \cdot x + c) + a \cdot \cos(d \cdot x + c)) \cdot \sqrt{a^2 - b^2}}{(a^2 \cdot \cos(d \cdot x + c))^2 - 2 \cdot a \cdot b \cdot \sin(d \cdot x + c) - a^2 - b^2)}\right) / ((a^3 - a \cdot b^2) \cdot d), \frac{(a^2 - b^2) \cdot d \cdot x + \sqrt{-a^2 + b^2} \cdot b \cdot \arctan\left(\frac{\sqrt{-a^2 + b^2} \cdot (b \cdot \sin(d \cdot x + c) + a)}{(a^2 - b^2) \cdot \cos(d \cdot x + c)}\right)}{(a^3 - a \cdot b^2) \cdot d} \right]$

giac [A] time = 0.32, size = 77, normalized size = 1.35

$$-\frac{2 \left(\pi \left\lfloor \frac{dx+c}{2\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(b) + \arctan\left(\frac{b \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + a}{\sqrt{-a^2 + b^2}}\right) \right) b}{\sqrt{-a^2 + b^2} a} - \frac{dx+c}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*csc(d*x+c)),x, algorithm="giac")

[Out] $-(2 \cdot (\pi \cdot \text{floor}(1/2 \cdot (d \cdot x + c) / \pi + 1/2) \cdot \operatorname{sgn}(b) + \arctan((b \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c) + a) / \sqrt{-a^2 + b^2}))) \cdot b / (\sqrt{-a^2 + b^2} \cdot a) - (d \cdot x + c) / a) / d$

maple [A] time = 0.57, size = 70, normalized size = 1.23

$$-\frac{2b \arctan\left(\frac{2b \tan\left(\frac{dx}{2} + \frac{c}{2}\right) + 2a}{2\sqrt{-a^2 + b^2}}\right)}{da\sqrt{-a^2 + b^2}} + \frac{2 \arctan\left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*csc(d*x+c)),x)

[Out] $-2/d/a \cdot b / (-a^2 + b^2)^{(1/2)} \cdot \arctan(1/2 \cdot (2 \cdot b \cdot \tan(1/2 \cdot d \cdot x + 1/2 \cdot c) + 2 \cdot a) / (-a^2 + b^2)^{(1/2)}) + 2/d/a \cdot \arctan(\tan(1/2 \cdot d \cdot x + 1/2 \cdot c))$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*csc(d*x+c)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* h

elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details) Is $4a^2-4b^2$ positive or negative?

mupad [B] time = 0.45, size = 184, normalized size = 3.23

$$\frac{x}{a} \frac{2b \operatorname{atanh} \left(\frac{2a^2 \sin\left(\frac{c}{2} + \frac{dx}{2}\right)(a^2 - b^2) - 2b^4 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) - 2b^2 \sin\left(\frac{c}{2} + \frac{dx}{2}\right)(a^2 - b^2) + ab^3 \cos\left(\frac{c}{2} + \frac{dx}{2}\right) + 3a^2 b^2 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) + ab \cos\left(\frac{c}{2} + \frac{dx}{2}\right)(a^2 - b^2)}{a \left(2 \sin\left(\frac{c}{2} + \frac{dx}{2}\right) a^2 + b \cos\left(\frac{c}{2} + \frac{dx}{2}\right) a \right) \sqrt{a^2 - b^2}}}{ad \sqrt{a^2 - b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a + b/sin(c + d*x)),x)`

[Out] $x/a - (2*b*\operatorname{atanh}((2*a^2*\sin(c/2 + (d*x)/2)*(a^2 - b^2) - 2*b^4*\sin(c/2 + (d*x)/2) - 2*b^2*\sin(c/2 + (d*x)/2)*(a^2 - b^2) + a*b^3*\cos(c/2 + (d*x)/2) + 3*a^2*b^2*\sin(c/2 + (d*x)/2) + a*b*\cos(c/2 + (d*x)/2)*(a^2 - b^2))/(a*(2*a^2*\sin(c/2 + (d*x)/2) + a*b*\cos(c/2 + (d*x)/2))*(a^2 - b^2)^{(1/2)})))/(a*d*(a^2 - b^2)^{(1/2)})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b \csc(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a+b*csc(d*x+c)),x)`

[Out] `Integral(1/(a + b*csc(c + d*x)), x)`

$$3.45 \quad \int \frac{\sin(x)}{a+b \csc(x)} dx$$

Optimal. Leaf size=61

$$-\frac{2b^2 \tanh^{-1}\left(\frac{a+b \tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}} - \frac{bx}{a^2} - \frac{\cos(x)}{a}$$

[Out] $-b*x/a^2 - \cos(x)/a - 2*b^2*\operatorname{arctanh}\left(\frac{a+b*\tan(1/2*x)}{(a^2-b^2)^{(1/2)}}\right)/a^2/(a^2-b^2)^{(1/2)}$

Rubi [A] time = 0.11, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {3853, 12, 3783, 2660, 618, 206}

$$-\frac{2b^2 \tanh^{-1}\left(\frac{a+b \tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^2\sqrt{a^2-b^2}} - \frac{bx}{a^2} - \frac{\cos(x)}{a}$$

Antiderivative was successfully verified.

[In] `Int[Sin[x]/(a + b*Csc[x]),x]`

[Out] $-\left(\frac{b*x}{a^2}\right) - \left(\frac{2*b^2*\operatorname{ArcTanh}\left[\frac{a + b*\tan[x/2]}{\sqrt{a^2 - b^2}}\right]}{a^2*\sqrt{a^2 - b^2}}\right) - \operatorname{Cos}[x]/a$

Rule 12

`Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]`

Rule 206

`Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])`

Rule 618

`Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]`

Rule 2660

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3783

```
Int[(csc[(c_) + (d_)*(x_)]*(b_) + (a_))^-1, x_Symbol] := Simp[x/a, x] - Dist[1/a, Int[1/(1 + (a*Sin[c + d*x])/b), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3853

```
Int[(csc[(e_) + (f_)*(x_)]*(d_))^(n_)/(csc[(e_) + (f_)*(x_)]*(b_) + (a_)), x_Symbol] := Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n), x] - Dist[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[b*n - a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}
 \int \frac{\sin(x)}{a + b \csc(x)} dx &= -\frac{\cos(x)}{a} - \frac{\int \frac{b}{a + b \csc(x)} dx}{a} \\
 &= -\frac{\cos(x)}{a} - \frac{b \int \frac{1}{a + b \csc(x)} dx}{a} \\
 &= -\frac{bx}{a^2} - \frac{\cos(x)}{a} + \frac{b \int \frac{1}{1 + \frac{a \sin(x)}{b}} dx}{a^2} \\
 &= -\frac{bx}{a^2} - \frac{\cos(x)}{a} + \frac{(2b) \operatorname{Subst} \left(\int \frac{1}{1 + \frac{2ax}{b} + x^2} dx, x, \tan\left(\frac{x}{2}\right) \right)}{a^2} \\
 &= -\frac{bx}{a^2} - \frac{\cos(x)}{a} - \frac{(4b) \operatorname{Subst} \left(\int \frac{1}{-4\left(1 - \frac{a^2}{b^2}\right) - x^2} dx, x, \frac{2a}{b} + 2 \tan\left(\frac{x}{2}\right) \right)}{a^2} \\
 &= -\frac{bx}{a^2} - \frac{2b^2 \tanh^{-1} \left(\frac{b\left(\frac{a}{b} + \tan\left(\frac{x}{2}\right)\right)}{\sqrt{a^2 - b^2}} \right)}{a^2 \sqrt{a^2 - b^2}} - \frac{\cos(x)}{a}
 \end{aligned}$$

Mathematica [A] time = 0.10, size = 56, normalized size = 0.92

$$-\frac{2b^2 \tan^{-1}\left(\frac{a+b \tan\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}}\right)}{\sqrt{b^2-a^2}} + a \cos(x) + bx$$

$$a^2$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]/(a + b*Csc[x]), x]

[Out] -((b*x - (2*b^2*ArcTan[(a + b*Tan[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + a*Cos[x])/a^2)

fricas [A] time = 0.49, size = 235, normalized size = 3.85

$$\frac{\sqrt{a^2 - b^2} b^2 \log\left(-\frac{(a^2 - 2b^2) \cos(x)^2 + 2ab \sin(x) + a^2 + b^2 - 2(b \cos(x) \sin(x) + a \cos(x)) \sqrt{a^2 - b^2}}{a^2 \cos(x)^2 - 2ab \sin(x) - a^2 - b^2}\right) - 2(a^2 b - b^3)x - 2(a^3 - ab^2) \cos(x)}{2(a^4 - a^2 b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(a+b*csc(x)), x, algorithm="fricas")

[Out] [1/2*(sqrt(a^2 - b^2)*b^2*log(-((a^2 - 2*b^2)*cos(x)^2 + 2*a*b*sin(x) + a^2 + b^2 - 2*(b*cos(x)*sin(x) + a*cos(x))*sqrt(a^2 - b^2))/(a^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) - 2*(a^2*b - b^3)*x - 2*(a^3 - a*b^2)*cos(x))/(a^4 - a^2*b^2), -(sqrt(-a^2 + b^2)*b^2*arctan(-sqrt(-a^2 + b^2)*(b*sin(x) + a)/((a^2 - b^2)*cos(x))) + (a^2*b - b^3)*x + (a^3 - a*b^2)*cos(x))/(a^4 - a^2*b^2)]

giac [A] time = 0.52, size = 77, normalized size = 1.26

$$\frac{2\left(\pi\left[\frac{x}{2\pi} + \frac{1}{2}\right] \operatorname{sgn}(b) + \arctan\left(\frac{b \tan\left(\frac{1}{2}x\right) + a}{\sqrt{-a^2 + b^2}}\right)\right) b^2}{\sqrt{-a^2 + b^2} a^2} - \frac{bx}{a^2} - \frac{2}{\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right) a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(a+b*csc(x)), x, algorithm="giac")

[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(b) + arctan((b*tan(1/2*x) + a)/sqrt(-a^2 + b^2)))*b^2/(sqrt(-a^2 + b^2)*a^2) - b*x/a^2 - 2/((tan(1/2*x)^2 + 1)*a)

maple [A] time = 0.46, size = 72, normalized size = 1.18

$$\frac{2b^2 \arctan\left(\frac{2 \tan\left(\frac{x}{2}\right)b+2a}{2\sqrt{-a^2+b^2}}\right)}{a^2\sqrt{-a^2+b^2}} - \frac{2}{a\left(\tan^2\left(\frac{x}{2}\right)+1\right)} - \frac{2b \arctan\left(\tan\left(\frac{x}{2}\right)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(x)/(a+b*csc(x)),x)`

[Out] `2*b^2/a^2/(-a^2+b^2)^(1/2)*arctan(1/2*(2*tan(1/2*x)*b+2*a)/(-a^2+b^2)^(1/2))-2/a/(tan(1/2*x)^2+1)-2/a^2*b*arctan(tan(1/2*x))`

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)/(a+b*csc(x)),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details)Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 0.65, size = 766, normalized size = 12.56

$$\frac{b^2 \operatorname{atan} \left(\frac{b^2 \sqrt{a^2 - b^2} \left(\frac{32 b^4}{a} - \frac{32 \tan\left(\frac{x}{2}\right) (2 a b^5 - 2 a^3 b^3)}{a^3} + \frac{b^2 \sqrt{a^2 - b^2} \left(32 a^2 b^2 + 64 a b^3 \tan\left(\frac{x}{2}\right) + \frac{32 a^3 b^2 + 32 \tan\left(\frac{x}{2}\right)}{a^4 - a^2 b^2} \right)}{a^4 - a^2 b^2} \right)}{a^4 - a^2 b^2} \right)}{a \left(\tan\left(\frac{x}{2}\right)^2 + 1 \right) - \frac{b x}{a^2} - \frac{128 b^5 \tan\left(\frac{x}{2}\right)}{a^3} + \frac{b^2 \sqrt{a^2 - b^2} \left(\frac{32 b^4}{a} - \frac{32 \tan\left(\frac{x}{2}\right) (2 a b^5 - 2 a^3 b^3)}{a^3} + \frac{b^2 \sqrt{a^2 - b^2} \left(32 a^2 b^2 + 64 a b^3 \tan\left(\frac{x}{2}\right) + \frac{32 a^3 b^2 + 32 \tan\left(\frac{x}{2}\right)}{a^4 - a^2 b^2} \right)}{a^4 - a^2 b^2} \right)}{a^4 - a^2 b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(x)/(a + b/sin(x)),x)`

[Out] $-2/(a \cdot (\tan(x/2)^2 + 1)) - (b \cdot x)/a^2 - (b^2 \cdot \operatorname{atan}(((b^2 \cdot (a^2 - b^2))^{1/2}) \cdot ((32 \cdot b^4)/a - (32 \cdot \tan(x/2) \cdot (2 \cdot a \cdot b^5 - 2 \cdot a^3 \cdot b^3))/a^3 + (b^2 \cdot (a^2 - b^2))^{1/2} \cdot (32 \cdot a^2 \cdot b^2 + 64 \cdot a \cdot b^3 \cdot \tan(x/2) + (b^2 \cdot (a^2 - b^2))^{1/2} \cdot (32 \cdot a^3 \cdot b^2 + (32 \cdot \tan(x/2) \cdot (3 \cdot a^7 \cdot b - 2 \cdot a^5 \cdot b^3))/a^3))/a^3)))/(a^4 - a^2 \cdot b^2)))/(a^4 - a^2 \cdot b^2)) \cdot 1i)/(a^4 - a^2 \cdot b^2) - (b^2 \cdot (a^2 - b^2))^{1/2} \cdot ((32 \cdot \tan(x/2) \cdot (2 \cdot a \cdot b^5 - 2 \cdot a^3 \cdot b^3))/a^3 - (32 \cdot b^4)/a + (b^2 \cdot (a^2 - b^2))^{1/2} \cdot (32 \cdot a^2 \cdot b^2 + 64 \cdot a \cdot b^3 \cdot \tan(x/2) - (b^2 \cdot (a^2 - b^2))^{1/2} \cdot (32 \cdot a^3 \cdot b^2 + (32 \cdot \tan(x/2) \cdot (3 \cdot a^7 \cdot b - 2 \cdot a^5 \cdot b^3))/a^3)))/(a^4 - a^2 \cdot b^2)))/(a^4 - a^2 \cdot b^2)) \cdot 1i)/(a^4 - a^2 \cdot b^2)))/((128 \cdot b^5 \cdot \tan(x/2))/a^3 + (b^2 \cdot (a^2 - b^2))^{1/2} \cdot ((32 \cdot b^4)/a - (32 \cdot \tan(x/2) \cdot (2 \cdot a \cdot b^5 - 2 \cdot a^3 \cdot b^3))/a^3 + (b^2 \cdot (a^2 - b^2))^{1/2} \cdot (32 \cdot a^2 \cdot b^2 + 64 \cdot a \cdot b^3 \cdot \tan(x/2) + (b^2 \cdot (a^2 - b^2))^{1/2} \cdot (32 \cdot a^3 \cdot b^2 + (32 \cdot \tan(x/2) \cdot (3 \cdot a^7 \cdot b - 2 \cdot a^5 \cdot b^3))/a^3)))/(a^4 - a^2 \cdot b^2)))/(a^4 - a^2 \cdot b^2)))/(a^4 - a^2 \cdot b^2) + (b^2 \cdot (a^2 - b^2))^{1/2} \cdot ((32 \cdot \tan(x/2) \cdot (2 \cdot a \cdot b^5 - 2 \cdot a^3 \cdot b^3))/a^3 - (32 \cdot b^4)/a + (b^2 \cdot (a^2 - b^2))^{1/2} \cdot (32 \cdot a^2 \cdot b^2 + 64 \cdot a \cdot b^3 \cdot \tan(x/2) - (b^2 \cdot (a^2 - b^2))^{1/2} \cdot (32 \cdot a^3 \cdot b^2 + (32 \cdot \tan(x/2) \cdot (3 \cdot a^7 \cdot b - 2 \cdot a^5 \cdot b^3))/a^3)))/(a^4 - a^2 \cdot b^2)))/(a^4 - a^2 \cdot b^2)) \cdot (a^2 - b^2)^{1/2} \cdot 2i)/(a^4 - a^2 \cdot b^2)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(x)}{a + b \csc(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)/(a+b*csc(x)),x)

[Out] Integral(sin(x)/(a + b*csc(x)), x)

$$3.46 \quad \int \frac{\sin^2(x)}{a+b \csc(x)} dx$$

Optimal. Leaf size=82

$$\frac{b \cos(x)}{a^2} + \frac{x(a^2 + 2b^2)}{2a^3} + \frac{2b^3 \tanh^{-1}\left(\frac{a+b \tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^3 \sqrt{a^2-b^2}} - \frac{\sin(x) \cos(x)}{2a}$$

[Out] 1/2*(a^2+2*b^2)*x/a^3+b*cos(x)/a^2-1/2*cos(x)*sin(x)/a+2*b^3*arctanh((a+b*tan(1/2*x))/(a^2-b^2)^(1/2))/a^3/(a^2-b^2)^(1/2)

Rubi [A] time = 0.26, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3853, 4104, 3919, 3831, 2660, 618, 206}

$$\frac{x(a^2 + 2b^2)}{2a^3} + \frac{2b^3 \tanh^{-1}\left(\frac{a+b \tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^3 \sqrt{a^2-b^2}} + \frac{b \cos(x)}{a^2} - \frac{\sin(x) \cos(x)}{2a}$$

Antiderivative was successfully verified.

[In] Int[Sin[x]^2/(a + b*Csc[x]),x]

[Out] ((a^2 + 2*b^2)*x)/(2*a^3) + (2*b^3*ArcTanh[(a + b*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^3*Sqrt[a^2 - b^2]) + (b*Cos[x])/a^2 - (Cos[x]*Sin[x])/(2*a)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 2660

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3831

Int[csc[(e_.) + (f_.)*(x_.)]/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3853

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n), x] - Dist[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[b*n - a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]

Rule 3919

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 4104

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_), x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \frac{\sin^2(x)}{a + b \csc(x)} dx &= -\frac{\cos(x) \sin(x)}{2a} + \frac{\int \frac{(-2b+a \csc(x)+b \csc^2(x)) \sin(x)}{a+b \csc(x)} dx}{2a} \\
&= \frac{b \cos(x)}{a^2} - \frac{\cos(x) \sin(x)}{2a} - \frac{\int \frac{-a^2-2b^2-ab \csc(x)}{a+b \csc(x)} dx}{2a^2} \\
&= \frac{(a^2 + 2b^2)x}{2a^3} + \frac{b \cos(x)}{a^2} - \frac{\cos(x) \sin(x)}{2a} - \frac{b^3 \int \frac{\csc(x)}{a+b \csc(x)} dx}{a^3} \\
&= \frac{(a^2 + 2b^2)x}{2a^3} + \frac{b \cos(x)}{a^2} - \frac{\cos(x) \sin(x)}{2a} - \frac{b^2 \int \frac{1}{1+\frac{a \sin(x)}{b}} dx}{a^3} \\
&= \frac{(a^2 + 2b^2)x}{2a^3} + \frac{b \cos(x)}{a^2} - \frac{\cos(x) \sin(x)}{2a} - \frac{(2b^2) \text{Subst} \left(\int \frac{1}{1+\frac{2ax}{b}+x^2} dx, x, \tan\left(\frac{x}{2}\right) \right)}{a^3} \\
&= \frac{(a^2 + 2b^2)x}{2a^3} + \frac{b \cos(x)}{a^2} - \frac{\cos(x) \sin(x)}{2a} + \frac{(4b^2) \text{Subst} \left(\int \frac{1}{-4\left(1-\frac{a^2}{b^2}\right)-x^2} dx, x, \frac{2a}{b} + 2 \tan\left(\frac{x}{2}\right) \right)}{a^3} \\
&= \frac{(a^2 + 2b^2)x}{2a^3} + \frac{2b^3 \tanh^{-1} \left(\frac{b\left(\frac{a}{b} + \tan\left(\frac{x}{2}\right)\right)}{\sqrt{a^2-b^2}} \right)}{a^3 \sqrt{a^2-b^2}} + \frac{b \cos(x)}{a^2} - \frac{\cos(x) \sin(x)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 78, normalized size = 0.95

$$\frac{8b^3 \tan^{-1} \left(\frac{a+b \tan\left(\frac{x}{2}\right)}{\sqrt{b^2-a^2}} \right)}{\sqrt{b^2-a^2}} + \frac{2a^2x - a^2 \sin(2x) + 4ab \cos(x) + 4b^2x}{4a^3}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]^2/(a + b*Csc[x]), x]

[Out] (2*a^2*x + 4*b^2*x - (8*b^3*ArcTan[(a + b*Tan[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + 4*a*b*Cos[x] - a^2*Sin[2*x])/(4*a^3)

fricas [A] time = 0.68, size = 285, normalized size = 3.48

$$\left[\frac{\sqrt{a^2 - b^2} b^3 \log \left(\frac{(a^2 - 2b^2) \cos(x)^2 + 2ab \sin(x) + a^2 + b^2 + 2(b \cos(x) \sin(x) + a \cos(x)) \sqrt{a^2 - b^2}}{a^2 \cos(x)^2 - 2ab \sin(x) - a^2 - b^2} \right) - (a^4 - a^2 b^2) \cos(x) \sin(x) + (a^4 + a^2 b^2)}{2(a^5 - a^3 b^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^2/(a+b*csc(x)),x, algorithm="fricas")

[Out] [1/2*(sqrt(a^2 - b^2)*b^3*log(((a^2 - 2*b^2)*cos(x)^2 + 2*a*b*sin(x) + a^2 + b^2 + 2*(b*cos(x)*sin(x) + a*cos(x))*sqrt(a^2 - b^2))/(a^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) - (a^4 - a^2*b^2)*cos(x)*sin(x) + (a^4 + a^2*b^2 - 2*b^4)*x + 2*(a^3*b - a*b^3)*cos(x))/(a^5 - a^3*b^2), 1/2*(2*sqrt(-a^2 + b^2)*b^3*arctan(-sqrt(-a^2 + b^2)*(b*sin(x) + a)/((a^2 - b^2)*cos(x))) - (a^4 - a^2*b^2)*cos(x)*sin(x) + (a^4 + a^2*b^2 - 2*b^4)*x + 2*(a^3*b - a*b^3)*cos(x))/(a^5 - a^3*b^2)]

giac [A] time = 0.64, size = 112, normalized size = 1.37

$$\frac{2 \left(\pi \left\lfloor \frac{x}{2\pi} + \frac{1}{2} \right\rfloor \operatorname{sgn}(b) + \arctan \left(\frac{b \tan\left(\frac{1}{2}x\right) + a}{\sqrt{-a^2 + b^2}} \right) \right) b^3}{\sqrt{-a^2 + b^2} a^3} + \frac{(a^2 + 2b^2)x}{2a^3} + \frac{a \tan\left(\frac{1}{2}x\right)^3 + 2b \tan\left(\frac{1}{2}x\right)^2 - a \tan\left(\frac{1}{2}x\right) + 2b}{\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right)^2 a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^2/(a+b*csc(x)),x, algorithm="giac")

[Out] -2*(pi*floor(1/2*x/pi + 1/2)*sgn(b) + arctan((b*tan(1/2*x) + a)/sqrt(-a^2 + b^2)))*b^3/(sqrt(-a^2 + b^2)*a^3) + 1/2*(a^2 + 2*b^2)*x/a^3 + (a*tan(1/2*x)^3 + 2*b*tan(1/2*x)^2 - a*tan(1/2*x) + 2*b)/((tan(1/2*x)^2 + 1)^2*a^2)

maple [A] time = 0.49, size = 142, normalized size = 1.73

$$-\frac{2b^3 \arctan\left(\frac{2 \tan\left(\frac{x}{2}\right)b + 2a}{2\sqrt{-a^2 + b^2}}\right)}{a^3 \sqrt{-a^2 + b^2}} + \frac{\tan^3\left(\frac{x}{2}\right)}{a \left(\tan^2\left(\frac{x}{2}\right) + 1\right)^2} + \frac{2b \left(\tan^2\left(\frac{x}{2}\right)\right)}{a^2 \left(\tan^2\left(\frac{x}{2}\right) + 1\right)^2} - \frac{\tan\left(\frac{x}{2}\right)}{a \left(\tan^2\left(\frac{x}{2}\right) + 1\right)^2} + \frac{2b}{a^2 \left(\tan^2\left(\frac{x}{2}\right) + 1\right)^2} + \frac{2 \arctan\left(\frac{x}{2}\right)}{a^2 \left(\tan^2\left(\frac{x}{2}\right) + 1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)^2/(a+b*csc(x)),x)

[Out] -2/a^3*b^3/(-a^2+b^2)^(1/2)*arctan(1/2*(2*tan(1/2*x)*b+2*a)/(-a^2+b^2)^(1/2))+1/a/(tan(1/2*x)^2+1)^2*tan(1/2*x)^3+2/a^2/(tan(1/2*x)^2+1)^2*b*tan(1/2*x)^2-1/a/(tan(1/2*x)^2+1)^2*tan(1/2*x)+2/a^2/(tan(1/2*x)^2+1)^2*b+2/a^3*arctan(tan(1/2*x))*b^2+1/2*x/a

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^2/(a+b*csc(x)),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details)Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 0.84, size = 1147, normalized size = 13.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(x)^2/(a + b/sin(x)),x)

[Out]
$$\begin{aligned} & ((2*b)/a^2 - \tan(x/2)/a + \tan(x/2)^3/a + (2*b*\tan(x/2)^2)/a^2)/(2*\tan(x/2)^2 + \tan(x/2)^4 + 1) - (\operatorname{atan}((40*b^3*\tan(x/2))/(8*a^2*b + 40*b^3 + (48*b^5)/a^2) + (48*b^5*\tan(x/2))/(8*a^4*b + 48*b^5 + 40*a^2*b^3) + (8*a*b*\tan(x/2))/(8*a*b + (40*b^3)/a + (48*b^5)/a^3))*(a^2*i + b^2*2i)*i)/a^3 + (b^3*\operatorname{atan}(((b^3*(a^2 - b^2)^{(1/2)}*((8*(4*a^2*b^6 + 4*a^4*b^4 + a^6*b^2))/a^5 + (8*\tan(x/2)*(2*a^8*b - 8*a^2*b^7 + 4*a^4*b^5 + 7*a^6*b^3))/a^6 + (b^3*(a^2 - b^2)^{(1/2)}*(64*b^4*\tan(x/2) + (8*(2*a^8*b + 2*a^6*b^3))/a^5 + (b^3*(a^2 - b^2)^{(1/2)}*(32*a^3*b^2 + (8*\tan(x/2)*(12*a^{10}*b - 8*a^8*b^3))/a^6)))/(a^5 - a^3*b^2))))/(a^5 - a^3*b^2))*i)/(a^5 - a^3*b^2) + (b^3*(a^2 - b^2)^{(1/2)}*((8*(4*a^2*b^6 + 4*a^4*b^4 + a^6*b^2))/a^5 + (8*\tan(x/2)*(2*a^8*b - 8*a^2*b^7 + 4*a^4*b^5 + 7*a^6*b^3))/a^6 - (b^3*(a^2 - b^2)^{(1/2)}*(64*b^4*\tan(x/2) + (8*(2*a^8*b + 2*a^6*b^3))/a^5 - (b^3*(a^2 - b^2)^{(1/2)}*(32*a^3*b^2 + (8*\tan(x/2)*(12*a^{10}*b - 8*a^8*b^3))/a^6)))/(a^5 - a^3*b^2))))/(a^5 - a^3*b^2))*i)/(a^5 - a^3*b^2))/((16*(2*b^7 + a^2*b^5))/a^5 + (16*\tan(x/2)*(8*b^8 + 8*a^2*b^6 + 2*a^4*b^4))/a^6 + (b^3*(a^2 - b^2)^{(1/2)}*((8*(4*a^2*b^6 + 4*a^4*b^4 + a^6*b^2))/a^5 + (8*\tan(x/2)*(2*a^8*b - 8*a^2*b^7 + 4*a^4*b^5 + 7*a^6*b^3))/a^6 + (b^3*(a^2 - b^2)^{(1/2)}*(64*b^4*\tan(x/2) + (8*(2*a^8*b + 2*a^6*b^3))/a^5 + (b^3*(a^2 - b^2)^{(1/2)}*(32*a^3*b^2 + (8*\tan(x/2)*(12*a^{10}*b - 8*a^8*b^3))/a^6)))/(a^5 - a^3*b^2))))/(a^5 - a^3*b^2) - (b^3*(a^2 - b^2)^{(1/2)}*((8*(4*a^2*b^6 + 4*a^4*b^4 + a^6*b^2))/a^5 + (8*\tan(x/2)*(2*a^8*b - 8*a^2*b^7 + 4*a^4*b^5 + 7*a^6*b^3))/a^6 - (b^3*(a^2 - b^2)^{(1/2)}*(64*b^4*\tan(x/2) + (8*(2*a^8*b + 2*a^6*b^3))/a^5 - (b^3*(a^2 - b^2)^{(1/2)}*(32*a^3*b^2 + (8*\tan(x/2)*(12*a^{10}*b - 8*a^8*b^3))/a^6)))/(a^5 - a^3*b^2))))/(a^5 - a^3*b^2))*i)/(a^5 - a^3*b^2) \end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(x)}{a + b \csc(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a+b*csc(x)),x)
```

```
[Out] Integral(sin(x)**2/(a + b*csc(x)), x)
```

$$3.47 \quad \int \frac{\sin^3(x)}{a+b \csc(x)} dx$$

Optimal. Leaf size=110

$$\frac{b \sin(x) \cos(x)}{2a^2} - \frac{bx(a^2 + 2b^2)}{2a^4} - \frac{2b^4 \tanh^{-1}\left(\frac{a+b \tan(\frac{x}{2})}{\sqrt{a^2-b^2}}\right)}{a^4 \sqrt{a^2-b^2}} - \frac{(2a^2 + 3b^2) \cos(x)}{3a^3} - \frac{\sin^2(x) \cos(x)}{3a}$$

[Out] $-1/2*b*(a^2+2*b^2)*x/a^4-1/3*(2*a^2+3*b^2)*\cos(x)/a^3+1/2*b*\cos(x)*\sin(x)/a^2-1/3*\cos(x)*\sin(x)^2/a-2*b^4*\operatorname{arctanh}((a+b*\tan(1/2*x))/\sqrt{a^2-b^2})/a^4/\sqrt{a^2-b^2}$

Rubi [A] time = 0.40, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3853, 4104, 3919, 3831, 2660, 618, 206}

$$-\frac{bx(a^2 + 2b^2)}{2a^4} - \frac{(2a^2 + 3b^2) \cos(x)}{3a^3} - \frac{2b^4 \tanh^{-1}\left(\frac{a+b \tan(\frac{x}{2})}{\sqrt{a^2-b^2}}\right)}{a^4 \sqrt{a^2-b^2}} + \frac{b \sin(x) \cos(x)}{2a^2} - \frac{\sin^2(x) \cos(x)}{3a}$$

Antiderivative was successfully verified.

[In] Int[Sin[x]^3/(a + b*Csc[x]),x]

[Out] $-(b*(a^2 + 2*b^2)*x)/(2*a^4) - (2*b^4*ArcTanh[(a + b*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^4*Sqrt[a^2 - b^2]) - ((2*a^2 + 3*b^2)*Cos[x])/(3*a^3) + (b*Cos[x]*Sin[x])/(2*a^2) - (Cos[x]*Sin[x]^2)/(3*a)$

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 2660

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*

e^{2*x^2}), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3831

Int[csc[(e_.) + (f_.)*(x_.)]/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3853

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n), x] - Dist[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[b*n - a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]

Rule 3919

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 4104

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.), x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \frac{\sin^3(x)}{a + b \csc(x)} dx &= -\frac{\cos(x) \sin^2(x)}{3a} + \frac{\int \frac{(-3b+2a \csc(x)+2b \csc^2(x)) \sin^2(x)}{a+b \csc(x)} dx}{3a} \\
&= \frac{b \cos(x) \sin(x)}{2a^2} - \frac{\cos(x) \sin^2(x)}{3a} - \frac{\int \frac{(-2(2a^2+3b^2)-ab \csc(x)+3b^2 \csc^2(x)) \sin(x)}{a+b \csc(x)} dx}{6a^2} \\
&= -\frac{(2a^2 + 3b^2) \cos(x)}{3a^3} + \frac{b \cos(x) \sin(x)}{2a^2} - \frac{\cos(x) \sin^2(x)}{3a} + \frac{\int \frac{-3b(a^2+2b^2)-3ab^2 \csc(x)}{a+b \csc(x)} dx}{6a^3} \\
&= -\frac{b(a^2 + 2b^2)x}{2a^4} - \frac{(2a^2 + 3b^2) \cos(x)}{3a^3} + \frac{b \cos(x) \sin(x)}{2a^2} - \frac{\cos(x) \sin^2(x)}{3a} + \frac{b^4 \int \frac{\csc(x)}{a+b \csc(x)} dx}{a^4} \\
&= -\frac{b(a^2 + 2b^2)x}{2a^4} - \frac{(2a^2 + 3b^2) \cos(x)}{3a^3} + \frac{b \cos(x) \sin(x)}{2a^2} - \frac{\cos(x) \sin^2(x)}{3a} + \frac{b^3 \int \frac{1}{1+\frac{a \sin(x)}{b}} dx}{a^4} \\
&= -\frac{b(a^2 + 2b^2)x}{2a^4} - \frac{(2a^2 + 3b^2) \cos(x)}{3a^3} + \frac{b \cos(x) \sin(x)}{2a^2} - \frac{\cos(x) \sin^2(x)}{3a} + \frac{(2b^3) \text{Subst} \left(\int \frac{1}{1+\frac{a \sin(x)}{b}} dx \right)}{a^4} \\
&= -\frac{b(a^2 + 2b^2)x}{2a^4} - \frac{(2a^2 + 3b^2) \cos(x)}{3a^3} + \frac{b \cos(x) \sin(x)}{2a^2} - \frac{\cos(x) \sin^2(x)}{3a} - \frac{(4b^3) \text{Subst} \left(\int \frac{1}{1+\frac{a \sin(x)}{b}} dx \right)}{a^4} \\
&= -\frac{b(a^2 + 2b^2)x}{2a^4} - \frac{2b^4 \tanh^{-1} \left(\frac{b \left(\frac{a}{b} + \tan \left(\frac{x}{2} \right) \right)}{\sqrt{a^2 - b^2}} \right)}{a^4 \sqrt{a^2 - b^2}} - \frac{(2a^2 + 3b^2) \cos(x)}{3a^3} + \frac{b \cos(x) \sin(x)}{2a^2} - \frac{\cos(x) \sin^2(x)}{3a}
\end{aligned}$$

Mathematica [A] time = 0.23, size = 98, normalized size = 0.89

$$\frac{a^3 \cos(3x) - 6bx(a^2 + 2b^2) - 3a(3a^2 + 4b^2) \cos(x) + \frac{24b^4 \tan^{-1} \left(\frac{a+b \tan \left(\frac{x}{2} \right)}{\sqrt{b^2 - a^2}} \right)}{\sqrt{b^2 - a^2}} + 3a^2 b \sin(2x)}{12a^4}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]^3/(a + b*Csc[x]),x]

[Out] (-6*b*(a^2 + 2*b^2)*x + (24*b^4*ArcTan[(a + b*Tan[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] - 3*a*(3*a^2 + 4*b^2)*Cos[x] + a^3*Cos[3*x] + 3*a^2*b*Sin[2*x])/(12*a^4)

fricas [A] time = 0.62, size = 329, normalized size = 2.99

$$\frac{3\sqrt{a^2 - b^2} b^4 \log\left(-\frac{(a^2 - 2b^2)\cos(x)^2 + 2ab\sin(x) + a^2 + b^2 - 2(b\cos(x)\sin(x) + a\cos(x))\sqrt{a^2 - b^2}}{a^2\cos(x)^2 - 2ab\sin(x) - a^2 - b^2}\right) + 2(a^5 - a^3b^2)\cos(x)^3 + 3(a^4b - a^2b^3)\cos(x)\sin(x) - 3(a^4b + a^2b^3 - 2b^5)x - 6(a^5 - a^4b)\cos(x)}{6(a^6 - a^4b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^3/(a+b*csc(x)),x, algorithm="fricas")

[Out] [1/6*(3*sqrt(a^2 - b^2)*b^4*log(-((a^2 - 2*b^2)*cos(x)^2 + 2*a*b*sin(x) + a^2 + b^2 - 2*(b*cos(x)*sin(x) + a*cos(x))*sqrt(a^2 - b^2))/(a^2*cos(x)^2 - 2*a*b*sin(x) - a^2 - b^2)) + 2*(a^5 - a^3*b^2)*cos(x)^3 + 3*(a^4*b - a^2*b^3)*cos(x)*sin(x) - 3*(a^4*b + a^2*b^3 - 2*b^5)*x - 6*(a^5 - a*b^4)*cos(x))/(a^6 - a^4*b^2), -1/6*(6*sqrt(-a^2 + b^2)*b^4*arctan(-sqrt(-a^2 + b^2)*(b*sin(x) + a)/((a^2 - b^2)*cos(x))) - 2*(a^5 - a^3*b^2)*cos(x)^3 - 3*(a^4*b - a^2*b^3)*cos(x)*sin(x) + 3*(a^4*b + a^2*b^3 - 2*b^5)*x + 6*(a^5 - a*b^4)*cos(x))/(a^6 - a^4*b^2)]

giac [A] time = 0.61, size = 149, normalized size = 1.35

$$\frac{2\left(\pi\left[\frac{x}{2\pi} + \frac{1}{2}\right]\operatorname{sgn}(b) + \arctan\left(\frac{b\tan\left(\frac{1}{2}x\right) + a}{\sqrt{-a^2 + b^2}}\right)\right)b^4}{\sqrt{-a^2 + b^2}a^4} - \frac{(a^2b + 2b^3)x}{2a^4} - \frac{3ab\tan\left(\frac{1}{2}x\right)^5 + 6b^2\tan\left(\frac{1}{2}x\right)^4 + 12a^2\tan\left(\frac{1}{2}x\right)^3}{3\left(\tan\left(\frac{1}{2}x\right)\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)^3/(a+b*csc(x)),x, algorithm="giac")

[Out] 2*(pi*floor(1/2*x/pi + 1/2)*sgn(b) + arctan((b*tan(1/2*x) + a)/sqrt(-a^2 + b^2)))*b^4/(sqrt(-a^2 + b^2)*a^4) - 1/2*(a^2*b + 2*b^3)*x/a^4 - 1/3*(3*a*b*tan(1/2*x)^5 + 6*b^2*tan(1/2*x)^4 + 12*a^2*tan(1/2*x)^3 + 12*b^2*tan(1/2*x)^2 - 3*a*b*tan(1/2*x) + 4*a^2 + 6*b^2)/((tan(1/2*x)^2 + 1)^3*a^3)

maple [B] time = 0.44, size = 213, normalized size = 1.94

$$\frac{2b^4 \arctan\left(\frac{2\tan\left(\frac{x}{2}\right)b + 2a}{2\sqrt{-a^2 + b^2}}\right)}{a^4\sqrt{-a^2 + b^2}} - \frac{\left(\tan^5\left(\frac{x}{2}\right)\right)b}{a^2\left(\tan^2\left(\frac{x}{2}\right) + 1\right)^3} - \frac{2b^2\left(\tan^4\left(\frac{x}{2}\right)\right)}{a^3\left(\tan^2\left(\frac{x}{2}\right) + 1\right)^3} - \frac{4\left(\tan^2\left(\frac{x}{2}\right)\right)}{a\left(\tan^2\left(\frac{x}{2}\right) + 1\right)^3} - \frac{4\left(\tan^2\left(\frac{x}{2}\right)\right)b^2}{a^3\left(\tan^2\left(\frac{x}{2}\right) + 1\right)^3} + \frac{b}{a^2\left(\tan^2\left(\frac{x}{2}\right) + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(x)^3/(a+b*csc(x)),x)`

[Out]
$$\frac{2a^4b^4}{(-a^2+b^2)^{1/2}} \arctan\left(\frac{1/2(2\tan(1/2x)b+2a)}{(-a^2+b^2)^{1/2}}\right) - \frac{1}{a^2} \frac{(\tan(1/2x)^2+1)^3 \tan(1/2x)^5 b - 2/a^3 (\tan(1/2x)^2+1)^3 b^2 \tan(1/2x)^4 - 4/a (\tan(1/2x)^2+1)^3 \tan(1/2x)^2 - 4/a^3 (\tan(1/2x)^2+1)^3 \tan(1/2x)^2 b^2 + 1/a^2 (\tan(1/2x)^2+1)^3 b \tan(1/2x) - 4/3/a (\tan(1/2x)^2+1)^3 - 2/a^3 (\tan(1/2x)^2+1)^3 b^2 - 1/a^2 b \arctan(\tan(1/2x)) - 2/a^4 \arctan(\tan(1/2x))}{b^3}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)^3/(a+b*csc(x)),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details) Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 0.95, size = 1218, normalized size = 11.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(x)^3/(a + b/sin(x)),x)`

[Out]
$$\begin{aligned} & - \left(\frac{2(2a^2 + 3b^2)}{3a^3} + \frac{(b \tan(x/2))^5}{a^2} + \frac{(2b^2 \tan(x/2))^4}{a^3} + \frac{4 \tan(x/2)^2 (a^2 + b^2)}{a^3} - \frac{(b \tan(x/2))^4}{a^2} \right) \frac{1}{(3 \tan(x/2)^2 + 3 \tan(x/2)^4 + \tan(x/2)^6 + 1)} \\ & - \frac{(b^4 \operatorname{atan}((b^4(a^2 - b^2)^{1/2} * ((8(4a^3b^8 + 4a^5b^6 + a^7b^4)) / a^8 + (8 \tan(x/2) * (4a^5b^7 - 8a^3b^9 + 7a^7b^5 + 2a^9b^3)) / a^9 + (b^4(a^2 - b^2)^{1/2} * ((8(2a^8b^4 + 2a^{10}b^2))) / a^8 + (64b^5 \tan(x/2)) / a + (b^4(a^2 - b^2)^{1/2} * (32a^3b^2 + (8 \tan(x/2) * (12a^{13}b - 8a^{11}b^3)) / a^9)) / (a^6 - a^4b^2))) / (a^6 - a^4b^2)) * 1i)}{(a^6 - a^4b^2)} \\ & + \frac{(b^4(a^2 - b^2)^{1/2} * ((8(4a^3b^8 + 4a^5b^6 + a^7b^4)) / a^8 + (8 \tan(x/2) * (4a^5b^7 - 8a^3b^9 + 7a^7b^5 + 2a^9b^3)) / a^9 - (b^4(a^2 - b^2)^{1/2} * ((8(2a^8b^4 + 2a^{10}b^2)) / a^8 + (64b^5 \tan(x/2)) / a - (b^4(a^2 - b^2)^{1/2} * (32a^3b^2 + (8 \tan(x/2) * (12a^{13}b - 8a^{11}b^3)) / a^9)) / (a^6 - a^4b^2))) / (a^6 - a^4b^2)) * 1i)}{(a^6 - a^4b^2)} \\ & + \frac{(16 * (2b^{10} + a^2b^8)) / a^8 + (16 \tan(x/2) * (8b^{11} + 8a^2b^9 + 2a^4b^7)) / a^9 + (b^4(a^2 - b^2)^{1/2} * ((8(4a^3b^8 + 4a^5b^6 + a^7b^4)) / a^8 + (8 \tan(x/2) * (4a^5b^7 - 8a^3b^9 + 7a^7b^5 + 2a^9b^3)) / a^9 + (b^4(a^2 - b^2)^{1/2} * ((8(2a^8b^4 + 2a^{10}b^2)) / a^8 + (64b^5 \tan(x/2)) / a + (b^4(a^2 - b^2)^{1/2} * (32a^3b^2 + (8 \tan(x/2) * (12a^{13}b - 8a^{11}b^3)) / a^9)) / (a^6 - a^4b^2))) / (a^6 - a^4b^2)) * 1i)}{(a^6 - a^4b^2)} \end{aligned}$$

$$3.48 \quad \int \frac{\sin^4(x)}{a+b \csc(x)} dx$$

Optimal. Leaf size=144

$$\frac{b \sin^2(x) \cos(x)}{3a^2} + \frac{2b^5 \tanh^{-1}\left(\frac{a+b \tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^5 \sqrt{a^2-b^2}} + \frac{b(2a^2+3b^2) \cos(x)}{3a^4} - \frac{(3a^2+4b^2) \sin(x) \cos(x)}{8a^3} + \frac{x(3a^4+4a^2b^2+8b^4)}{8a^5}$$

[Out] 1/8*(3*a^4+4*a^2*b^2+8*b^4)*x/a^5+1/3*b*(2*a^2+3*b^2)*cos(x)/a^4-1/8*(3*a^2+4*b^2)*cos(x)*sin(x)/a^3+1/3*b*cos(x)*sin(x)^2/a^2-1/4*cos(x)*sin(x)^3/a+2*b^5*arctanh((a+b*tan(1/2*x))/(a^2-b^2)^(1/2))/a^5/(a^2-b^2)^(1/2)

Rubi [A] time = 0.59, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3853, 4104, 3919, 3831, 2660, 618, 206}

$$\frac{x(4a^2b^2+3a^4+8b^4)}{8a^5} + \frac{b(2a^2+3b^2) \cos(x)}{3a^4} - \frac{(3a^2+4b^2) \sin(x) \cos(x)}{8a^3} + \frac{2b^5 \tanh^{-1}\left(\frac{a+b \tan\left(\frac{x}{2}\right)}{\sqrt{a^2-b^2}}\right)}{a^5 \sqrt{a^2-b^2}} + \frac{b \sin^2(x) \cos(x)}{3a^2}$$

Antiderivative was successfully verified.

[In] Int[Sin[x]^4/(a + b*Csc[x]), x]

[Out] ((3*a^4 + 4*a^2*b^2 + 8*b^4)*x)/(8*a^5) + (2*b^5*ArcTanh[(a + b*Tan[x/2])/Sqrt[a^2 - b^2]])/(a^5*Sqrt[a^2 - b^2]) + (b*(2*a^2 + 3*b^2)*Cos[x])/(3*a^4) - ((3*a^2 + 4*b^2)*Cos[x]*Sin[x])/(8*a^3) + (b*Cos[x]*Sin[x]^2)/(3*a^2) - (Cos[x]*Sin[x]^3)/(4*a)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 2660

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*

e^{2x^2}), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3831

Int[csc[(e_.) + (f_.)*(x_.)]/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3853

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Simp[(Cot[e + f*x]*(d*Csc[e + f*x])^n)/(a*f*n), x] - Dist[1/(a*d*n), Int[((d*Csc[e + f*x])^(n + 1)*Simp[b*n - a*(n + 1)*Csc[e + f*x] - b*(n + 1)*Csc[e + f*x]^2, x])/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1] && IntegerQ[2*n]

Rule 3919

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 4104

Int[((A_.) + csc[(e_.) + (f_.)*(x_.)]*(B_.) + csc[(e_.) + (f_.)*(x_.)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^(m_.), x_Symbol] := Simp[(A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n)/(a*f*n), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]

Rubi steps

$$\begin{aligned}
\int \frac{\sin^4(x)}{a + b \csc(x)} dx &= -\frac{\cos(x) \sin^3(x)}{4a} + \frac{\int \frac{(-4b+3a \csc(x)+3b \csc^2(x)) \sin^3(x)}{a+b \csc(x)} dx}{4a} \\
&= \frac{b \cos(x) \sin^2(x)}{3a^2} - \frac{\cos(x) \sin^3(x)}{4a} - \frac{\int \frac{(-3(3a^2+4b^2)-ab \csc(x)+8b^2 \csc^2(x)) \sin^2(x)}{a+b \csc(x)} dx}{12a^2} \\
&= -\frac{(3a^2 + 4b^2) \cos(x) \sin(x)}{8a^3} + \frac{b \cos(x) \sin^2(x)}{3a^2} - \frac{\cos(x) \sin^3(x)}{4a} + \frac{\int \frac{(-8b(2a^2+3b^2)+a(9a^2-4b^2)) \csc(x)}{a+b \csc(x)} dx}{2a^2} \\
&= \frac{b(2a^2 + 3b^2) \cos(x)}{3a^4} - \frac{(3a^2 + 4b^2) \cos(x) \sin(x)}{8a^3} + \frac{b \cos(x) \sin^2(x)}{3a^2} - \frac{\cos(x) \sin^3(x)}{4a} - \frac{\int \frac{-3b(2a^2+3b^2)+a(9a^2-4b^2)}{a+b \csc(x)} dx}{2a^2} \\
&= \frac{(3a^4 + 4a^2b^2 + 8b^4)x}{8a^5} + \frac{b(2a^2 + 3b^2) \cos(x)}{3a^4} - \frac{(3a^2 + 4b^2) \cos(x) \sin(x)}{8a^3} + \frac{b \cos(x) \sin^2(x)}{3a^2} \\
&= \frac{(3a^4 + 4a^2b^2 + 8b^4)x}{8a^5} + \frac{b(2a^2 + 3b^2) \cos(x)}{3a^4} - \frac{(3a^2 + 4b^2) \cos(x) \sin(x)}{8a^3} + \frac{b \cos(x) \sin^2(x)}{3a^2} \\
&= \frac{(3a^4 + 4a^2b^2 + 8b^4)x}{8a^5} + \frac{b(2a^2 + 3b^2) \cos(x)}{3a^4} - \frac{(3a^2 + 4b^2) \cos(x) \sin(x)}{8a^3} + \frac{b \cos(x) \sin^2(x)}{3a^2} \\
&= \frac{(3a^4 + 4a^2b^2 + 8b^4)x}{8a^5} + \frac{b(2a^2 + 3b^2) \cos(x)}{3a^4} - \frac{(3a^2 + 4b^2) \cos(x) \sin(x)}{8a^3} + \frac{b \cos(x) \sin^2(x)}{3a^2} \\
&= \frac{(3a^4 + 4a^2b^2 + 8b^4)x}{8a^5} + \frac{2b^5 \tanh^{-1}\left(\frac{b\left(\frac{a}{b} + \tan\left(\frac{x}{2}\right)\right)}{\sqrt{a^2 - b^2}}\right)}{a^5 \sqrt{a^2 - b^2}} + \frac{b(2a^2 + 3b^2) \cos(x)}{3a^4} - \frac{(3a^2 + 4b^2) \cos(x) \sin(x)}{8a^3} + \frac{b \cos(x) \sin^2(x)}{3a^2}
\end{aligned}$$

Mathematica [A] time = 0.31, size = 129, normalized size = 0.90

$$\frac{36a^4x - 24a^4 \sin(2x) + 3a^4 \sin(4x) - 8a^3b \cos(3x) + 48a^2b^2x - 24a^2b^2 \sin(2x) + 24ab(3a^2 + 4b^2) \cos(x) - \frac{192b^5}{96a^5}}{96a^5}$$

Antiderivative was successfully verified.

[In] Integrate[Sin[x]^4/(a + b*Csc[x]),x]

[Out] $(36a^4x + 48a^2b^2x + 96b^4x - (192b^5 \operatorname{ArcTan}[(a + b \tan(x/2))/\sqrt{-a^2 + b^2}])/\sqrt{-a^2 + b^2})/\sqrt{-a^2 + b^2} + 24ab(3a^2 + 4b^2)\cos(x) - 8a^3b \cos(3x) - 24a^4 \sin(2x) - 24a^2b^2 \sin(2x) + 3a^4 \sin(4x))/(96a^5)$

fricas [A] time = 1.13, size = 410, normalized size = 2.85

$$\frac{12\sqrt{a^2 - b^2} b^5 \log\left(\frac{(a^2 - 2b^2)\cos(x)^2 + 2ab\sin(x) + a^2 + b^2 + 2(b\cos(x)\sin(x) + a\cos(x))\sqrt{a^2 - b^2}}{a^2\cos(x)^2 - 2ab\sin(x) - a^2 - b^2}\right) - 8(a^5b - a^3b^3)\cos(x)^3 + 3(3a^6 - a^4b^2 + 4a^2b^4 - 8b^6)x + 24(a^5b - ab^5)\cos(x) + 3(2(a^6 - a^4b^2)\cos(x)^3 - (5a^6 - a^4b^2 - 4a^2b^4)\cos(x))\sin(x)}{24(a^7 - a^5b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)^4/(a+b*csc(x)),x, algorithm="fricas")`

[Out] $[1/24*(12*\sqrt{a^2 - b^2}*b^5*\log(((a^2 - 2*b^2)*\cos(x)^2 + 2*a*b*\sin(x) + a^2 + b^2 + 2*(b*\cos(x)*\sin(x) + a*\cos(x))*\sqrt{a^2 - b^2}))/((a^2*\cos(x)^2 - 2*a*b*\sin(x) - a^2 - b^2)) - 8*(a^5*b - a^3*b^3)*\cos(x)^3 + 3*(3*a^6 + a^4*b^2 + 4*a^2*b^4 - 8*b^6)*x + 24*(a^5*b - a*b^5)*\cos(x) + 3*(2*(a^6 - a^4*b^2)*\cos(x)^3 - (5*a^6 - a^4*b^2 - 4*a^2*b^4)*\cos(x))*\sin(x))/(a^7 - a^5*b^2), 1/24*(24*\sqrt{-a^2 + b^2}*b^5*\arctan(-\sqrt{-a^2 + b^2}*(b*\sin(x) + a)/((a^2 - b^2)*\cos(x))) - 8*(a^5*b - a^3*b^3)*\cos(x)^3 + 3*(3*a^6 + a^4*b^2 + 4*a^2*b^4 - 8*b^6)*x + 24*(a^5*b - a*b^5)*\cos(x) + 3*(2*(a^6 - a^4*b^2)*\cos(x)^3 - (5*a^6 - a^4*b^2 - 4*a^2*b^4)*\cos(x))*\sin(x))/(a^7 - a^5*b^2)]$

giac [A] time = 0.66, size = 252, normalized size = 1.75

$$\frac{2\left(\pi\left[\frac{x}{2\pi} + \frac{1}{2}\right] \operatorname{sgn}(b) + \arctan\left(\frac{b \tan\left(\frac{1}{2}x\right) + a}{\sqrt{-a^2 + b^2}}\right)\right) b^5}{\sqrt{-a^2 + b^2} a^5} + \frac{(3a^4 + 4a^2b^2 + 8b^4)x}{8a^5} + \frac{9a^3 \tan\left(\frac{1}{2}x\right)^7 + 12ab^2 \tan\left(\frac{1}{2}x\right)^7}{8a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)^4/(a+b*csc(x)),x, algorithm="giac")`

[Out] $-2*(\pi*\operatorname{floor}(1/2*x/\pi + 1/2)*\operatorname{sgn}(b) + \arctan((b*\tan(1/2*x) + a)/\sqrt{-a^2 + b^2}))*b^5/(\sqrt{-a^2 + b^2}*a^5) + 1/8*(3*a^4 + 4*a^2*b^2 + 8*b^4)*x/a^5 + 1/12*(9*a^3*\tan(1/2*x)^7 + 12*a*b^2*\tan(1/2*x)^7 + 24*b^3*\tan(1/2*x)^6 + 33*a^3*\tan(1/2*x)^5 + 12*a*b^2*\tan(1/2*x)^5 + 48*a^2*b*\tan(1/2*x)^4 + 72*b^3*\tan(1/2*x)^4 - 33*a^3*\tan(1/2*x)^3 - 12*a*b^2*\tan(1/2*x)^3 + 64*a^2*b*\tan(1/2*x)^2 + 72*b^3*\tan(1/2*x)^2 - 9*a^3*\tan(1/2*x) - 12*a*b^2*\tan(1/2*x) + 16*a^2*b + 24*b^3)/((\tan(1/2*x)^2 + 1)^4*a^4)$

maple [B] time = 0.38, size = 405, normalized size = 2.81

$$-\frac{2b^5 \arctan\left(\frac{2\tan\left(\frac{x}{2}\right)b+2a}{2\sqrt{-a^2+b^2}}\right)}{a^5\sqrt{-a^2+b^2}} + \frac{3\left(\tan^7\left(\frac{x}{2}\right)\right)}{4a\left(\tan^2\left(\frac{x}{2}\right)+1\right)^4} + \frac{\left(\tan^7\left(\frac{x}{2}\right)\right)b^2}{a^3\left(\tan^2\left(\frac{x}{2}\right)+1\right)^4} + \frac{2b^3\left(\tan^6\left(\frac{x}{2}\right)\right)}{a^4\left(\tan^2\left(\frac{x}{2}\right)+1\right)^4} + \frac{\left(\tan^5\left(\frac{x}{2}\right)\right)b^2}{a^3\left(\tan^2\left(\frac{x}{2}\right)+1\right)^4} + \frac{11}{4a\left(\tan^2\left(\frac{x}{2}\right)+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(x)^4/(a+b*csc(x)),x)`

[Out] $-2*b^5/a^5/(-a^2+b^2)^{(1/2)}*\arctan(1/2*(2*\tan(1/2*x)*b+2*a)/(-a^2+b^2)^{(1/2)})+3/4/a/(\tan(1/2*x)^2+1)^4*\tan(1/2*x)^7+1/a^3/(\tan(1/2*x)^2+1)^4*\tan(1/2*x)^7*b^2+2/a^4/(\tan(1/2*x)^2+1)^4*b^3*\tan(1/2*x)^6+1/a^3/(\tan(1/2*x)^2+1)^4*\tan(1/2*x)^5*b^2+11/4/a/(\tan(1/2*x)^2+1)^4*\tan(1/2*x)^5+4/a^2/(\tan(1/2*x)^2+1)^4*\tan(1/2*x)^4*b+6/a^4/(\tan(1/2*x)^2+1)^4*\tan(1/2*x)^4*b^3-1/a^3/(\tan(1/2*x)^2+1)^4*\tan(1/2*x)^3*b^2-11/4/a/(\tan(1/2*x)^2+1)^4*\tan(1/2*x)^3+6/a^4/(\tan(1/2*x)^2+1)^4*\tan(1/2*x)^2*b^3+16/3/a^2/(\tan(1/2*x)^2+1)^4*\tan(1/2*x)^2*b-3/4/a/(\tan(1/2*x)^2+1)^4*\tan(1/2*x)-1/a^3/(\tan(1/2*x)^2+1)^4*\tan(1/2*x)*b^2+4/3/a^2/(\tan(1/2*x)^2+1)^4*b+2/a^4/(\tan(1/2*x)^2+1)^4*b^3+2/a^5*\arctan(\tan(1/2*x))*b^4+3/4/a*\arctan(\tan(1/2*x))+1/a^3*\arctan(\tan(1/2*x))*b^2$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sin(x)^4/(a+b*csc(x)),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see 'assume?' for more details)Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 1.26, size = 1639, normalized size = 11.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(x)^4/(a + b/sin(x)),x)`

[Out] $((2*(2*a^2*b + 3*b^3))/(3*a^4) - (\tan(x/2)*(3*a^2 + 4*b^2))/(4*a^3) + (\tan(x/2)^7*(3*a^2 + 4*b^2))/(4*a^3) - (\tan(x/2)^3*(11*a^2 + 4*b^2))/(4*a^3) + (\tan(x/2)^5*(11*a^2 + 4*b^2))/(4*a^3) + (2*b^3*\tan(x/2)^6)/a^4 + (2*\tan(x/2)^4*(2*a^2*b + 3*b^3))/a^4 + (2*\tan(x/2)^2*(8*a^2*b + 9*b^3))/(3*a^4))/(4*a^4)$

$$\begin{aligned} & n(x/2)^2 + 6*\tan(x/2)^4 + 4*\tan(x/2)^6 + \tan(x/2)^8 + 1) - (\operatorname{atan}((81*b^3*\tan(x/2))/ \\ & (8*((27*a^2*b)/8 + (81*b^3)/8 + (63*b^5)/(2*a^2) + (35*b^7)/a^4 + (40*b^9)/a^6)) + \\ & (63*b^5*\tan(x/2))/(2*((27*a^4*b)/8 + (63*b^5)/2 + (81*a^2*b^3)/8 + (35*b^7)/a^2 + \\ & (40*b^9)/a^4)) + (35*b^7*\tan(x/2))/((27*a^6*b)/8 + 35*b^7 + (63*a^2*b^5)/2 + \\ & (81*a^4*b^3)/8 + (40*b^9)/a^2) + (40*b^9*\tan(x/2))/((27*a^8*b)/8 + 40*b^9 + \\ & 35*a^2*b^7 + (63*a^4*b^5)/2 + (81*a^6*b^3)/8) + (27*a*b*\tan(x/2))/(8*((27*a*b)/8 + \\ & (81*b^3)/(8*a) + (63*b^5)/(2*a^3) + (35*b^7)/a^5 + (40*b^9)/a^7))) * (a^4*3i + b^4*8i + a^2*b^2*4i) * 1i) / (4*a^5) + \\ & (b^5 * \operatorname{atan}(((b^5*(a^2 - b^2)^{(1/2)} * ((32*a^4*b^10 + 32*a^6*b^8 + 32*a^8*b^6 + 12*a^10*b^4 + \\ & (9*a^12*b^2)/2)/a^11 + (\tan(x/2)*(18*a^14*b - 128*a^4*b^11 + 64*a^6*b^9 + \\ & 64*a^8*b^7 + 104*a^10*b^5 + 39*a^12*b^3))/(2*a^12) + (b^5*(a^2 - b^2)^{(1/2)} * \\ & ((12*a^14*b + 16*a^10*b^5 + 4*a^12*b^3)/a^11 + (64*b^6*\tan(x/2))/a^2 + (b^5*(a^2 - \\ & b^2)^{(1/2)} * (32*a^3*b^2 + (\tan(x/2)*(192*a^16*b - 128*a^14*b^3)))/(2*a^12)))/ \\ & (a^7 - a^5*b^2)))/((a^7 - a^5*b^2)) * 1i) / (a^7 - a^5*b^2) + (b^5*(a^2 - b^2)^{(1/2)} * \\ & ((32*a^4*b^10 + 32*a^6*b^8 + 32*a^8*b^6 + 12*a^10*b^4 + (9*a^12*b^2)/2)/a^11 + \\ & (\tan(x/2)*(18*a^14*b - 128*a^4*b^11 + 64*a^6*b^9 + 64*a^8*b^7 + 104*a^10*b^5 + \\ & 39*a^12*b^3))/(2*a^12) - (b^5*(a^2 - b^2)^{(1/2)} * ((12*a^14*b + 16*a^10*b^5 + \\ & 4*a^12*b^3)/a^11 + (64*b^6*\tan(x/2))/a^2 - (b^5*(a^2 - b^2)^{(1/2)} * (32*a^3*b^2 + \\ & (\tan(x/2)*(192*a^16*b - 128*a^14*b^3)))/(2*a^12)))/((a^7 - a^5*b^2)))/ \\ & (a^7 - a^5*b^2)) * 1i) / (a^7 - a^5*b^2) / ((32*b^13 + 40*a^2*b^11 + 24*a^4*b^9 + \\ & 9*a^6*b^7)/a^11 + (\tan(x/2)*(128*b^14 + 128*a^2*b^12 + 128*a^4*b^10 + 48*a^6*b^8 + \\ & 18*a^8*b^6))/a^12 + (b^5*(a^2 - b^2)^{(1/2)} * ((32*a^4*b^10 + 32*a^6*b^8 + 32*a^8*b^6 + \\ & 12*a^10*b^4 + (9*a^12*b^2)/2)/a^11 + (\tan(x/2)*(18*a^14*b - 128*a^4*b^11 + 64*a^6*b^9 + \\ & 64*a^8*b^7 + 104*a^10*b^5 + 39*a^12*b^3))/(2*a^12) + (b^5*(a^2 - b^2)^{(1/2)} * \\ & ((12*a^14*b + 16*a^10*b^5 + 4*a^12*b^3)/a^11 + (64*b^6*\tan(x/2))/a^2 + (b^5*(a^2 - \\ & b^2)^{(1/2)} * (32*a^3*b^2 + (\tan(x/2)*(192*a^16*b - 128*a^14*b^3)))/(2*a^12)))/ \\ & (a^7 - a^5*b^2)))/((a^7 - a^5*b^2)) - (b^5*(a^2 - b^2)^{(1/2)} * ((32*a^4*b^10 + \\ & 32*a^6*b^8 + 32*a^8*b^6 + 12*a^10*b^4 + (9*a^12*b^2)/2)/a^11 + (\tan(x/2)*(18*a^14*b - \\ & 128*a^4*b^11 + 64*a^6*b^9 + 64*a^8*b^7 + 104*a^10*b^5 + 39*a^12*b^3))/(2*a^12) - \\ & (b^5*(a^2 - b^2)^{(1/2)} * ((12*a^14*b + 16*a^10*b^5 + 4*a^12*b^3)/a^11 + (64*b^6*\tan(x/2))/a^2 - \\ & (b^5*(a^2 - b^2)^{(1/2)} * (32*a^3*b^2 + (\tan(x/2)*(192*a^16*b - 128*a^14*b^3)))/(2*a^12)))/ \\ & (a^7 - a^5*b^2)))/((a^7 - a^5*b^2)) * (a^2 - b^2)^{(1/2)} * 2i) / (a^7 - a^5*b^2) \end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^4(x)}{a + b \csc(x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(x)**4/(a+b*csc(x)),x)

[Out] Integral(sin(x)**4/(a + b*csc(x)), x)

$$3.49 \quad \int \frac{1}{(a+b \csc(c+dx))^2} dx$$

Optimal. Leaf size=108

$$\frac{2b(2a^2 - b^2) \tanh^{-1}\left(\frac{a+b \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2 d (a^2 - b^2)^{3/2}} - \frac{b^2 \cot(c+dx)}{ad (a^2 - b^2) (a + b \csc(c+dx))} + \frac{x}{a^2}$$

[Out] $x/a^2 + 2*b*(2*a^2 - b^2)*\operatorname{arctanh}((a+b*\tan(1/2*d*x+1/2*c))/(a^2-b^2)^{(1/2)})/a^2 / (a^2-b^2)^{(3/2)} / d - b^2*\cot(d*x+c)/a/(a^2-b^2)/d/(a+b*\csc(d*x+c))$

Rubi [A] time = 0.17, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3785, 3919, 3831, 2660, 618, 206}

$$\frac{2b(2a^2 - b^2) \tanh^{-1}\left(\frac{a+b \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2 d (a^2 - b^2)^{3/2}} - \frac{b^2 \cot(c+dx)}{ad (a^2 - b^2) (a + b \csc(c+dx))} + \frac{x}{a^2}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + b*\operatorname{Csc}[c + d*x])^{-2}, x]$

[Out] $x/a^2 + (2*b*(2*a^2 - b^2)*\operatorname{ArcTanh}[(a + b*\operatorname{Tan}[(c + d*x)/2])/ \operatorname{Sqrt}[a^2 - b^2]]) / (a^2*(a^2 - b^2)^{(3/2)*d} - (b^2*\operatorname{Cot}[c + d*x]) / (a*(a^2 - b^2)*d*(a + b*\operatorname{Csc}[c + d*x])))$

Rule 206

$\operatorname{Int}[(a_.) + (b_.)*(x_.)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[\operatorname{Rt}[-b, 2]*x] / \operatorname{Rt}[a, 2]]) / (\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /;$ $\operatorname{FreeQ}[\{a, b\}, x] \ \&\& \ \operatorname{NegQ}[a/b] \ \&\& \ (\operatorname{Gt} Q[a, 0] \ || \ \operatorname{Lt} Q[b, 0])$

Rule 618

$\operatorname{Int}[(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^{-1}, x_Symbol] \rightarrow \operatorname{Dist}[-2, \operatorname{Subst}[\operatorname{Int}[1/\operatorname{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /;$ $\operatorname{FreeQ}[\{a, b, c\}, x] \ \&\& \ \operatorname{NeQ}[b^2 - 4*a*c, 0]$

Rule 2660

$\operatorname{Int}[(a_.) + (b_.)*\sin[(c_.) + (d_.)*(x_.)])^{-1}, x_Symbol] \rightarrow \operatorname{With}[\{e = \operatorname{FreeFactors}[\operatorname{Tan}[(c + d*x)/2], x]\}, \operatorname{Dist}[(2*e)/d, \operatorname{Subst}[\operatorname{Int}[1/(a + 2*b*e*x + a*$

e^{2*x^2}), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3785

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.) + (a_.))^(n_.), x_Symbol] := Simp[(b^2*Cot[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dist[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]

Rule 3831

Int[csc[(e_.) + (f_.)*(x_.)]/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]

Rule 3919

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.) + (c_.))/(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.)), x_Symbol] := Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + b \csc(c + dx))^2} dx &= -\frac{b^2 \cot(c + dx)}{a(a^2 - b^2)d(a + b \csc(c + dx))} - \frac{\int \frac{-a^2 + b^2 + ab \csc(c + dx)}{a + b \csc(c + dx)} dx}{a(a^2 - b^2)} \\
&= \frac{x}{a^2} - \frac{b^2 \cot(c + dx)}{a(a^2 - b^2)d(a + b \csc(c + dx))} - \frac{(b(2a^2 - b^2)) \int \frac{\csc(c + dx)}{a + b \csc(c + dx)} dx}{a^2(a^2 - b^2)} \\
&= \frac{x}{a^2} - \frac{b^2 \cot(c + dx)}{a(a^2 - b^2)d(a + b \csc(c + dx))} - \frac{(2a^2 - b^2) \int \frac{1}{1 + \frac{a \sin(c + dx)}{b}} dx}{a^2(a^2 - b^2)} \\
&= \frac{x}{a^2} - \frac{b^2 \cot(c + dx)}{a(a^2 - b^2)d(a + b \csc(c + dx))} - \frac{(2(2a^2 - b^2)) \text{Subst}\left(\int \frac{1}{1 + \frac{2ax}{b} + x^2} dx, x, \tan\left(\frac{1}{2}\right)\right)}{a^2(a^2 - b^2)d} \\
&= \frac{x}{a^2} - \frac{b^2 \cot(c + dx)}{a(a^2 - b^2)d(a + b \csc(c + dx))} + \frac{(4(2a^2 - b^2)) \text{Subst}\left(\int \frac{1}{-4\left(1 - \frac{a^2}{b^2}\right) - x^2} dx, x, \frac{2a}{b}\right)}{a^2(a^2 - b^2)d} \\
&= \frac{x}{a^2} + \frac{2b(2a^2 - b^2) \tanh^{-1}\left(\frac{b\left(\frac{a}{b} + \tan\left(\frac{1}{2}(c + dx)\right)\right)}{\sqrt{a^2 - b^2}}\right)}{a^2(a^2 - b^2)^{3/2}d} - \frac{b^2 \cot(c + dx)}{a(a^2 - b^2)d(a + b \csc(c + dx))}
\end{aligned}$$

Mathematica [A] time = 0.44, size = 139, normalized size = 1.29

$$\frac{\csc(c + dx)(a \sin(c + dx) + b) \left(-\frac{2b(b^2 - 2a^2) \tan^{-1}\left(\frac{a + b \tan\left(\frac{1}{2}(c + dx)\right)}{\sqrt{b^2 - a^2}}\right)(a + b \csc(c + dx))}{(b^2 - a^2)^{3/2}} + \frac{ab^2 \cot(c + dx)}{(b - a)(a + b)} + (c + dx)(a + b \csc(c + dx)) \right)}{a^2 d (a + b \csc(c + dx))^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Csc[c + d*x])^(-2), x]

[Out] (Csc[c + d*x]*((a*b^2*Cot[c + d*x])/((-a + b)*(a + b)) + (c + d*x)*(a + b*Csc[c + d*x]) - (2*b*(-2*a^2 + b^2)*ArcTan[(a + b*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]])*(a + b*Csc[c + d*x]))/(-a^2 + b^2)^(3/2)*(b + a*Sin[c + d*x]))/(a^2*d*(a + b*Csc[c + d*x])^2)

fricas [B] time = 0.61, size = 493, normalized size = 4.56

$$\frac{2(a^5 - 2a^3b^2 + ab^4)dx \sin(dx + c) + 2(a^4b - 2a^2b^3 + b^5)dx + (2a^2b^2 - b^4 + (2a^3b - ab^3) \sin(dx + c))\sqrt{a^2 - b^2}}{2((a^7 - 2a^5b^2 + a^3b^4)d \sin(dx + c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*csc(d*x+c))^2,x, algorithm="fricas")

[Out] [1/2*(2*(a^5 - 2*a^3*b^2 + a*b^4)*d*x*sin(d*x + c) + 2*(a^4*b - 2*a^2*b^3 + b^5)*d*x + (2*a^2*b^2 - b^4 + (2*a^3*b - a*b^3)*sin(d*x + c))*sqrt(a^2 - b^2)*log(((a^2 - 2*b^2)*cos(d*x + c)^2 + 2*a*b*sin(d*x + c) + a^2 + b^2 + 2*(b*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c))*sqrt(a^2 - b^2))/(a^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2)) - 2*(a^3*b^2 - a*b^4)*cos(d*x + c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*sin(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d), ((a^5 - 2*a^3*b^2 + a*b^4)*d*x*sin(d*x + c) + (a^4*b - 2*a^2*b^3 + b^5)*d*x + (2*a^2*b^2 - b^4 + (2*a^3*b - a*b^3)*sin(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*sin(d*x + c) + a)/((a^2 - b^2)*cos(d*x + c)))) - (a^3*b^2 - a*b^4)*cos(d*x + c))/((a^7 - 2*a^5*b^2 + a^3*b^4)*d*sin(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d)]

giac [A] time = 0.53, size = 158, normalized size = 1.46

$$\frac{2(2a^2b - b^3) \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(b) + \arctan \left(\frac{b \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + a}{\sqrt{-a^2 + b^2}} \right) \right)}{(a^4 - a^2b^2) \sqrt{-a^2 + b^2}} + \frac{2(ab \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + b^2)}{(a^3 - ab^2) \left(b \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 + 2a \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + b \right)} - \frac{dx+c}{a^2}$$

d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*csc(d*x+c))^2,x, algorithm="giac")

[Out] -(2*(2*a^2*b - b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(b) + arctan((b*tan(1/2*d*x + 1/2*c) + a)/sqrt(-a^2 + b^2)))/((a^4 - a^2*b^2)*sqrt(-a^2 + b^2)) + 2*(a*b*tan(1/2*d*x + 1/2*c) + b^2)/((a^3 - a*b^2)*(b*tan(1/2*d*x + 1/2*c)^2 + 2*a*tan(1/2*d*x + 1/2*c) + b)) - (d*x + c)/a^2)/d

maple [B] time = 0.69, size = 247, normalized size = 2.29

$$\frac{2b \tan \left(\frac{dx}{2} + \frac{c}{2} \right)}{d \left(\left(\tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) b + 2a \tan \left(\frac{dx}{2} + \frac{c}{2} \right) + b \right) (a^2 - b^2)} - \frac{2b^2}{da \left(\left(\tan^2 \left(\frac{dx}{2} + \frac{c}{2} \right) \right) b + 2a \tan \left(\frac{dx}{2} + \frac{c}{2} \right) + b \right) (a^2 - b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(a+b*\text{csc}(d*x+c))^2,x)$

[Out] $-2/d*b/(\tan(1/2*d*x+1/2*c)^2*b+2*a*\tan(1/2*d*x+1/2*c)+b)/(a^2-b^2)*\tan(1/2*d*x+1/2*c)-2/d*b^2/a/(\tan(1/2*d*x+1/2*c)^2*b+2*a*\tan(1/2*d*x+1/2*c)+b)/(a^2-b^2)-4/d*b/(a^2-b^2)/(-a^2+b^2)^{(1/2)}*\arctan(1/2*(2*b*\tan(1/2*d*x+1/2*c)+2*a)/(-a^2+b^2)^{(1/2)})+2/d*b^3/a^2/(a^2-b^2)/(-a^2+b^2)^{(1/2)}*\arctan(1/2*(2*b*\tan(1/2*d*x+1/2*c)+2*a)/(-a^2+b^2)^{(1/2)})+2/d/a^2*\arctan(\tan(1/2*d*x+1/2*c))$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/(a+b*\text{csc}(d*x+c))^2,x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see 'assume?' for more details)Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 4.18, size = 2677, normalized size = 24.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(a + b/\sin(c + d*x))^2,x)$

[Out] $(b*\text{atan}(((b*(2*a^2 - b^2))*((a + b)^3*(a - b)^3)^{(1/2)}*((32*(a*b^6 - 2*a^3*b^4 + a^5*b^2))/(a^6 + a^2*b^4 - 2*a^4*b^2) - (32*\tan(c/2 + (d*x)/2)*(2*a*b^7 - 2*a^7*b - 8*a^3*b^5 + 9*a^5*b^3)))/(a^7 + a^3*b^4 - 2*a^5*b^2) + (b*(2*a^2 - b^2))*((a + b)^3*(a - b)^3)^{(1/2)}*((32*(a^8*b - a^6*b^3))/(a^6 + a^2*b^4 - 2*a^4*b^2) + (32*\tan(c/2 + (d*x)/2)*(2*a^4*b^6 - 6*a^6*b^4 + 4*a^8*b^2)))/(a^7 + a^3*b^4 - 2*a^5*b^2) + (b*(2*a^2 - b^2))*((a + b)^3*(a - b)^3)^{(1/2)}*((32*(a^5*b^6 - 2*a^7*b^4 + a^9*b^2))/(a^6 + a^2*b^4 - 2*a^4*b^2) + (32*\tan(c/2 + (d*x)/2)*(3*a^11*b - 2*a^5*b^7 + 7*a^7*b^5 - 8*a^9*b^3)))/(a^7 + a^3*b^4 - 2*a^5*b^2)))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2) - (b*(2*a^2 - b^2))*((a + b)^3*(a - b)^3)^{(1/2)}*((32*\tan(c/2 + (d*x)/2)*(2*a*b^7 - 2*a^7*b - 8*a^3*b^5 + 9*a^5*b^3))/(a^7 + a^3*b^4 - 2*a^5*b^2) - (32*(a*b^6 - 2*a^3*b^4 + a^5*b^2))/(a^6 + a^2*b^4 - 2*a^4*b^2) + (b*(2*a^2 - b^2))*((a + b)^3*(a - b)^3)^{(1/2)}*((32*(a^8*b - a^6*b^3))/(a^6 + a^2*b^4 - 2*a^4*b^2) + (32*\tan(c/2 + (d*x)/2)*(2*a^4*b^6 - 6*a^6*b^4 + 4*a^8*b^2)))/(a^7 +$

$$\begin{aligned}
& a^3 b^4 - 2 a^5 b^2) - (b(2 a^2 - b^2) * ((a + b)^3 * (a - b)^3)^{(1/2)} * ((32 * (a \\
& ^5 b^6 - 2 a^7 b^4 + a^9 b^2)) / (a^6 + a^2 b^4 - 2 a^4 b^2) + (32 * \tan(c/2 + \\
& (d*x)/2) * (3 a^{11} b - 2 a^5 b^7 + 7 a^7 b^5 - 8 a^9 b^3)) / (a^7 + a^3 b^4 - 2 \\
& * a^5 b^2))) / (a^8 - a^2 b^6 + 3 a^4 b^4 - 3 a^6 b^2)) / (a^8 - a^2 b^6 + 3 a^4 \\
& 4 b^4 - 3 a^6 b^2)) * 1i) / (a^8 - a^2 b^6 + 3 a^4 b^4 - 3 a^6 b^2)) / ((64 * (b^5 \\
& - 2 a^2 b^3)) / (a^6 + a^2 b^4 - 2 a^4 b^2) + (64 * \tan(c/2 + (d*x)/2) * (2 b^6 - \\
& 6 a^2 b^4 + 4 a^4 b^2)) / (a^7 + a^3 b^4 - 2 a^5 b^2) + (b(2 a^2 - b^2) * ((a \\
& + b)^3 * (a - b)^3)^{(1/2)} * ((32 * (a b^6 - 2 a^3 b^4 + a^5 b^2)) / (a^6 + a^2 b^4 \\
& - 2 a^4 b^2) - (32 * \tan(c/2 + (d*x)/2) * (2 a^2 b^7 - 2 a^7 b - 8 a^3 b^5 + 9 a \\
& ^5 b^3)) / (a^7 + a^3 b^4 - 2 a^5 b^2) + (b(2 a^2 - b^2) * ((a + b)^3 * (a - b)^ \\
& 3)^{(1/2)} * ((32 * (a^8 b - a^6 b^3)) / (a^6 + a^2 b^4 - 2 a^4 b^2) + (32 * \tan(c/2 \\
& + (d*x)/2) * (2 a^4 b^6 - 6 a^6 b^4 + 4 a^8 b^2)) / (a^7 + a^3 b^4 - 2 a^5 b^2) \\
& + (b(2 a^2 - b^2) * ((a + b)^3 * (a - b)^3)^{(1/2)} * ((32 * (a^5 b^6 - 2 a^7 b^4 + \\
& a^9 b^2)) / (a^6 + a^2 b^4 - 2 a^4 b^2) + (32 * \tan(c/2 + (d*x)/2) * (3 a^{11} b - \\
& 2 a^5 b^7 + 7 a^7 b^5 - 8 a^9 b^3)) / (a^7 + a^3 b^4 - 2 a^5 b^2))) / (a^8 - a \\
& ^2 b^6 + 3 a^4 b^4 - 3 a^6 b^2)) / (a^8 - a^2 b^6 + 3 a^4 b^4 - 3 a^6 b^2)) \\
& / (a^8 - a^2 b^6 + 3 a^4 b^4 - 3 a^6 b^2) + (b(2 a^2 - b^2) * ((a + b)^3 * (a - \\
& b)^3)^{(1/2)} * ((32 * \tan(c/2 + (d*x)/2) * (2 a^2 b^7 - 2 a^7 b - 8 a^3 b^5 + 9 a^5 \\
& * b^3)) / (a^7 + a^3 b^4 - 2 a^5 b^2) - (32 * (a b^6 - 2 a^3 b^4 + a^5 b^2)) / (a^ \\
& 6 + a^2 b^4 - 2 a^4 b^2) + (b(2 a^2 - b^2) * ((a + b)^3 * (a - b)^3)^{(1/2)} * ((3 \\
& 2 * (a^8 b - a^6 b^3)) / (a^6 + a^2 b^4 - 2 a^4 b^2) + (32 * \tan(c/2 + (d*x)/2) * (\\
& 2 a^4 b^6 - 6 a^6 b^4 + 4 a^8 b^2)) / (a^7 + a^3 b^4 - 2 a^5 b^2) - (b(2 a^2 \\
& - b^2) * ((a + b)^3 * (a - b)^3)^{(1/2)} * ((32 * (a^5 b^6 - 2 a^7 b^4 + a^9 b^2)) / (\\
& a^6 + a^2 b^4 - 2 a^4 b^2) + (32 * \tan(c/2 + (d*x)/2) * (3 a^{11} b - 2 a^5 b^7 + \\
& 7 a^7 b^5 - 8 a^9 b^3)) / (a^7 + a^3 b^4 - 2 a^5 b^2))) / (a^8 - a^2 b^6 + 3 a \\
& ^4 b^4 - 3 a^6 b^2)) / (a^8 - a^2 b^6 + 3 a^4 b^4 - 3 a^6 b^2)) / (a^8 - a^2 * \\
& b^6 + 3 a^4 b^4 - 3 a^6 b^2)) * (2 a^2 - b^2) * ((a + b)^3 * (a - b)^3)^{(1/2)} * 2i \\
&) / (d * (a^8 - a^2 b^6 + 3 a^4 b^4 - 3 a^6 b^2)) - ((2 * b * \tan(c/2 + (d*x)/2)) / (\\
& a^2 - b^2) + (2 * b^2) / (a * (a^2 - b^2))) / (d * (b + 2 * a * \tan(c/2 + (d*x)/2) + b * \tan \\
& (c/2 + (d*x)/2)^2)) - (2 * \operatorname{atan}((64 * a^5 * b * \tan(c/2 + (d*x)/2)) / ((64 * a^3 * b^9) / \\
& (a^6 + a^2 b^4 - 2 a^4 b^2) - (192 * a^5 * b^7) / (a^6 + a^2 b^4 - 2 a^4 b^2) + (\\
& 128 * a^7 * b^5) / (a^6 + a^2 b^4 - 2 a^4 b^2) + (64 * a^9 * b^3) / (a^6 + a^2 b^4 - 2 * \\
& a^4 b^2) - (64 * a^{11} * b) / (a^6 + a^2 b^4 - 2 a^4 b^2)) - (64 * a * b^5 * \tan(c/2 + (\\
& d*x)/2)) / ((64 * a^3 * b^9) / (a^6 + a^2 b^4 - 2 a^4 b^2) - (192 * a^5 * b^7) / (a^6 + a \\
& ^2 b^4 - 2 a^4 b^2) + (128 * a^7 * b^5) / (a^6 + a^2 b^4 - 2 a^4 b^2) + (64 * a^9 * b \\
& ^3) / (a^6 + a^2 b^4 - 2 a^4 b^2) - (64 * a^{11} * b) / (a^6 + a^2 b^4 - 2 a^4 b^2)) \\
& + (64 * a^3 * b^3 * \tan(c/2 + (d*x)/2)) / ((64 * a^3 * b^9) / (a^6 + a^2 b^4 - 2 a^4 b^2) \\
& - (192 * a^5 * b^7) / (a^6 + a^2 b^4 - 2 a^4 b^2) + (128 * a^7 * b^5) / (a^6 + a^2 b^4 \\
& - 2 a^4 b^2) + (64 * a^9 * b^3) / (a^6 + a^2 b^4 - 2 a^4 b^2) - (64 * a^{11} * b) / (a^6 \\
& + a^2 b^4 - 2 a^4 b^2)))) / (a^2 * d)
\end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \csc(c + dx))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csc(d*x+c))**2,x)
```

```
[Out] Integral((a + b*csc(c + d*x))**(-2), x)
```

$$3.50 \quad \int \frac{1}{(a+b \csc(c+dx))^3} dx$$

Optimal. Leaf size=170

$$\frac{x}{a^3} - \frac{b^2(5a^2 - 2b^2) \cot(c+dx)}{2a^2d(a^2 - b^2)^2(a+b \csc(c+dx))} - \frac{b^2 \cot(c+dx)}{2ad(a^2 - b^2)(a+b \csc(c+dx))^2} + \frac{b(6a^4 - 5a^2b^2 + 2b^4) \tanh^{-1}\left(\frac{a+b \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3d(a^2 - b^2)^{5/2}}$$

[Out] $x/a^3 + b*(6*a^4 - 5*a^2*b^2 + 2*b^4)*\operatorname{arctanh}\left(\frac{a+b*\tan(1/2*d*x+1/2*c)}{(a^2-b^2)^{1/2}}\right)/a^3/(a^2-b^2)^{5/2}/d - 1/2*b^2*\cot(d*x+c)/a/(a^2-b^2)/d/(a+b*\csc(d*x+c))^2 - 1/2*b^2*(5*a^2-2*b^2)*\cot(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*\csc(d*x+c))$

Rubi [A] time = 0.32, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3785, 4060, 3919, 3831, 2660, 618, 206}

$$\frac{b(-5a^2b^2 + 6a^4 + 2b^4) \tanh^{-1}\left(\frac{a+b \tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3d(a^2 - b^2)^{5/2}} - \frac{b^2(5a^2 - 2b^2) \cot(c+dx)}{2a^2d(a^2 - b^2)^2(a+b \csc(c+dx))} - \frac{b^2 \cot(c+dx)}{2ad(a^2 - b^2)(a+b \csc(c+dx))}$$

Antiderivative was successfully verified.

[In] $\operatorname{Int}[(a + b*\operatorname{Csc}[c + d*x])^{-3}, x]$

[Out] $x/a^3 + (b*(6*a^4 - 5*a^2*b^2 + 2*b^4)*\operatorname{ArcTanh}[(a + b*\operatorname{Tan}[(c + d*x)/2])/ \operatorname{Sqrt}[a^2 - b^2]])/(a^3*(a^2 - b^2)^{5/2}*d) - (b^2*\operatorname{Cot}[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*\operatorname{Csc}[c + d*x])^2) - (b^2*(5*a^2 - 2*b^2)*\operatorname{Cot}[c + d*x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*\operatorname{Csc}[c + d*x]))$

Rule 206

$\operatorname{Int}[(a + b*x)^{-1}, x_Symbol] \rightarrow \operatorname{Simp}[(1*\operatorname{ArcTanh}[(\operatorname{Rt}[-b, 2]*x)/\operatorname{Rt}[a, 2]])/(\operatorname{Rt}[a, 2]*\operatorname{Rt}[-b, 2]), x] /; \operatorname{FreeQ}\{a, b\}, x \ \&\& \operatorname{NegQ}[a/b] \ \&\& (\operatorname{GtQ}[a, 0] \ || \ \operatorname{LtQ}[b, 0])$

Rule 618

$\operatorname{Int}[(a + b*x + c*x^2)^{-1}, x_Symbol] \rightarrow \operatorname{Dist}[-2, \operatorname{Subst}[\operatorname{Int}[1/\operatorname{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \operatorname{FreeQ}\{a, b, c\}, x \ \&\& \operatorname{NeQ}[b^2 - 4*a*c, 0]$

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] := Simp[(b^2*Cot[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dist[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 3831

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] := Dist[1/b, Int[1/(1 + (a*Sin[e + f*x])/b), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3919

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] := Simp[(c*x)/a, x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 4060

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] := Simp[((A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(a*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a + b \csc(c + dx))^3} dx &= -\frac{b^2 \cot(c + dx)}{2a(a^2 - b^2)d(a + b \csc(c + dx))^2} - \frac{\int \frac{-2(a^2 - b^2) + 2ab \csc(c + dx) - b^2 \csc^2(c + dx)}{(a + b \csc(c + dx))^2} dx}{2a(a^2 - b^2)} \\
&= -\frac{b^2 \cot(c + dx)}{2a(a^2 - b^2)d(a + b \csc(c + dx))^2} - \frac{b^2(5a^2 - 2b^2) \cot(c + dx)}{2a^2(a^2 - b^2)^2 d(a + b \csc(c + dx))} + \frac{\int \frac{2(a^2 - b^2)}{(a + b \csc(c + dx))^2} dx}{2a(a^2 - b^2)} \\
&= \frac{x}{a^3} - \frac{b^2 \cot(c + dx)}{2a(a^2 - b^2)d(a + b \csc(c + dx))^2} - \frac{b^2(5a^2 - 2b^2) \cot(c + dx)}{2a^2(a^2 - b^2)^2 d(a + b \csc(c + dx))} - \frac{b(6a^4 - 5a^2b^2 + 2b^4)}{a^3(a^2 - b^2)^{5/2}d} \\
&= \frac{x}{a^3} - \frac{b^2 \cot(c + dx)}{2a(a^2 - b^2)d(a + b \csc(c + dx))^2} - \frac{b^2(5a^2 - 2b^2) \cot(c + dx)}{2a^2(a^2 - b^2)^2 d(a + b \csc(c + dx))} - \frac{b(6a^4 - 5a^2b^2 + 2b^4)}{a^3(a^2 - b^2)^{5/2}d} \\
&= \frac{x}{a^3} - \frac{b^2 \cot(c + dx)}{2a(a^2 - b^2)d(a + b \csc(c + dx))^2} - \frac{b^2(5a^2 - 2b^2) \cot(c + dx)}{2a^2(a^2 - b^2)^2 d(a + b \csc(c + dx))} - \frac{b(6a^4 - 5a^2b^2 + 2b^4)}{a^3(a^2 - b^2)^{5/2}d} \\
&= \frac{x}{a^3} - \frac{b^2 \cot(c + dx)}{2a(a^2 - b^2)d(a + b \csc(c + dx))^2} - \frac{b^2(5a^2 - 2b^2) \cot(c + dx)}{2a^2(a^2 - b^2)^2 d(a + b \csc(c + dx))} + \frac{b(6a^4 - 5a^2b^2 + 2b^4) \tanh^{-1}\left(\frac{b\left(\frac{a}{b} + \tan\left(\frac{1}{2}(c + dx)\right)\right)}{\sqrt{a^2 - b^2}}\right)}{a^3(a^2 - b^2)^{5/2}d} - \frac{b^2 \cot(c + dx)}{2a(a^2 - b^2)d(a + b \csc(c + dx))^2}
\end{aligned}$$

Mathematica [A] time = 1.08, size = 216, normalized size = 1.27

$$\frac{\csc^2(c + dx)(a \sin(c + dx) + b) \left(-\frac{3ab^2(2a^2 - b^2) \cot(c + dx)(a \sin(c + dx) + b)}{(a - b)^2(a + b)^2} - \frac{2b(6a^4 - 5a^2b^2 + 2b^4) \csc(c + dx)(a \sin(c + dx) + b)^2 \tan^{-1}\left(\frac{a + b \tan\left(\frac{1}{2}(c + dx)\right)}{\sqrt{a^2 - b^2}}\right)}{(b^2 - a^2)^{5/2}} \right)}{2a^3d(a + b \csc(c + dx))^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Csc[c + d*x])^(-3), x]

```
[Out] (Csc[c + d*x]^2*(b + a*Sin[c + d*x])*((a*b^3*Cot[c + d*x])/((a - b)*(a + b)
) - (3*a*b^2*(2*a^2 - b^2)*Cot[c + d*x]*(b + a*Sin[c + d*x]))/((a - b)^2*(a
+ b)^2) + 2*(c + d*x)*Csc[c + d*x]*(b + a*Sin[c + d*x])^2 - (2*b*(6*a^4 -
5*a^2*b^2 + 2*b^4)*ArcTan[(a + b*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]]*Csc[c
+ d*x]*(b + a*Sin[c + d*x])^2)/(-a^2 + b^2)^(5/2)))/(2*a^3*d*(a + b*Csc[c +
d*x])^3)
```

fricas [B] time = 0.75, size = 933, normalized size = 5.49

$$\left[\frac{4(a^8 - 3a^6b^2 + 3a^4b^4 - a^2b^6)dx \cos(dx + c)^2 - 4(a^8 - 2a^6b^2 + 2a^2b^6 - b^8)dx - (6a^6b + a^4b^3 - 3a^2b^5 + 2b^7)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*csc(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*x*cos(d*x + c)^2 - 4*(a^8
- 2*a^6*b^2 + 2*a^2*b^6 - b^8)*d*x - (6*a^6*b + a^4*b^3 - 3*a^2*b^5 + 2*b^7
- (6*a^6*b - 5*a^4*b^3 + 2*a^2*b^5)*cos(d*x + c)^2 + 2*(6*a^5*b^2 - 5*a^3
*b^4 + 2*a*b^6)*sin(d*x + c))*sqrt(a^2 - b^2)*log(((a^2 - 2*b^2)*cos(d*x +
c)^2 + 2*a*b*sin(d*x + c) + a^2 + b^2 + 2*(b*cos(d*x + c)*sin(d*x + c) + a
cos(d*x + c))*sqrt(a^2 - b^2))/(a^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a
^2 - b^2)) + 2*(5*a^5*b^3 - 7*a^3*b^5 + 2*a*b^7)*cos(d*x + c) - 2*(4*(a^7*b
- 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x - 3*(2*a^6*b^2 - 3*a^4*b^4 + a^2*b^6)
*cos(d*x + c))*sin(d*x + c))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cos
(d*x + c)^2 - 2*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*sin(d*x + c)
- (a^11 - 2*a^9*b^2 + 2*a^5*b^6 - a^3*b^8)*d), 1/2*(2*(a^8 - 3*a^6*b^2 + 3*
a^4*b^4 - a^2*b^6)*d*x*cos(d*x + c)^2 - 2*(a^8 - 2*a^6*b^2 + 2*a^2*b^6 - b^8
)*d*x - (6*a^6*b + a^4*b^3 - 3*a^2*b^5 + 2*b^7 - (6*a^6*b - 5*a^4*b^3 + 2*
a^2*b^5)*cos(d*x + c)^2 + 2*(6*a^5*b^2 - 5*a^3*b^4 + 2*a*b^6)*sin(d*x + c))
*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*sin(d*x + c) + a)/((a^2 - b^2
)*cos(d*x + c))) + (5*a^5*b^3 - 7*a^3*b^5 + 2*a*b^7)*cos(d*x + c) - (4*(a^7
*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x - 3*(2*a^6*b^2 - 3*a^4*b^4 + a^2*b^6)
*cos(d*x + c))*sin(d*x + c))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cos
(d*x + c)^2 - 2*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*sin(d*x + c)
) - (a^11 - 2*a^9*b^2 + 2*a^5*b^6 - a^3*b^8)*d]]
```

giac [A] time = 0.38, size = 297, normalized size = 1.75

$$\frac{(6a^4b - 5a^2b^3 + 2b^5) \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(b) + \arctan \left(\frac{b \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + a}{\sqrt{-a^2 + b^2}} \right) \right)}{(a^7 - 2a^5b^2 + a^3b^4) \sqrt{-a^2 + b^2}} + \frac{4a^3b^2 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^3 - ab^4 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^3 + 10a^4b \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^2 + a^2b^3 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)}{(a^6 - 2a^4b^2 + a^2b^4) \left(b \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + a \right)}$$

d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*csc(d*x+c))^3,x, algorithm="giac")

[Out] $-\left(\left(6a^4b - 5a^2b^3 + 2b^5\right)\left(\pi\operatorname{floor}\left(\frac{1}{2}(dx + c)\right)/\pi + \frac{1}{2}\right)\operatorname{sgn}(b) + \arctan\left(\frac{b\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + a}{\sqrt{-a^2 + b^2}}\right)\right) / \left(\left(a^7 - 2a^5b^2 + a^3b^4\right)\sqrt{-a^2 + b^2}\right) + \left(4a^3b^2\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - a^2b^4\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 10a^4b\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + a^2b^3\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 2b^5\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 16a^3b^2\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 7a^2b^4\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 5a^2b^3 - 2b^5\right) / \left(\left(a^6 - 2a^4b^2 + a^2b^4\right)\left(b\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 + 2a\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + b\right)^2 - (dx + c)/a^3\right) / d$

maple [B] time = 0.78, size = 796, normalized size = 4.68

$$\frac{4ab^2\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{d\left(\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)b + 2a\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + b\right)^2(a^4 - 2a^2b^2 + b^4)} + \frac{b^4\left(\tan^3\left(\frac{dx}{2} + \frac{c}{2}\right)\right)}{da\left(\left(\tan^2\left(\frac{dx}{2} + \frac{c}{2}\right)\right)b + 2a\tan\left(\frac{dx}{2} + \frac{c}{2}\right) + b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*csc(d*x+c))^3,x)

[Out] $-4/dab^2/(\tan(1/2dx+1/2c)^{2b+2a\tan(1/2dx+1/2c)+b})^2/(a^4-2a^2b^2+b^4)*\tan(1/2dx+1/2c)^3+1/dab^4/(\tan(1/2dx+1/2c)^{2b+2a\tan(1/2dx+1/2c)+b})^2/(a^4-2a^2b^2+b^4)*\tan(1/2dx+1/2c)^3-10/dab^2/(\tan(1/2dx+1/2c)^{2b+2a\tan(1/2dx+1/2c)+b})^2/(a^4-2a^2b^2+b^4)*\tan(1/2dx+1/2c)^2-1/db^3/(\tan(1/2dx+1/2c)^{2b+2a\tan(1/2dx+1/2c)+b})^2/(a^4-2a^2b^2+b^4)*\tan(1/2dx+1/2c)^2+2/dab^5/(\tan(1/2dx+1/2c)^{2b+2a\tan(1/2dx+1/2c)+b})^2/(a^4-2a^2b^2+b^4)*\tan(1/2dx+1/2c)^2-16/dab^2/(\tan(1/2dx+1/2c)^{2b+2a\tan(1/2dx+1/2c)+b})^2/(a^4-2a^2b^2+b^4)*\tan(1/2dx+1/2c)+7/dab^4/(\tan(1/2dx+1/2c)^{2b+2a\tan(1/2dx+1/2c)+b})^2/(a^4-2a^2b^2+b^4)*\tan(1/2dx+1/2c)-5/db^3/(\tan(1/2dx+1/2c)^{2b+2a\tan(1/2dx+1/2c)+b})^2/(a^4-2a^2b^2+b^4)+2/dab^5/(\tan(1/2dx+1/2c)^{2b+2a\tan(1/2dx+1/2c)+b})^2/(a^4-2a^2b^2+b^4)-6/dab/(a^4-2a^2b^2+b^4)/(-a^2+b^2)^{(1/2)}*\arctan(1/2*(2b*\tan(1/2dx+1/2c)+2a)/(-a^2+b^2)^{(1/2)})+5/dab^3/(a^4-2a^2b^2+b^4)/(-a^2+b^2)^{(1/2)}*\arctan(1/2*(2b*\tan(1/2dx+1/2c)+2a)/(-a^2+b^2)^{(1/2)})-2/dab^3b^5/(a^4-2a^2b^2+b^4)/(-a^2+b^2)^{(1/2)}*\arctan(1/2*(2b*\tan(1/2dx+1/2c)+2a)/(-a^2+b^2)^{(1/2)})+2/dab^3*\arctan(\tan(1/2dx+1/2c))$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

$$\begin{aligned}
& 8*b^4 + 4*a^{10}*b^2)) / (a^{13} + a^5*b^8 - 4*a^7*b^6 + 6*a^9*b^4 - 4*a^{11}*b^2) \\
& - (((8*(4*a^{14}*b + 2*a^6*b^9 - 4*a^8*b^7 + 6*a^{10}*b^5 - 8*a^{12}*b^3)) / (a^{13} \\
& + a^5*b^8 - 4*a^7*b^6 + 6*a^9*b^4 - 4*a^{11}*b^2) - ((8*(4*a^8*b^{10} - 16*a^{10}*b^8 + 24*a^{12}*b^6 - 16*a^{14}*b^4 + 4*a^{16}*b^2)) / (a^{13} + a^5*b^8 - 4*a^7*b^6 + 6*a^9*b^4 - 4*a^{11}*b^2) + (8*\tan(c/2 + (d*x)/2)*(12*a^{18}*b - 8*a^8*b^{11} + 44*a^{10}*b^9 - 96*a^{12}*b^7 + 104*a^{14}*b^5 - 56*a^{16}*b^3)) / (a^{14} + a^6*b^8 - 4*a^8*b^6 + 6*a^{10}*b^4 - 4*a^{12}*b^2))*1i) / a^3 + (8*\tan(c/2 + (d*x)/2)*(8*a^6*b^{10} - 36*a^8*b^8 + 72*a^{10}*b^6 - 68*a^{12}*b^4 + 24*a^{14}*b^2)) / (a^{14} + a^6*b^8 - 4*a^8*b^6 + 6*a^{10}*b^4 - 4*a^{12}*b^2))*1i) / a^3 + (8*\tan(c/2 + (d*x)/2)*(8*a^{12}*b - 8*a^2*b^{11} + 44*a^4*b^9 - 105*a^6*b^7 + 124*a^8*b^5 - 72*a^{10}*b^3)) / (a^{14} + a^6*b^8 - 4*a^8*b^6 + 6*a^{10}*b^4 - 4*a^{12}*b^2))*1i) / a^3 + (((8*(4*a^2*b^{10} - 16*a^4*b^8 + 24*a^6*b^6 - 16*a^8*b^4 + 4*a^{10}*b^2)) / (a^{13} + a^5*b^8 - 4*a^7*b^6 + 6*a^9*b^4 - 4*a^{11}*b^2) + (((((8*(4*a^8*b^{10} - 16*a^{10}*b^8 + 24*a^{12}*b^6 - 16*a^{14}*b^4 + 4*a^{16}*b^2)) / (a^{13} + a^5*b^8 - 4*a^7*b^6 + 6*a^9*b^4 - 4*a^{11}*b^2) + (8*\tan(c/2 + (d*x)/2)*(12*a^{18}*b - 8*a^8*b^{11} + 44*a^{10}*b^9 - 96*a^{12}*b^7 + 104*a^{14}*b^5 - 56*a^{16}*b^3)) / (a^{14} + a^6*b^8 - 4*a^8*b^6 + 6*a^{10}*b^4 - 4*a^{12}*b^2))*1i) / a^3 + (8*(4*a^{14}*b + 2*a^6*b^9 - 4*a^8*b^7 + 6*a^{10}*b^5 - 8*a^{12}*b^3)) / (a^{13} + a^5*b^8 - 4*a^7*b^6 + 6*a^9*b^4 - 4*a^{11}*b^2) + (8*\tan(c/2 + (d*x)/2)*(8*a^6*b^{10} - 36*a^8*b^8 + 72*a^{10}*b^6 - 68*a^{12}*b^4 + 24*a^{14}*b^2)) / (a^{14} + a^6*b^8 - 4*a^8*b^6 + 6*a^{10}*b^4 - 4*a^{12}*b^2))*1i) / a^3 + (8*\tan(c/2 + (d*x)/2)*(8*a^{12}*b - 8*a^2*b^{11} + 44*a^4*b^9 - 105*a^6*b^7 + 124*a^8*b^5 - 72*a^{10}*b^3)) / (a^{14} + a^6*b^8 - 4*a^8*b^6 + 6*a^{10}*b^4 - 4*a^{12}*b^2))*1i) / a^3 + (16*\tan(c/2 + (d*x)/2)*(8*b^{10} - 36*a^2*b^8 + 72*a^4*b^6 - 68*a^6*b^4 + 24*a^8*b^2)) / (a^{14} + a^6*b^8 - 4*a^8*b^6 + 6*a^{10}*b^4 - 4*a^{12}*b^2)))) / (a^3*d) + (b*atan(((b*((a + b)^5*(a - b)^5)^(1/2)*(6*a^4 + 2*b^4 - 5*a^2*b^2))*((8*(4*a^2*b^{10} - 16*a^4*b^8 + 24*a^6*b^6 - 16*a^8*b^4 + 4*a^{10}*b^2)) / (a^{13} + a^5*b^8 - 4*a^7*b^6 + 6*a^9*b^4 - 4*a^{11}*b^2) + (8*\tan(c/2 + (d*x)/2)*(8*a^{12}*b - 8*a^2*b^{11} + 44*a^4*b^9 - 105*a^6*b^7 + 124*a^8*b^5 - 72*a^{10}*b^3)) / (a^{14} + a^6*b^8 - 4*a^8*b^6 + 6*a^{10}*b^4 - 4*a^{12}*b^2) - (b*((a + b)^5*(a - b)^5)^(1/2))*((8*(4*a^{14}*b + 2*a^6*b^9 - 4*a^8*b^7 + 6*a^{10}*b^5 - 8*a^{12}*b^3)) / (a^{13} + a^5*b^8 - 4*a^7*b^6 + 6*a^9*b^4 - 4*a^{11}*b^2) + (8*\tan(c/2 + (d*x)/2)*(8*a^6*b^{10} - 36*a^8*b^8 + 72*a^{10}*b^6 - 68*a^{12}*b^4 + 24*a^{14}*b^2)) / (a^{14} + a^6*b^8 - 4*a^8*b^6 + 6*a^{10}*b^4 - 4*a^{12}*b^2) - (b*((8*(4*a^8*b^{10} - 16*a^{10}*b^8 + 24*a^{12}*b^6 - 16*a^{14}*b^4 + 4*a^{16}*b^2)) / (a^{13} + a^5*b^8 - 4*a^7*b^6 + 6*a^9*b^4 - 4*a^{11}*b^2) + (8*\tan(c/2 + (d*x)/2)*(12*a^{18}*b - 8*a^8*b^{11} + 44*a^{10}*b^9 - 96*a^{12}*b^7 + 104*a^{14}*b^5 - 56*a^{16}*b^3)) / (a^{14} + a^6*b^8 - 4*a^8*b^6 + 6*a^{10}*b^4 - 4*a^{12}*b^2))*((a + b)^5*(a - b)^5)^(1/2)*(6*a^4 + 2*b^4 - 5*a^2*b^2)) / (2*(a^{13} - a^3*b^{10} + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^{11}*b^2))))*(6*a^4 + 2*b^4 - 5*a^2*b^2)) / (2*(a^{13} - a^3*b^{10} + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^{11}*b^2)))*1i) / (2*(a^{13} - a^3*b^{10} + 5*a^5*b^8 - 10*a^7*b^6 + 10*a^9*b^4 - 5*a^{11}*b^2)) + (b*((a + b)^5*(a - b)^5)^(1/2)*(6*a^4 + 2*b^4 - 5*a^2*b^2))*((8*(4*a^2*b^{10} - 16*a^4*b^8 + 24*a^6*b^6 - 16*a^8*b^4 + 4*a^{10}*b^2)) / (a^{13} + a^5*b^8 - 4*a^7*b^6 + 6*a^9*b^4 - 4*a^{11}*b^2) + (8*\tan(c/2 + (d*x)/2)*(8*a^{12}*b - 8*a^2*b^{11} + 44*a^4*b^9 - 105*a^6*b^7 + 1
\end{aligned}$$

$$4 + a^6 b^8 - 4 a^8 b^6 + 6 a^{10} b^4 - 4 a^{12} b^2) * ((a + b)^5 * (a - b)^5)^{(1/2)} * (6 a^4 + 2 b^4 - 5 a^2 b^2) / (2 * (a^{13} - a^3 b^{10} + 5 a^5 b^8 - 10 a^7 b^6 + 10 a^9 b^4 - 5 a^{11} b^2)) * (6 a^4 + 2 b^4 - 5 a^2 b^2) / (2 * (a^{13} - a^3 b^{10} + 5 a^5 b^8 - 10 a^7 b^6 + 10 a^9 b^4 - 5 a^{11} b^2))) / (2 * (a^{13} - a^3 b^{10} + 5 a^5 b^8 - 10 a^7 b^6 + 10 a^9 b^4 - 5 a^{11} b^2))) * ((a + b)^5 * (a - b)^5)^{(1/2)} * (6 a^4 + 2 b^4 - 5 a^2 b^2) * i / (d * (a^{13} - a^3 b^{10} + 5 a^5 b^8 - 10 a^7 b^6 + 10 a^9 b^4 - 5 a^{11} b^2))$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \csc(c + dx))^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*csc(d*x+c))**3,x)

[Out] Integral((a + b*csc(c + d*x))**(-3), x)

$$3.51 \quad \int \frac{1}{(a+b \csc(c+dx))^4} dx$$

Optimal. Leaf size=239

$$\frac{x}{a^4} - \frac{b^2 (8a^2 - 3b^2) \cot(c + dx)}{6a^2 d (a^2 - b^2)^2 (a + b \csc(c + dx))^2} - \frac{b^2 \cot(c + dx)}{3ad (a^2 - b^2) (a + b \csc(c + dx))^3} + \frac{b (8a^6 - 8a^4 b^2 + 7a^2 b^4 - 2b^6) \tanh^{-1} \left(\frac{a + b \tan \left(\frac{1}{2}(c + dx) \right)}{\sqrt{a^2 - b^2}} \right)}{a^4 d (a^2 - b^2)^{7/2}}$$

[Out] $x/a^4 + b*(8*a^6 - 8*a^4*b^2 + 7*a^2*b^4 - 2*b^6)*\text{arctanh}((a+b*\tan(1/2*d*x+1/2*c))/(a^2-b^2)^{(1/2}))/a^4/(a^2-b^2)^{(7/2)}/d - 1/3*b^2*\cot(d*x+c)/a/(a^2-b^2)/d/(a+b*\csc(d*x+c))^3 - 1/6*b^2*(8*a^2-3*b^2)*\cot(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*\csc(d*x+c))^2 - 1/6*b^2*(26*a^4-17*a^2*b^2+6*b^4)*\cot(d*x+c)/a^3/(a^2-b^2)^3/d/(a+b*\csc(d*x+c))$

Rubi [A] time = 0.50, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3785, 4060, 3919, 3831, 2660, 618, 206}

$$\frac{b (-8a^4 b^2 + 7a^2 b^4 + 8a^6 - 2b^6) \tanh^{-1} \left(\frac{a + b \tan \left(\frac{1}{2}(c + dx) \right)}{\sqrt{a^2 - b^2}} \right)}{a^4 d (a^2 - b^2)^{7/2}} - \frac{b^2 (-17a^2 b^2 + 26a^4 + 6b^4) \cot(c + dx)}{6a^3 d (a^2 - b^2)^3 (a + b \csc(c + dx))} - \frac{b^2 (8a^2 - 3b^2) \cot(c + dx)}{6a^2 d (a^2 - b^2)^2 (a + b \csc(c + dx))}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + b*\text{Csc}[c + d*x])^{-4}, x]$

[Out] $x/a^4 + (b*(8*a^6 - 8*a^4*b^2 + 7*a^2*b^4 - 2*b^6)*\text{ArcTanh}[(a + b*\text{Tan}[(c + d*x)/2])/ \text{Sqrt}[a^2 - b^2]])/(a^4*(a^2 - b^2)^{(7/2)*d}) - (b^2*\text{Cot}[c + d*x])/ (3*a*(a^2 - b^2)*d*(a + b*\text{Csc}[c + d*x])^3) - (b^2*(8*a^2 - 3*b^2)*\text{Cot}[c + d*x])/ (6*a^2*(a^2 - b^2)^2*d*(a + b*\text{Csc}[c + d*x])^2) - (b^2*(26*a^4 - 17*a^2*b^2 + 6*b^4)*\text{Cot}[c + d*x])/ (6*a^3*(a^2 - b^2)^3*d*(a + b*\text{Csc}[c + d*x]))$

Rule 206

$\text{Int}[(a + b*x^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}\{a, b, x\} \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 618

$\text{Int}[(a + b*x + c*x^2)^{-1}, x_Symbol] \rightarrow \text{Dist}[-2, \text{Subst}[\text{Int}[1/\text{Simp}[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; \text{FreeQ}\{a, b, c\},$

$x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 2660

$\text{Int}[(a_.) + (b_.)\sin[(c_.) + (d_.)x]^{-1}, x_Symbol] \rightarrow \text{With}\{e = \text{FreeFactors}[\text{Tan}[(c + dx)/2], x]\}, \text{Dist}[(2e)/d, \text{Subst}[\text{Int}[1/(a + 2bex + ae^2x^2), x], x, \text{Tan}[(c + dx)/2]/e], x] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[a^2 - b^2, 0]$

Rule 3785

$\text{Int}[(\csc[(c_.) + (d_.)x] * (b_.) + (a_.)^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(b^2 \cot[c + dx] * (a + b \csc[c + dx])^{(n+1)}) / (a d (n+1) (a^2 - b^2)), x] + \text{Dist}[1 / (a (n+1) (a^2 - b^2)), \text{Int}[(a + b \csc[c + dx])^{(n+1)} * \text{Simp}[(a^2 - b^2) * (n+1) - a b (n+1) \csc[c + dx] + b^2 (n+2) \csc[c + dx]^2, x], x], x] /; \text{FreeQ}\{a, b, c, d\}, x] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{LtQ}[n, -1] \&\& \text{IntegerQ}[2*n]$

Rule 3831

$\text{Int}[\csc[(e_.) + (f_.)x] / (\csc[(e_.) + (f_.)x] * (b_.) + (a_)), x_Symbol] \rightarrow \text{Dist}[1/b, \text{Int}[1 / (1 + (a \sin[e + fx])/b), x], x] /; \text{FreeQ}\{a, b, e, f\}, x] \&\& \text{NeQ}[a^2 - b^2, 0]$

Rule 3919

$\text{Int}[(\csc[(e_.) + (f_.)x] * (d_.) + (c_.) / (\csc[(e_.) + (f_.)x] * (b_.) + (a_)), x_Symbol] \rightarrow \text{Simp}[(c*x)/a, x] - \text{Dist}[(b*c - a*d)/a, \text{Int}[\csc[e + fx] / (a + b \csc[e + fx]), x], x] /; \text{FreeQ}\{a, b, c, d, e, f\}, x] \&\& \text{NeQ}[b*c - a*d, 0]$

Rule 4060

$\text{Int}[(A_.) + \csc[(e_.) + (f_.)x] * (B_.) + \csc[(e_.) + (f_.)x]^2 * (C_.) * (\csc[(e_.) + (f_.)x] * (b_.) + (a_))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[(A*b^2 - a*b*B + a^2*C) \cot[e + fx] * (a + b \csc[e + fx])^{(m+1)} / (a*f*(m+1)*(a^2 - b^2)), x] + \text{Dist}[1 / (a*(m+1)*(a^2 - b^2)), \text{Int}[(a + b \csc[e + fx])^{(m+1)} * \text{Simp}[A*(a^2 - b^2)*(m+1) - a*(A*b - a*B + b*C)*(m+1) \csc[e + fx] + (A*b^2 - a*b*B + a^2*C)*(m+2) \csc[e + fx]^2, x], x], x] /; \text{FreeQ}\{a, b, e, f, A, B, C\}, x] \&\& \text{NeQ}[a^2 - b^2, 0] \&\& \text{LtQ}[m, -1]$

Rubi steps

$$\begin{aligned} \int \frac{1}{(a + b \csc(c + dx))^4} dx &= -\frac{b^2 \cot(c + dx)}{3a(a^2 - b^2)d(a + b \csc(c + dx))^3} - \frac{\int \frac{-3(a^2 - b^2) + 3ab \csc(c + dx) - 2b^2 \csc^2(c + dx)}{(a + b \csc(c + dx))^3} dx}{3a(a^2 - b^2)} \\ &= -\frac{b^2 \cot(c + dx)}{3a(a^2 - b^2)d(a + b \csc(c + dx))^3} - \frac{b^2(8a^2 - 3b^2)\cot(c + dx)}{6a^2(a^2 - b^2)^2 d(a + b \csc(c + dx))^2} + \frac{\int \frac{6(a^2 - b^2)}{(a + b \csc(c + dx))^2} dx}{6a^2(a^2 - b^2)^2} \\ &= -\frac{b^2 \cot(c + dx)}{3a(a^2 - b^2)d(a + b \csc(c + dx))^3} - \frac{b^2(8a^2 - 3b^2)\cot(c + dx)}{6a^2(a^2 - b^2)^2 d(a + b \csc(c + dx))^2} - \frac{b^2(26a^4 - 15ab^2)}{6a^3(a^2 - b^2)^2} \\ &= \frac{x}{a^4} - \frac{b^2 \cot(c + dx)}{3a(a^2 - b^2)d(a + b \csc(c + dx))^3} - \frac{b^2(8a^2 - 3b^2)\cot(c + dx)}{6a^2(a^2 - b^2)^2 d(a + b \csc(c + dx))^2} - \frac{b^2(26a^4 - 15ab^2)}{6a^3(a^2 - b^2)^2} \\ &= \frac{x}{a^4} - \frac{b^2 \cot(c + dx)}{3a(a^2 - b^2)d(a + b \csc(c + dx))^3} - \frac{b^2(8a^2 - 3b^2)\cot(c + dx)}{6a^2(a^2 - b^2)^2 d(a + b \csc(c + dx))^2} - \frac{b^2(26a^4 - 15ab^2)}{6a^3(a^2 - b^2)^2} \\ &= \frac{x}{a^4} - \frac{b^2 \cot(c + dx)}{3a(a^2 - b^2)d(a + b \csc(c + dx))^3} - \frac{b^2(8a^2 - 3b^2)\cot(c + dx)}{6a^2(a^2 - b^2)^2 d(a + b \csc(c + dx))^2} - \frac{b^2(26a^4 - 15ab^2)}{6a^3(a^2 - b^2)^2} \\ &= \frac{x}{a^4} - \frac{b^2 \cot(c + dx)}{3a(a^2 - b^2)d(a + b \csc(c + dx))^3} - \frac{b^2(8a^2 - 3b^2)\cot(c + dx)}{6a^2(a^2 - b^2)^2 d(a + b \csc(c + dx))^2} - \frac{b^2(26a^4 - 15ab^2)}{6a^3(a^2 - b^2)^2} \\ &= \frac{x}{a^4} + \frac{b(8a^6 - 8a^4b^2 + 7a^2b^4 - 2b^6) \tanh^{-1}\left(\frac{b\left(\frac{a}{b} + \tan\left(\frac{1}{2}(c + dx)\right)\right)}{\sqrt{a^2 - b^2}}\right)}{a^4(a^2 - b^2)^{7/2}d} - \frac{b^2 \cot(c + dx)}{3a(a^2 - b^2)d(a + b \csc(c + dx))^3} \end{aligned}$$

Mathematica [A] time = 2.02, size = 279, normalized size = 1.17

$$\csc^3(c + dx)(a \sin(c + dx) + b) \left(\frac{ab^3(12a^2 - 7b^2)\cot(c + dx)(a \sin(c + dx) + b)}{(a - b)^2(a + b)^2} - \frac{ab^2(36a^4 - 32a^2b^2 + 11b^4)\cot(c + dx)(a \sin(c + dx) + b)^2}{(a - b)^3(a + b)^3} - \frac{6b(-8a^4d(a + b) + 6a^4d(a + b) + 6a^4d(a + b) + 6a^4d(a + b))}{6a^3(a^2 - b^2)^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Csc[c + d*x])^(-4),x]

[Out] (Csc[c + d*x]^3*(b + a*Sin[c + d*x])*((2*a*b^4*Cot[c + d*x])/((-a + b)*(a + b)) + (a*b^3*(12*a^2 - 7*b^2)*Cot[c + d*x]*(b + a*Sin[c + d*x]))/((a - b)^2*(a + b)^2) - (a*b^2*(36*a^4 - 32*a^2*b^2 + 11*b^4)*Cot[c + d*x]*(b + a*Sin[c + d*x])^2)/((a - b)^3*(a + b)^3) + 6*(c + d*x)*Csc[c + d*x]*(b + a*Sin[c + d*x])^3 - (6*b*(-8*a^6 + 8*a^4*b^2 - 7*a^2*b^4 + 2*b^6)*ArcTan[(a + b*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]]*Csc[c + d*x]*(b + a*Sin[c + d*x])^3)/(-a^2 + b^2)^(7/2))/((6*a^4*d*(a + b*Csc[c + d*x])^4)

fricas [B] time = 0.75, size = 1554, normalized size = 6.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*csc(d*x+c))^4,x, algorithm="fricas")

[Out] [1/12*(36*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*x*cos(d*x + c)^2 - 2*(36*a^9*b^2 - 68*a^7*b^4 + 43*a^5*b^6 - 11*a^3*b^8)*cos(d*x + c)^3 - 12*(3*a^10*b - 11*a^8*b^3 + 14*a^6*b^5 - 6*a^4*b^7 - a^2*b^9 + b^11)*d*x - 3*(24*a^8*b^2 - 16*a^6*b^4 + 13*a^4*b^6 + a^2*b^8 - 2*b^10 - 3*(8*a^8*b^2 - 8*a^6*b^4 + 7*a^4*b^6 - 2*a^2*b^8)*cos(d*x + c)^2 + (8*a^9*b + 16*a^7*b^3 - 17*a^5*b^5 + 19*a^3*b^7 - 6*a*b^9 - (8*a^9*b - 8*a^7*b^3 + 7*a^5*b^5 - 2*a^3*b^7)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 - b^2)*log(((a^2 - 2*b^2)*cos(d*x + c)^2 + 2*a*b*sin(d*x + c) + a^2 + b^2 + 2*(b*cos(d*x + c))*sin(d*x + c) + a*cos(d*x + c))*sqrt(a^2 - b^2))/(a^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c) - a^2 - b^2) + 12*(6*a^9*b^2 - 7*a^7*b^4 + 2*a^3*b^8 - a*b^10)*cos(d*x + c) + 6*(2*(a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*x*cos(d*x + c)^2 - 2*(a^11 - a^9*b^2 - 6*a^7*b^4 + 14*a^5*b^6 - 11*a^3*b^8 + 3*a*b^10)*d*x + 5*(4*a^8*b^3 - 7*a^6*b^5 + 4*a^4*b^7 - a^2*b^9)*cos(d*x + c))*sin(d*x + c))/(3*(a^14*b - 4*a^12*b^3 + 6*a^10*b^5 - 4*a^8*b^7 + a^6*b^9)*d*cos(d*x + c)^2 - (3*a^14*b - 11*a^12*b^3 + 14*a^10*b^5 - 6*a^8*b^7 - a^6*b^9 + a^4*b^11)*d + ((a^15 - 4*a^13*b^2 + 6*a^11*b^4 - 4*a^9*b^6 + a^7*b^8)*d*cos(d*x + c)^2 - (a^15 - a^13*b^2 - 6*a^11*b^4 + 14*a^9*b^6 - 11*a^7*b^8 + 3*a^5*b^10)*d)*sin(d*x + c)), 1/6*(18*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*x*cos(d*x + c)^2 - (36*a^9*b^2 - 68*a^7*b^4 + 43*a^5*b^6 - 11*a^3*b^8)*cos(d*x + c)^3 - 6*(3*a^10*b - 11*a^8*b^3 + 14*a^6*b^5 - 6*a^4*b^7 - a^2*b^9 + b^11)*d*x - 3*(24*a^8*b^2 - 16*a^6*b^4 + 13*a^4*b^6 + a^2*b^8 - 2*b^10 - 3*(8*a^8*b^2 - 8*a^6*b^4 + 7*a^4*b^6 - 2*a^2*b^8)*cos(d*x + c)^2 + (8*a^9*b + 16*a^7*b^3 - 17*a^5*b^5 + 19*a^3*b^7 - 6*a*b^9 - (8*a^9*b - 8*a^7*b^3 + 7*a^5*b^5 - 2*a^3*b^7)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*sin(d*x + c) + a)/((a^2 - b^2)*cos(d*x + c))) + 6*(6*a^9*b^2 - 7*a^7*b^4 + 2*a^3*b^8 - a*b^10)*cos(d*x + c) + 3*(2*(a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*x*cos(d*x + c)^2 - 2*(a^11 - a^9*b^2 - 6*a^7*b^4 + 14*a^5*b^6 - 11*a^3*b^8 + 3*a

```
*b^10)*d*x + 5*(4*a^8*b^3 - 7*a^6*b^5 + 4*a^4*b^7 - a^2*b^9)*cos(d*x + c))*
sin(d*x + c))/(3*(a^14*b - 4*a^12*b^3 + 6*a^10*b^5 - 4*a^8*b^7 + a^6*b^9)*d
*cos(d*x + c)^2 - (3*a^14*b - 11*a^12*b^3 + 14*a^10*b^5 - 6*a^8*b^7 - a^6*b
^9 + a^4*b^11)*d + ((a^15 - 4*a^13*b^2 + 6*a^11*b^4 - 4*a^9*b^6 + a^7*b^8)*
d*cos(d*x + c)^2 - (a^15 - a^13*b^2 - 6*a^11*b^4 + 14*a^9*b^6 - 11*a^7*b^8
+ 3*a^5*b^10)*d)*sin(d*x + c))]
```

giac [B] time = 0.38, size = 535, normalized size = 2.24

$$\frac{3(8a^6b - 8a^4b^3 + 7a^2b^5 - 2b^7) \left(\pi \left[\frac{dx+c}{2\pi} + \frac{1}{2} \right] \operatorname{sgn}(b) + \arctan \left(\frac{b \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + a}{\sqrt{-a^2 + b^2}} \right) \right)}{(a^{10} - 3a^8b^2 + 3a^6b^4 - a^4b^6) \sqrt{-a^2 + b^2}} + \frac{18a^5b^3 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^5 - 6a^3b^5 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^5 + 3ab^7 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right)^5}{\sqrt{-a^2 + b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*csc(d*x+c))^4,x, algorithm="giac")

```
[Out] -1/3*(3*(8*a^6*b - 8*a^4*b^3 + 7*a^2*b^5 - 2*b^7)*(pi*floor(1/2*(d*x + c)/p
i + 1/2)*sgn(b) + arctan((b*tan(1/2*d*x + 1/2*c) + a)/sqrt(-a^2 + b^2)))/((
a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*sqrt(-a^2 + b^2)) + (18*a^5*b^3*tan
(1/2*d*x + 1/2*c)^5 - 6*a^3*b^5*tan(1/2*d*x + 1/2*c)^5 + 3*a*b^7*tan(1/2*d*
x + 1/2*c)^5 + 84*a^6*b^2*tan(1/2*d*x + 1/2*c)^4 - 12*a^4*b^4*tan(1/2*d*x +
1/2*c)^4 - 3*a^2*b^6*tan(1/2*d*x + 1/2*c)^4 + 6*b^8*tan(1/2*d*x + 1/2*c)^4
+ 104*a^7*b*tan(1/2*d*x + 1/2*c)^3 + 88*a^5*b^3*tan(1/2*d*x + 1/2*c)^3 - 7
8*a^3*b^5*tan(1/2*d*x + 1/2*c)^3 + 36*a*b^7*tan(1/2*d*x + 1/2*c)^3 + 228*a^
6*b^2*tan(1/2*d*x + 1/2*c)^2 - 114*a^4*b^4*tan(1/2*d*x + 1/2*c)^2 + 24*a^2*
b^6*tan(1/2*d*x + 1/2*c)^2 + 12*b^8*tan(1/2*d*x + 1/2*c)^2 + 138*a^5*b^3*ta
n(1/2*d*x + 1/2*c) - 96*a^3*b^5*tan(1/2*d*x + 1/2*c) + 33*a*b^7*tan(1/2*d*x
+ 1/2*c) + 26*a^4*b^4 - 17*a^2*b^6 + 6*b^8)/((a^9 - 3*a^7*b^2 + 3*a^5*b^4
- a^3*b^6)*(b*tan(1/2*d*x + 1/2*c)^2 + 2*a*tan(1/2*d*x + 1/2*c) + b)^3) - 3
*(d*x + c)/a^4)/d
```

maple [B] time = 0.82, size = 1912, normalized size = 8.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b*csc(d*x+c))^4,x)

```
[Out] -88/3/d*a^2*b^3/(tan(1/2*d*x+1/2*c)^2*b+2*a*tan(1/2*d*x+1/2*c)+b)^3/(a^6-3*
a^4*b^2+3*a^2*b^4-b^6)*tan(1/2*d*x+1/2*c)^3-46/d*a^2*b^3/(tan(1/2*d*x+1/2*c
)^2*b+2*a*tan(1/2*d*x+1/2*c)+b)^3/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)*tan(1/2*d*x
+1/2*c)+4/d*a*b^4/(tan(1/2*d*x+1/2*c)^2*b+2*a*tan(1/2*d*x+1/2*c)+b)^3/(a^6-
```


$$\begin{aligned}
& 3a^4b^2+3a^2b^4-b^6) \tan(1/2dx+1/2c)^4 + 2/d/a^4 \arctan(\tan(1/2dx+1/2c)) \\
& - 8/d/a^6b^6 / (\tan(1/2dx+1/2c)^{2b+2a} \tan(1/2dx+1/2c) + b)^3 / (a^6-3a^4b^2+3a^2b^4-b^6) \\
& \tan(1/2dx+1/2c)^2 - 8/d/a^2b^6 / (a^6-3a^4b^2+3a^2b^4-b^6) \\
& \tan(1/2dx+1/2c)^5 + 38/d/a^4b^4 / (\tan(1/2dx+1/2c)^{2b+2a} \tan(1/2dx+1/2c) + b)^3 \\
& / (a^6-3a^4b^2+3a^2b^4-b^6) \tan(1/2dx+1/2c)^2 - 1/d/a^2b^7 / (\tan(1/2dx+1/2c)^{2b+2a} \tan(1/2dx+1/2c) + b)^3 \\
& / (a^6-3a^4b^2+3a^2b^4-b^6) \tan(1/2dx+1/2c)^5 - 104/3/d/a^4b^4 / (\tan(1/2dx+1/2c)^{2b+2a} \tan(1/2dx+1/2c) + b)^3 \\
& / (a^6-3a^4b^2+3a^2b^4-b^6) \tan(1/2dx+1/2c)^3 - 7/d/a^2b^5 / (a^6-3a^4b^2+3a^2b^4-b^6) \\
& / (-a^2+b^2)^{(1/2)} \arctan(1/2(2b \tan(1/2dx+1/2c) + 2a) / (-a^2+b^2)^{(1/2)}) - 2/d/a^3b^8 / (\tan(1/2dx+1/2c)^{2b+2a} \tan(1/2dx+1/2c) + b)^3 \\
& / (a^6-3a^4b^2+3a^2b^4-b^6) \tan(1/2dx+1/2c)^4 - 11/d/a^2b^7 / (\tan(1/2dx+1/2c)^{2b+2a} \tan(1/2dx+1/2c) + b)^3 \\
& / (a^6-3a^4b^2+3a^2b^4-b^6) \tan(1/2dx+1/2c) - 28/d/a^3b^2 / (\tan(1/2dx+1/2c)^{2b+2a} \tan(1/2dx+1/2c) + b)^3 \\
& / (a^6-3a^4b^2+3a^2b^4-b^6) \tan(1/2dx+1/2c)^4 - 4/d/a^3b^8 / (\tan(1/2dx+1/2c)^{2b+2a} \tan(1/2dx+1/2c) + b)^3 \\
& / (a^6-3a^4b^2+3a^2b^4-b^6) \tan(1/2dx+1/2c)^2 - 26/3/d/a^4b^4 / (\tan(1/2dx+1/2c)^{2b+2a} \tan(1/2dx+1/2c) + b)^3 \\
& / (a^6-3a^4b^2+3a^2b^4-b^6) + 17/3/d/a^6b^6 / (\tan(1/2dx+1/2c)^{2b+2a} \tan(1/2dx+1/2c) + b)^3 \\
& / (a^6-3a^4b^2+3a^2b^4-b^6) + 2/d/a^4b^7 / (a^6-3a^4b^2+3a^2b^4-b^6) / (-a^2+b^2)^{(1/2)} \arctan(1/2(2b \tan(1/2dx+1/2c) + 2a) / (-a^2+b^2)^{(1/2)}) \\
& + 1/d/a^6b^6 / (\tan(1/2dx+1/2c)^{2b+2a} \tan(1/2dx+1/2c) + b)^3 / (a^6-3a^4b^2+3a^2b^4-b^6) \tan(1/2dx+1/2c)^4 - 12/d/a^2b^7 / (\tan(1/2dx+1/2c)^{2b+2a} \tan(1/2dx+1/2c) + b)^3 \\
& / (a^6-3a^4b^2+3a^2b^4-b^6) \tan(1/2dx+1/2c)^2 - 2/d/a^3b^8 / (\tan(1/2dx+1/2c)^{2b+2a} \tan(1/2dx+1/2c) + b)^3 \\
& / (a^6-3a^4b^2+3a^2b^4-b^6) + 8/d/b^3 / (a^6-3a^4b^2+3a^2b^4-b^6) / (-a^2+b^2)^{(1/2)} \arctan(1/2(2b \tan(1/2dx+1/2c) + 2a) / (-a^2+b^2)^{(1/2)}) + 2/d/b^5 / (\tan(1/2dx+1/2c)^{2b+2a} \tan(1/2dx+1/2c) + b)^3 \\
& / (a^6-3a^4b^2+3a^2b^4-b^6) \tan(1/2dx+1/2c)^5 + 32/d/b^5 / (\tan(1/2dx+1/2c)^{2b+2a} \tan(1/2dx+1/2c) + b)^3 \\
& / (a^6-3a^4b^2+3a^2b^4-b^6) \tan(1/2dx+1/2c) + 26/d/b^5 / (\tan(1/2dx+1/2c)^{2b+2a} \tan(1/2dx+1/2c) + b)^3 \\
& / (a^6-3a^4b^2+3a^2b^4-b^6) \tan(1/2dx+1/2c)^3
\end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*csc(dx+c))^4,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* h

elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more details) Is 4*a^2-4*b^2 positive or negative?

mupad [B] time = 12.38, size = 8167, normalized size = 34.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b/sin(c + d*x))^4,x)

[Out]
$$\begin{aligned} & (2*\operatorname{atan}\left(\frac{(8*(4*a^3*b^{14} - 24*a^5*b^{12} + 60*a^7*b^{10} - 80*a^9*b^8 + 60*a^{11}*b^6 - 24*a^{13}*b^4 + 4*a^{15}*b^2))}{(a^{20} + a^8*b^{12} - 6*a^{10}*b^{10} + 15*a^{12}*b^8 - 20*a^{14}*b^6 + 15*a^{16}*b^4 - 6*a^{18}*b^2)} - \frac{(8*(4*a^{20}*b + 2*a^8*b^{13} - 14*a^{10}*b^{11} + 30*a^{12}*b^9 - 30*a^{14}*b^7 + 20*a^{16}*b^5 - 12*a^{18}*b^3))}{(a^{20} + a^8*b^{12} - 6*a^{10}*b^{10} + 15*a^{12}*b^8 - 20*a^{14}*b^6 + 15*a^{16}*b^4 - 6*a^{18}*b^2)} - \frac{(8*(4*a^{11}*b^{14} - 24*a^{13}*b^{12} + 60*a^{15}*b^{10} - 80*a^{17}*b^8 + 60*a^{19}*b^6 - 24*a^{21}*b^4 + 4*a^{23}*b^2))}{(a^{20} + a^8*b^{12} - 6*a^{10}*b^{10} + 15*a^{12}*b^8 - 20*a^{14}*b^6 + 15*a^{16}*b^4 - 6*a^{18}*b^2)} + (8*\tan(c/2 + (d*x)/2)*(12*a^{25}*b - 8*a^{11}*b^{15} + 60*a^{13}*b^{13} - 192*a^{15}*b^{11} + 340*a^{17}*b^9 - 360*a^{19}*b^7 + 228*a^{21}*b^5 - 80*a^{23}*b^3))}{(a^{21} + a^9*b^{12} - 6*a^{11}*b^{10} + 15*a^{13}*b^8 - 20*a^{15}*b^6 + 15*a^{17}*b^4 - 6*a^{19}*b^2)}\right)*i)/a^4 + (8*\tan(c/2 + (d*x)/2)*(8*a^8*b^{14} - 52*a^{10}*b^{12} + 140*a^{12}*b^{10} - 220*a^{14}*b^8 + 220*a^{16}*b^6 - 128*a^{18}*b^4 + 32*a^{20}*b^2))/(a^{21} + a^9*b^{12} - 6*a^{11}*b^{10} + 15*a^{13}*b^8 - 20*a^{15}*b^6 + 15*a^{17}*b^4 - 6*a^{19}*b^2))*i)/a^4 + (8*\tan(c/2 + (d*x)/2)*(8*a^{17}*b - 8*a^3*b^{15} + 60*a^5*b^{13} - 189*a^7*b^{11} + 344*a^9*b^9 - 396*a^{11}*b^7 + 272*a^{13}*b^5 - 116*a^{15}*b^3))/(a^{21} + a^9*b^{12} - 6*a^{11}*b^{10} + 15*a^{13}*b^8 - 20*a^{15}*b^6 + 15*a^{17}*b^4 - 6*a^{19}*b^2))/a^4 + (((((8*(4*a^{11}*b^{14} - 24*a^{13}*b^{12} + 60*a^{15}*b^{10} - 80*a^{17}*b^8 + 60*a^{19}*b^6 - 24*a^{21}*b^4 + 4*a^{23}*b^2))/(a^{20} + a^8*b^{12} - 6*a^{10}*b^{10} + 15*a^{12}*b^8 - 20*a^{14}*b^6 + 15*a^{16}*b^4 - 6*a^{18}*b^2) + (8*\tan(c/2 + (d*x)/2)*(12*a^{25}*b - 8*a^{11}*b^{15} + 60*a^{13}*b^{13} - 192*a^{15}*b^{11} + 340*a^{17}*b^9 - 360*a^{19}*b^7 + 228*a^{21}*b^5 - 80*a^{23}*b^3))/(a^{21} + a^9*b^{12} - 6*a^{11}*b^{10} + 15*a^{13}*b^8 - 20*a^{15}*b^6 + 15*a^{17}*b^4 - 6*a^{19}*b^2))*i)/a^4 + (8*(4*a^{20}*b + 2*a^8*b^{13} - 14*a^{10}*b^{11} + 30*a^{12}*b^9 - 30*a^{14}*b^7 + 20*a^{16}*b^5 - 12*a^{18}*b^3))/(a^{20} + a^8*b^{12} - 6*a^{10}*b^{10} + 15*a^{12}*b^8 - 20*a^{14}*b^6 + 15*a^{16}*b^4 - 6*a^{18}*b^2) + (8*\tan(c/2 + (d*x)/2)*(8*a^8*b^{14} - 52*a^{10}*b^{12} + 140*a^{12}*b^{10} - 220*a^{14}*b^8 + 220*a^{16}*b^6 - 128*a^{18}*b^4 + 32*a^{20}*b^2))/(a^{21} + a^9*b^{12} - 6*a^{11}*b^{10} + 15*a^{13}*b^8 - 20*a^{15}*b^6 + 15*a^{17}*b^4 - 6*a^{19}*b^2))*i)/a^4 + (8*(4*a^3*b^{14} - 24*a^5*b^{12} + 60*a^7*b^{10} - 80*a^9*b^8 + 60*a^{11}*b^6 - 24*a^{13}*b^4 + 4*a^{15}*b^2))/(a^{20} + a^8*b^{12} - 6*a^{10}*b^{10} + 15*a^{12}*b^8 - 20*a^{14}*b^6 + 15*a^{16}*b^4 - 6*a^{18}*b^2) + (8*\tan(c/2 + (d*x)/2)*(8*a^{17}*b - 8*a^3*b^{15} + 60*a^5*b^{13} - 189*a^7*b^{11} + 344*a^9*b^9 - 396*a^{11}*b^7 + 272*a^{13}*b^5 - 116*a^{15}*b^3))/(a^{21} + a^9*b^{12} - 6*a^{11}*b^{10} + 15*a^{13}*b^8 - 20*a^{15}*b^6 + 15*a^{17}*b^4 - 6*a^{19}*b^2))/a^4)/((16*(2*b^{13} - 11*a^2*b^{11} + 34*a^4*b^9 - 66*a^6*b^7 + 64*a^8*b^5 - 48*a^{10}*b^3))/(a^{20} + a$$

$$\begin{aligned}
& 8*b^{12} - 6*a^{10}*b^{10} + 15*a^{12}*b^8 - 20*a^{14}*b^6 + 15*a^{16}*b^4 - 6*a^{18}*b^2) - (((8*(4*a^3*b^{14} - 24*a^5*b^{12} + 60*a^7*b^{10} - 80*a^9*b^8 + 60*a^{11}*b^6 - 24*a^{13}*b^4 + 4*a^{15}*b^2)))/(a^{20} + a^8*b^{12} - 6*a^{10}*b^{10} + 15*a^{12}*b^8 - 20*a^{14}*b^6 + 15*a^{16}*b^4 - 6*a^{18}*b^2) - (((8*(4*a^{20}*b + 2*a^8*b^{13} - 14*a^{10}*b^{11} + 30*a^{12}*b^9 - 30*a^{14}*b^7 + 20*a^{16}*b^5 - 12*a^{18}*b^3)))/(a^{20} + a^8*b^{12} - 6*a^{10}*b^{10} + 15*a^{12}*b^8 - 20*a^{14}*b^6 + 15*a^{16}*b^4 - 6*a^{18}*b^2) - (((8*(4*a^{11}*b^{14} - 24*a^{13}*b^{12} + 60*a^{15}*b^{10} - 80*a^{17}*b^8 + 60*a^{19}*b^6 - 24*a^{21}*b^4 + 4*a^{23}*b^2)))/(a^{20} + a^8*b^{12} - 6*a^{10}*b^{10} + 15*a^{12}*b^8 - 20*a^{14}*b^6 + 15*a^{16}*b^4 - 6*a^{18}*b^2) + (8*\tan(c/2 + (d*x)/2) * (12*a^{25}*b - 8*a^{11}*b^{15} + 60*a^{13}*b^{13} - 192*a^{15}*b^{11} + 340*a^{17}*b^9 - 360*a^{19}*b^7 + 228*a^{21}*b^5 - 80*a^{23}*b^3)))/(a^{21} + a^9*b^{12} - 6*a^{11}*b^{10} + 15*a^{13}*b^8 - 20*a^{15}*b^6 + 15*a^{17}*b^4 - 6*a^{19}*b^2))*1i)/a^4 + (8*\tan(c/2 + (d*x)/2)*(8*a^8*b^{14} - 52*a^{10}*b^{12} + 140*a^{12}*b^{10} - 220*a^{14}*b^8 + 220*a^{16}*b^6 - 128*a^{18}*b^4 + 32*a^{20}*b^2))/(a^{21} + a^9*b^{12} - 6*a^{11}*b^{10} + 15*a^{13}*b^8 - 20*a^{15}*b^6 + 15*a^{17}*b^4 - 6*a^{19}*b^2))*1i)/a^4 + (8*\tan(c/2 + (d*x)/2)*(8*a^{17}*b - 8*a^3*b^{15} + 60*a^5*b^{13} - 189*a^7*b^{11} + 344*a^9*b^9 - 396*a^{11}*b^7 + 272*a^{13}*b^5 - 116*a^{15}*b^3))/(a^{21} + a^9*b^{12} - 6*a^{11}*b^{10} + 15*a^{13}*b^8 - 20*a^{15}*b^6 + 15*a^{17}*b^4 - 6*a^{19}*b^2))*1i)/a^4 + (((((((8*(4*a^{11}*b^{14} - 24*a^{13}*b^{12} + 60*a^{15}*b^{10} - 80*a^{17}*b^8 + 60*a^{19}*b^6 - 24*a^{21}*b^4 + 4*a^{23}*b^2)))/(a^{20} + a^8*b^{12} - 6*a^{10}*b^{10} + 15*a^{12}*b^8 - 20*a^{14}*b^6 + 15*a^{16}*b^4 - 6*a^{18}*b^2) + (8*\tan(c/2 + (d*x)/2)*(12*a^{25}*b - 8*a^{11}*b^{15} + 60*a^{13}*b^{13} - 192*a^{15}*b^{11} + 340*a^{17}*b^9 - 360*a^{19}*b^7 + 228*a^{21}*b^5 - 80*a^{23}*b^3)))/(a^{21} + a^9*b^{12} - 6*a^{11}*b^{10} + 15*a^{13}*b^8 - 20*a^{15}*b^6 + 15*a^{17}*b^4 - 6*a^{19}*b^2))*1i)/a^4 + (8*(4*a^{20}*b + 2*a^8*b^{13} - 14*a^{10}*b^{11} + 30*a^{12}*b^9 - 30*a^{14}*b^7 + 20*a^{16}*b^5 - 12*a^{18}*b^3))/(a^{20} + a^8*b^{12} - 6*a^{10}*b^{10} + 15*a^{12}*b^8 - 20*a^{14}*b^6 + 15*a^{16}*b^4 - 6*a^{18}*b^2) + (8*\tan(c/2 + (d*x)/2)*(8*a^8*b^{14} - 52*a^{10}*b^{12} + 140*a^{12}*b^{10} - 220*a^{14}*b^8 + 220*a^{16}*b^6 - 128*a^{18}*b^4 + 32*a^{20}*b^2)))/(a^{21} + a^9*b^{12} - 6*a^{11}*b^{10} + 15*a^{13}*b^8 - 20*a^{15}*b^6 + 15*a^{17}*b^4 - 6*a^{19}*b^2))*1i)/a^4 + (8*(4*a^3*b^{14} - 24*a^5*b^{12} + 60*a^7*b^{10} - 80*a^9*b^8 + 60*a^{11}*b^6 - 24*a^{13}*b^4 + 4*a^{15}*b^2))/(a^{20} + a^8*b^{12} - 6*a^{10}*b^{10} + 15*a^{12}*b^8 - 20*a^{14}*b^6 + 15*a^{16}*b^4 - 6*a^{18}*b^2) + (8*\tan(c/2 + (d*x)/2)*(8*a^{17}*b - 8*a^3*b^{15} + 60*a^5*b^{13} - 189*a^7*b^{11} + 344*a^9*b^9 - 396*a^{11}*b^7 + 272*a^{13}*b^5 - 116*a^{15}*b^3))/(a^{21} + a^9*b^{12} - 6*a^{11}*b^{10} + 15*a^{13}*b^8 - 20*a^{15}*b^6 + 15*a^{17}*b^4 - 6*a^{19}*b^2))*1i)/a^4 + (16*\tan(c/2 + (d*x)/2)*(8*b^{14} - 52*a^2*b^{12} + 140*a^4*b^{10} - 220*a^6*b^8 + 220*a^8*b^6 - 128*a^{10}*b^4 + 32*a^{12}*b^2))/(a^{21} + a^9*b^{12} - 6*a^{11}*b^{10} + 15*a^{13}*b^8 - 20*a^{15}*b^6 + 15*a^{17}*b^4 - 6*a^{19}*b^2)))/((a^4*d) - ((6*b^8 - 17*a^2*b^6 + 26*a^4*b^4)/(3*a^3*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (\tan(c/2 + (d*x)/2)*(11*b^7 - 32*a^2*b^5 + 46*a^4*b^3))/(a^2*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (\tan(c/2 + (d*x)/2)^4*(2*b^8 - a^2*b^6 - 4*a^4*b^4 + 28*a^6*b^2))/(a^3*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (2*\tan(c/2 + (d*x)/2)^2*(2*b^8 + 4*a^2*b^6 - 19*a^4*b^4 + 38*a^6*b^2))/(a^3*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (2*\tan(c/2 + (d*x)/2)^3*(2*a^2 + 3*b^2)*(26*a^4*b + 6*b^5 - 17*a^2*b^3))/(3*a^2*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (b^3*\tan(c/2 +
\end{aligned}$$

$$\begin{aligned}
& ((d*x)/2)^5*(6*a^4 + b^4 - 2*a^2*b^2)/(a^2*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)))/(d*(b^3*\tan(c/2 + (d*x)/2)^6 + \tan(c/2 + (d*x)/2)^3*(12*a*b^2 + 8*a^3 \\
&) + \tan(c/2 + (d*x)/2)^2*(12*a^2*b + 3*b^3) + \tan(c/2 + (d*x)/2)^4*(12*a^2*b + 3*b^3) + b^3 + 6*a*b^2*\tan(c/2 + (d*x)/2) + 6*a*b^2*\tan(c/2 + (d*x)/2)^5) \\
& + (b*\operatorname{atan}(((b*((a + b)^7*(a - b)^7)^{(1/2)}*((8*(4*a^3*b^14 - 24*a^5*b^12 + 60*a^7*b^10 - 80*a^9*b^8 + 60*a^11*b^6 - 24*a^13*b^4 + 4*a^15*b^2)))/(a^2 \\
& 0 + a^8*b^12 - 6*a^10*b^10 + 15*a^12*b^8 - 20*a^14*b^6 + 15*a^16*b^4 - 6*a^18*b^2) + (8*\tan(c/2 + (d*x)/2)*(8*a^17*b - 8*a^3*b^15 + 60*a^5*b^13 - 189*a^7*b^11 + 344*a^9*b^9 - 396*a^11*b^7 + 272*a^13*b^5 - 116*a^15*b^3)))/(a^21 \\
& + a^9*b^12 - 6*a^11*b^10 + 15*a^13*b^8 - 20*a^15*b^6 + 15*a^17*b^4 - 6*a^19*b^2) - (b*((a + b)^7*(a - b)^7)^{(1/2)}*((8*(4*a^20*b + 2*a^8*b^13 - 14*a^10*b^11 + 30*a^12*b^9 - 30*a^14*b^7 + 20*a^16*b^5 - 12*a^18*b^3)))/(a^20 + a^8*b^12 - 6*a^10*b^10 + 15*a^12*b^8 - 20*a^14*b^6 + 15*a^16*b^4 - 6*a^18*b^2) \\
&) + (8*\tan(c/2 + (d*x)/2)*(8*a^8*b^14 - 52*a^10*b^12 + 140*a^12*b^10 - 220*a^14*b^8 + 220*a^16*b^6 - 128*a^18*b^4 + 32*a^20*b^2)))/(a^21 + a^9*b^12 - 6*a^11*b^10 + 15*a^13*b^8 - 20*a^15*b^6 + 15*a^17*b^4 - 6*a^19*b^2) - (b*((8 \\
& *(4*a^11*b^14 - 24*a^13*b^12 + 60*a^15*b^10 - 80*a^17*b^8 + 60*a^19*b^6 - 24*a^21*b^4 + 4*a^23*b^2)))/(a^20 + a^8*b^12 - 6*a^10*b^10 + 15*a^12*b^8 - 20*a^14*b^6 + 15*a^16*b^4 - 6*a^18*b^2) + (8*\tan(c/2 + (d*x)/2)*(12*a^25*b - 8*a^11*b^15 + 60*a^13*b^13 - 192*a^15*b^11 + 340*a^17*b^9 - 360*a^19*b^7 + 228*a^21*b^5 - 80*a^23*b^3)))/(a^21 + a^9*b^12 - 6*a^11*b^10 + 15*a^13*b^8 - 20*a^15*b^6 + 15*a^17*b^4 - 6*a^19*b^2))*((a + b)^7*(a - b)^7)^{(1/2)}*(8*a^6 - 2*b^6 + 7*a^2*b^4 - 8*a^4*b^2))/(2*(a^18 - a^4*b^14 + 7*a^6*b^12 - 21*a^8*b^10 + 35*a^10*b^8 - 35*a^12*b^6 + 21*a^14*b^4 - 7*a^16*b^2)))*(8*a^6 - 2*b^6 + 7*a^2*b^4 - 8*a^4*b^2))/(2*(a^18 - a^4*b^14 + 7*a^6*b^12 - 21*a^8*b^10 + 35*a^10*b^8 - 35*a^12*b^6 + 21*a^14*b^4 - 7*a^16*b^2)))*(8*a^6 - 2*b^6 + 7*a^2*b^4 - 8*a^4*b^2)*i)/(2*(a^18 - a^4*b^14 + 7*a^6*b^12 - 21*a^8*b^10 + 35*a^10*b^8 - 35*a^12*b^6 + 21*a^14*b^4 - 7*a^16*b^2)) + (b*((a + b)^7*(a - b)^7)^{(1/2)}*((8*(4*a^3*b^14 - 24*a^5*b^12 + 60*a^7*b^10 - 80*a^9*b^8 + 60*a^11*b^6 - 24*a^13*b^4 + 4*a^15*b^2)))/(a^20 + a^8*b^12 - 6*a^10*b^10 + 15*a^12*b^8 - 20*a^14*b^6 + 15*a^16*b^4 - 6*a^18*b^2) + (8*\tan(c/2 + (d*x)/2)*(8*a^17*b - 8*a^3*b^15 + 60*a^5*b^13 - 189*a^7*b^11 + 344*a^9*b^9 - 396*a^11*b^7 + 272*a^13*b^5 - 116*a^15*b^3)))/(a^21 + a^9*b^12 - 6*a^11*b^10 + 15*a^13*b^8 - 20*a^15*b^6 + 15*a^17*b^4 - 6*a^19*b^2) + (b*((a + b)^7*(a - b)^7)^{(1/2)}*((8*(4*a^20*b + 2*a^8*b^13 - 14*a^10*b^11 + 30*a^12*b^9 - 30*a^14*b^7 + 20*a^16*b^5 - 12*a^18*b^3)))/(a^20 + a^8*b^12 - 6*a^10*b^10 + 15*a^12*b^8 - 20*a^14*b^6 + 15*a^16*b^4 - 6*a^18*b^2) + (8*\tan(c/2 + (d*x)/2)*(8*a^8*b^14 - 52*a^10*b^12 + 140*a^12*b^10 - 220*a^14*b^8 + 220*a^16*b^6 - 128*a^18*b^4 + 32*a^20*b^2)))/(a^21 + a^9*b^12 - 6*a^11*b^10 + 15*a^13*b^8 - 20*a^15*b^6 + 15*a^17*b^4 - 6*a^19*b^2) + (b*((8*(4*a^11*b^14 - 24*a^13*b^12 + 60*a^15*b^10 - 80*a^17*b^8 + 60*a^19*b^6 - 24*a^21*b^4 + 4*a^23*b^2)))/(a^20 + a^8*b^12 - 6*a^10*b^10 + 15*a^12*b^8 - 20*a^14*b^6 + 15*a^16*b^4 - 6*a^18*b^2) + (8*\tan(c/2 + (d*x)/2)*(12*a^25*b - 8*a^11*b^15 + 60*a^13*b^13 - 192*a^15*b^11 + 340*a^17*b^9 - 360*a^19*b^7 + 228*a^21*b^5 - 80*a^23*b^3)))/(a^21 + a^9*b^12 - 6*a^11*b^10 + 15*a^13*b^8 - 20*a^15*b^6 + 15*a^17*b^4 -
\end{aligned}$$

$$\begin{aligned}
& 6a^{19}b^2) * ((a + b)^7 * (a - b)^7)^{(1/2)} * (8a^6 - 2b^6 + 7a^2b^4 - 8a^4b^2) / (2(a^{18} - a^4b^{14} + 7a^6b^{12} - 21a^8b^{10} + 35a^{10}b^8 - 35a^{12}b^6 + 21a^{14}b^4 - 7a^{16}b^2)) * (8a^6 - 2b^6 + 7a^2b^4 - 8a^4b^2) / (2(a^{18} - a^4b^{14} + 7a^6b^{12} - 21a^8b^{10} + 35a^{10}b^8 - 35a^{12}b^6 + 21a^{14}b^4 - 7a^{16}b^2)) * (8a^6 - 2b^6 + 7a^2b^4 - 8a^4b^2) * i) / (2(a^{18} - a^4b^{14} + 7a^6b^{12} - 21a^8b^{10} + 35a^{10}b^8 - 35a^{12}b^6 + 21a^{14}b^4 - 7a^{16}b^2)) / ((16(2b^{13} - 11a^2b^{11} + 34a^4b^9 - 66a^6b^7 + 64a^8b^5 - 48a^{10}b^3)) / (a^{20} + a^8b^{12} - 6a^{10}b^{10} + 15a^{12}b^8 - 20a^{14}b^6 + 15a^{16}b^4 - 6a^{18}b^2) + (16 * \tan(c/2 + (d*x)/2) * (8b^{14} - 52a^2b^{12} + 140a^4b^{10} - 220a^6b^8 + 220a^8b^6 - 128a^{10}b^4 + 32a^{12}b^2)) / (a^{21} + a^9b^{12} - 6a^{11}b^{10} + 15a^{13}b^8 - 20a^{15}b^6 + 15a^{17}b^4 - 6a^{19}b^2) - (b * ((a + b)^7 * (a - b)^7)^{(1/2)} * ((8(4a^3b^{14} - 24a^5b^{12} + 60a^7b^{10} - 80a^9b^8 + 60a^{11}b^6 - 24a^{13}b^4 + 4a^{15}b^2)) / (a^{20} + a^8b^{12} - 6a^{10}b^{10} + 15a^{12}b^8 - 20a^{14}b^6 + 15a^{16}b^4 - 6a^{18}b^2) + (8 * \tan(c/2 + (d*x)/2) * (8a^{17}b - 8a^3b^{15} + 60a^5b^{13} - 189a^7b^{11} + 344a^9b^9 - 396a^{11}b^7 + 272a^{13}b^5 - 116a^{15}b^3)) / (a^{21} + a^9b^{12} - 6a^{11}b^{10} + 15a^{13}b^8 - 20a^{15}b^6 + 15a^{17}b^4 - 6a^{19}b^2) - (b * ((a + b)^7 * (a - b)^7)^{(1/2)} * ((8(4a^{20}b + 2a^8b^{13} - 14a^{10}b^{11} + 30a^{12}b^9 - 30a^{14}b^7 + 20a^{16}b^5 - 12a^{18}b^3)) / (a^{20} + a^8b^{12} - 6a^{10}b^{10} + 15a^{12}b^8 - 20a^{14}b^6 + 15a^{16}b^4 - 6a^{18}b^2) + (8 * \tan(c/2 + (d*x)/2) * (8a^8b^{14} - 52a^{10}b^{12} + 140a^{12}b^{10} - 220a^{14}b^8 + 220a^{16}b^6 - 128a^{18}b^4 + 32a^{20}b^2)) / (a^{21} + a^9b^{12} - 6a^{11}b^{10} + 15a^{13}b^8 - 20a^{15}b^6 + 15a^{17}b^4 - 6a^{19}b^2) - (b * ((8(4a^{11}b^{14} - 24a^{13}b^{12} + 60a^{15}b^{10} - 80a^{17}b^8 + 60a^{19}b^6 - 24a^{21}b^4 + 4a^{23}b^2)) / (a^{20} + a^8b^{12} - 6a^{10}b^{10} + 15a^{12}b^8 - 20a^{14}b^6 + 15a^{16}b^4 - 6a^{18}b^2) + (8 * \tan(c/2 + (d*x)/2) * (12a^{25}b - 8a^{11}b^{15} + 60a^{13}b^{13} - 192a^{15}b^{11} + 340a^{17}b^9 - 360a^{19}b^7 + 228a^{21}b^5 - 80a^{23}b^3)) / (a^{21} + a^9b^{12} - 6a^{11}b^{10} + 15a^{13}b^8 - 20a^{15}b^6 + 15a^{17}b^4 - 6a^{19}b^2)) * ((a + b)^7 * (a - b)^7)^{(1/2)} * (8a^6 - 2b^6 + 7a^2b^4 - 8a^4b^2) / (2(a^{18} - a^4b^{14} + 7a^6b^{12} - 21a^8b^{10} + 35a^{10}b^8 - 35a^{12}b^6 + 21a^{14}b^4 - 7a^{16}b^2)) * (8a^6 - 2b^6 + 7a^2b^4 - 8a^4b^2) / (2(a^{18} - a^4b^{14} + 7a^6b^{12} - 21a^8b^{10} + 35a^{10}b^8 - 35a^{12}b^6 + 21a^{14}b^4 - 7a^{16}b^2)) + (b * ((a + b)^7 * (a - b)^7)^{(1/2)} * ((8(4a^3b^{14} - 24a^5b^{12} + 60a^7b^{10} - 80a^9b^8 + 60a^{11}b^6 - 24a^{13}b^4 + 4a^{15}b^2)) / (a^{20} + a^8b^{12} - 6a^{10}b^{10} + 15a^{12}b^8 - 20a^{14}b^6 + 15a^{16}b^4 - 6a^{18}b^2) + (8 * \tan(c/2 + (d*x)/2) * (8a^{17}b - 8a^3b^{15} + 60a^5b^{13} - 189a^7b^{11} + 344a^9b^9 - 396a^{11}b^7 + 272a^{13}b^5 - 116a^{15}b^3)) / (a^{21} + a^9b^{12} - 6a^{11}b^{10} + 15a^{13}b^8 - 20a^{15}b^6 + 15a^{17}b^4 - 6a^{19}b^2) + (b * ((a + b)^7 * (a - b)^7)^{(1/2)} * ((8(4a^{20}b + 2a^8b^{13} - 14a^{10}b^{11} + 30a^{12}b^9 - 30a^{14}b^7 + 20a^{16}b^5 - 12a^{18}b^3)) / (a^{20} + a^8b^{12} - 6a^{10}b^{10} + 15a^{12}b^8 - 20a^{14}b^6 + 15a^{16}b^4 - 6a^{18}b^2) + (8 * \tan(c/2 + (d*x)/2) * (8a^8b^{14} - 52a^{10}b^{12} + 140a^{12}b^{10} - 220a^{14}b^8
\end{aligned}$$

```

+ 220*a^16*b^6 - 128*a^18*b^4 + 32*a^20*b^2))/(a^21 + a^9*b^12 - 6*a^11*b^1
0 + 15*a^13*b^8 - 20*a^15*b^6 + 15*a^17*b^4 - 6*a^19*b^2) + (b*((8*(4*a^11*
b^14 - 24*a^13*b^12 + 60*a^15*b^10 - 80*a^17*b^8 + 60*a^19*b^6 - 24*a^21*b^
4 + 4*a^23*b^2)))/(a^20 + a^8*b^12 - 6*a^10*b^10 + 15*a^12*b^8 - 20*a^14*b^6
+ 15*a^16*b^4 - 6*a^18*b^2) + (8*tan(c/2 + (d*x)/2)*(12*a^25*b - 8*a^11*b^
15 + 60*a^13*b^13 - 192*a^15*b^11 + 340*a^17*b^9 - 360*a^19*b^7 + 228*a^21*
b^5 - 80*a^23*b^3))/(a^21 + a^9*b^12 - 6*a^11*b^10 + 15*a^13*b^8 - 20*a^15*
b^6 + 15*a^17*b^4 - 6*a^19*b^2))*((a + b)^7*(a - b)^7)^(1/2)*(8*a^6 - 2*b^6
+ 7*a^2*b^4 - 8*a^4*b^2))/(2*(a^18 - a^4*b^14 + 7*a^6*b^12 - 21*a^8*b^10 +
35*a^10*b^8 - 35*a^12*b^6 + 21*a^14*b^4 - 7*a^16*b^2)))*(8*a^6 - 2*b^6 + 7
*a^2*b^4 - 8*a^4*b^2))/(2*(a^18 - a^4*b^14 + 7*a^6*b^12 - 21*a^8*b^10 + 35*
a^10*b^8 - 35*a^12*b^6 + 21*a^14*b^4 - 7*a^16*b^2)))*(8*a^6 - 2*b^6 + 7*a^2
*b^4 - 8*a^4*b^2))/(2*(a^18 - a^4*b^14 + 7*a^6*b^12 - 21*a^8*b^10 + 35*a^10
*b^8 - 35*a^12*b^6 + 21*a^14*b^4 - 7*a^16*b^2))))*((a + b)^7*(a - b)^7)^(1/
2)*(8*a^6 - 2*b^6 + 7*a^2*b^4 - 8*a^4*b^2)*1i)/(d*(a^18 - a^4*b^14 + 7*a^6*
b^12 - 21*a^8*b^10 + 35*a^10*b^8 - 35*a^12*b^6 + 21*a^14*b^4 - 7*a^16*b^2))

```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b \csc(c + dx))^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*csc(d*x+c))**4,x)

[Out] Integral((a + b*csc(c + d*x))**(-4), x)

$$3.52 \quad \int \frac{1}{3+5 \csc(c+dx)} dx$$

Optimal. Leaf size=31

$$-\frac{5 \tan^{-1}\left(\frac{\cos(c+dx)}{\sin(c+dx)+3}\right)}{6d} - \frac{x}{12}$$

[Out] $-1/12*x-5/6*\arctan(\cos(d*x+c)/(3+\sin(d*x+c)))/d$

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3783, 2657}

$$-\frac{5 \tan^{-1}\left(\frac{\cos(c+dx)}{\sin(c+dx)+3}\right)}{6d} - \frac{x}{12}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(3 + 5*\text{Csc}[c + d*x])^{-1}, x]$

[Out] $-x/12 - (5*\text{ArcTan}[\text{Cos}[c + d*x]/(3 + \text{Sin}[c + d*x])])/(6*d)$

Rule 2657

$\text{Int}[(a_ + (b_)*\sin[(c_.) + (d_)*(x_)])^{-1}, x_Symbol] \rightarrow \text{With}[\{q = \text{Rt}[a^2 - b^2, 2]\}, \text{Simp}[x/q, x] + \text{Simp}[(2*\text{ArcTan}[(b*\text{Cos}[c + d*x])/(a + q + b*\text{Sin}[c + d*x])])]/(d*q), x] \text{ /; } \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{GtQ}[a^2 - b^2, 0] \ \&\& \ \text{PosQ}[a]$

Rule 3783

$\text{Int}[(\text{csc}[(c_.) + (d_)*(x_)]*(b_.) + (a_))^{-1}, x_Symbol] \rightarrow \text{Simp}[x/a, x] - \text{Dist}[1/a, \text{Int}[1/(1 + (a*\text{Sin}[c + d*x])/b), x], x] \text{ /; } \text{FreeQ}[\{a, b, c, d\}, x] \ \&\& \ \text{NeQ}[a^2 - b^2, 0]$

Rubi steps

$$\begin{aligned} \int \frac{1}{3+5 \csc(c+dx)} dx &= \frac{x}{3} - \frac{1}{3} \int \frac{1}{1 + \frac{3}{5} \sin(c+dx)} dx \\ &= -\frac{x}{12} - \frac{5 \tan^{-1}\left(\frac{\cos(c+dx)}{3+\sin(c+dx)}\right)}{6d} \end{aligned}$$

Mathematica [B] time = 0.05, size = 66, normalized size = 2.13

$$\frac{2(c + dx) - 5 \tan^{-1} \left(\frac{2 \left(\sin\left(\frac{1}{2}(c+dx)\right) + \cos\left(\frac{1}{2}(c+dx)\right) \right)}{\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)} \right)}{6d}$$

Antiderivative was successfully verified.

[In] Integrate[(3 + 5*Csc[c + d*x])^(-1), x]

[Out] (2*(c + d*x) - 5*ArcTan[(2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])])/(6*d)

fricas [A] time = 0.51, size = 33, normalized size = 1.06

$$\frac{4 dx - 5 \arctan \left(\frac{5 \sin(dx+c)+3}{4 \cos(dx+c)} \right)}{12 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3+5*csc(d*x+c)), x, algorithm="fricas")

[Out] 1/12*(4*d*x - 5*arctan(1/4*(5*sin(d*x + c) + 3)/cos(d*x + c)))/d

giac [A] time = 0.59, size = 49, normalized size = 1.58

$$\frac{dx + c + 10 \arctan \left(-\frac{3 \cos(dx+c)+\sin(dx+c)+3}{\cos(dx+c)-3 \sin(dx+c)-9} \right)}{12 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3+5*csc(d*x+c)), x, algorithm="giac")

[Out] -1/12*(d*x + c + 10*arctan(-(3*cos(d*x + c) + sin(d*x + c) + 3)/(cos(d*x + c) - 3*sin(d*x + c) - 9)))/d

maple [A] time = 0.56, size = 36, normalized size = 1.16

$$-\frac{5 \arctan \left(\frac{5 \tan\left(\frac{dx}{2} + \frac{c}{2}\right)}{4} + \frac{3}{4} \right)}{6d} + \frac{2 \arctan \left(\tan\left(\frac{dx}{2} + \frac{c}{2}\right) \right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3+5*csc(d*x+c)), x)

[Out] $-5/6/d*\arctan(5/4*\tan(1/2*d*x+1/2*c))+3/4)+2/3/d*\arctan(\tan(1/2*d*x+1/2*c))$

maxima [A] time = 0.41, size = 49, normalized size = 1.58

$$\frac{5 \arctan\left(\frac{5 \sin(dx+c)}{4(\cos(dx+c)+1)} + \frac{3}{4}\right) - 4 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(3+5*csc(d*x+c)),x, algorithm="maxima")`

[Out] $-1/6*(5*\arctan(5/4*\sin(d*x + c)/(\cos(d*x + c) + 1) + 3/4) - 4*\arctan(\sin(d*x + c)/(\cos(d*x + c) + 1)))/d$

mupad [B] time = 0.25, size = 39, normalized size = 1.26

$$\frac{x}{3} - \frac{5 \operatorname{atan}\left(\frac{7 \tan\left(\frac{c}{2} + \frac{dx}{2}\right) - 15}{24 \tan\left(\frac{c}{2} + \frac{dx}{2}\right) + 20}\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(5/sin(c + d*x) + 3),x)`

[Out] $x/3 - (5*\operatorname{atan}((7*\tan(c/2 + (d*x)/2) - 15)/(24*\tan(c/2 + (d*x)/2) + 20)))/(6*d)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{5 \csc(c + dx) + 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(3+5*csc(d*x+c)),x)`

[Out] `Integral(1/(5*csc(c + d*x) + 3), x)`

$$3.53 \quad \int \frac{1}{5+3 \csc(c+dx)} dx$$

Optimal. Leaf size=68

$$\frac{3 \log \left(\sin \left(\frac{1}{2}(c+dx) \right) + 3 \cos \left(\frac{1}{2}(c+dx) \right) \right)}{20d} - \frac{3 \log \left(3 \sin \left(\frac{1}{2}(c+dx) \right) + \cos \left(\frac{1}{2}(c+dx) \right) \right)}{20d} + \frac{x}{5}$$

[Out] 1/5*x+3/20*ln(3*cos(1/2*d*x+1/2*c)+sin(1/2*d*x+1/2*c))/d-3/20*ln(cos(1/2*d*x+1/2*c)+3*sin(1/2*d*x+1/2*c))/d

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3783, 2660, 616, 31}

$$\frac{3 \log \left(\sin \left(\frac{1}{2}(c+dx) \right) + 3 \cos \left(\frac{1}{2}(c+dx) \right) \right)}{20d} - \frac{3 \log \left(3 \sin \left(\frac{1}{2}(c+dx) \right) + \cos \left(\frac{1}{2}(c+dx) \right) \right)}{20d} + \frac{x}{5}$$

Antiderivative was successfully verified.

[In] Int[(5 + 3*Csc[c + d*x])^(-1), x]

[Out] x/5 + (3*Log[3*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/(20*d) - (3*Log[Cos[(c + d*x)/2] + 3*Sin[(c + d*x)/2]])/(20*d)

Rule 31

Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 616

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q, Int[1/Simp[b/2 + q/2 + c*x, x], x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]

Rule 2660

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] := With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]

Rule 3783

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_.))^-1), x_Symbol] := Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a*Sin[c + d*x])/b), x], x] /; FreeQ[{a, b, c, d}, x]
] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{5 + 3 \csc(c + dx)} dx &= \frac{x}{5} - \frac{1}{5} \int \frac{1}{1 + \frac{5}{3} \sin(c + dx)} dx \\ &= \frac{x}{5} - \frac{2 \operatorname{Subst}\left(\int \frac{1}{1 + \frac{10x}{3} + x^2} dx, x, \tan\left(\frac{1}{2}(c + dx)\right)\right)}{5d} \\ &= \frac{x}{5} - \frac{3 \operatorname{Subst}\left(\int \frac{1}{\frac{1}{3} + x} dx, x, \tan\left(\frac{1}{2}(c + dx)\right)\right)}{20d} + \frac{3 \operatorname{Subst}\left(\int \frac{1}{3+x} dx, x, \tan\left(\frac{1}{2}(c + dx)\right)\right)}{20d} \\ &= \frac{x}{5} + \frac{3 \log\left(3 + \tan\left(\frac{1}{2}(c + dx)\right)\right)}{20d} - \frac{3 \log\left(1 + 3 \tan\left(\frac{1}{2}(c + dx)\right)\right)}{20d} \end{aligned}$$

Mathematica [A] time = 0.05, size = 67, normalized size = 0.99

$$\frac{4(c + dx) + 3 \log\left(\sin\left(\frac{1}{2}(c + dx)\right) + 3 \cos\left(\frac{1}{2}(c + dx)\right)\right) - 3 \log\left(3 \sin\left(\frac{1}{2}(c + dx)\right) + \cos\left(\frac{1}{2}(c + dx)\right)\right)}{20d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(5 + 3*Csc[c + d*x])^-1, x]
```

```
[Out] (4*(c + d*x) + 3*Log[3*Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 3*Log[Cos[(c + d*x)/2] + 3*Sin[(c + d*x)/2]])/(20*d)
```

fricas [A] time = 0.54, size = 52, normalized size = 0.76

$$\frac{8 dx + 3 \log(4 \cos(dx + c) + 3 \sin(dx + c) + 5) - 3 \log(-4 \cos(dx + c) + 3 \sin(dx + c) + 5)}{40 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(5+3*csc(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/40*(8*d*x + 3*log(4*cos(d*x + c) + 3*sin(d*x + c) + 5) - 3*log(-4*cos(d*x + c) + 3*sin(d*x + c) + 5))/d
```

giac [A] time = 0.51, size = 45, normalized size = 0.66

$$\frac{4 dx + 4c - 3 \log \left(\left| 3 \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + 1 \right| \right) + 3 \log \left(\left| \tan \left(\frac{1}{2} dx + \frac{1}{2} c \right) + 3 \right| \right)}{20d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5+3*csc(d*x+c)),x, algorithm="giac")

[Out] 1/20*(4*d*x + 4*c - 3*log(abs(3*tan(1/2*d*x + 1/2*c) + 1)) + 3*log(abs(tan(1/2*d*x + 1/2*c) + 3)))/d

maple [A] time = 0.64, size = 53, normalized size = 0.78

$$-\frac{3 \ln \left(1 + 3 \tan \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{20d} + \frac{2 \arctan \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) \right)}{5d} + \frac{3 \ln \left(\tan \left(\frac{dx}{2} + \frac{c}{2} \right) + 3 \right)}{20d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(5+3*csc(d*x+c)),x)

[Out] -3/20/d*ln(1+3*tan(1/2*d*x+1/2*c))+2/5/d*arctan(tan(1/2*d*x+1/2*c))+3/20/d*ln(tan(1/2*d*x+1/2*c)+3)

maxima [A] time = 0.41, size = 71, normalized size = 1.04

$$\frac{8 \arctan \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} \right) - 3 \log \left(\frac{3 \sin(dx+c)}{\cos(dx+c)+1} + 1 \right) + 3 \log \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} + 3 \right)}{20d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(5+3*csc(d*x+c)),x, algorithm="maxima")

[Out] 1/20*(8*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) - 3*log(3*sin(d*x + c)/(cos(d*x + c) + 1) + 1) + 3*log(sin(d*x + c)/(cos(d*x + c) + 1) + 3))/d

mupad [B] time = 0.29, size = 27, normalized size = 0.40

$$\frac{x}{5} - \frac{3 \operatorname{atanh} \left(\frac{1}{2 \left(\frac{200 \tan \left(\frac{c}{2} + \frac{dx}{2} \right)}{27} + \frac{20}{9} \right)} + \frac{41}{40} \right)}{10d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(3/sin(c + d*x) + 5),x)`

[Out] `x/5 - (3*atanh(1/(2*((200*tan(c/2 + (d*x)/2))/27 + 20/9)) + 41/40))/(10*d)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{3 \csc(c + dx) + 5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(5+3*csc(d*x+c)),x)`

[Out] `Integral(1/(3*csc(c + d*x) + 5), x)`

3.54 $\int \csc^3(e + fx)(a + b \csc(e + fx))^m dx$

Optimal. Leaf size=274

$$\frac{\sqrt{2} (a^2 + b^2(m + 1)) \cot(e + fx)(a + b \csc(e + fx))^m \left(\frac{a+b \csc(e+fx)}{a+b}\right)^{-m} F_1\left(\frac{1}{2}; \frac{1}{2}, -m; \frac{3}{2}; \frac{1}{2}(1 - \csc(e + fx)), \frac{b(1-\csc(e+fx))}{a+b}\right)}{b^2 f(m + 2) \sqrt{\csc(e + fx) + 1}}$$

[Out] $-\cot(f*x+e)*(a+b*\csc(f*x+e))^{(1+m)}/b/f/(2+m)+a*(a+b)*\text{AppellF1}(1/2, -1-m, 1/2, 3/2, b*(1-\csc(f*x+e))/(a+b), 1/2-1/2*\csc(f*x+e))*\cot(f*x+e)*(a+b*\csc(f*x+e))^{m*2^{(1/2)}/b^2/f/(2+m)}/(((a+b*\csc(f*x+e))/(a+b))^m)/(1+\csc(f*x+e))^{(1/2)}-(a^2+b^2*(1+m))*\text{AppellF1}(1/2, -m, 1/2, 3/2, b*(1-\csc(f*x+e))/(a+b), 1/2-1/2*\csc(f*x+e))*\cot(f*x+e)*(a+b*\csc(f*x+e))^{m*2^{(1/2)}/b^2/f/(2+m)}/(((a+b*\csc(f*x+e))/(a+b))^m)/(1+\csc(f*x+e))^{(1/2)}$

Rubi [A] time = 0.34, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3840, 4007, 3834, 139, 138}

$$\frac{\sqrt{2} (a^2 + b^2(m + 1)) \cot(e + fx)(a + b \csc(e + fx))^m \left(\frac{a+b \csc(e+fx)}{a+b}\right)^{-m} F_1\left(\frac{1}{2}; \frac{1}{2}, -m; \frac{3}{2}; \frac{1}{2}(1 - \csc(e + fx)), \frac{b(1-\csc(e+fx))}{a+b}\right)}{b^2 f(m + 2) \sqrt{\csc(e + fx) + 1}}$$

Antiderivative was successfully verified.

[In] Int[Csc[e + f*x]^3*(a + b*Csc[e + f*x])^m,x]

[Out] $-\left(\frac{\cot[e + f*x]*(a + b*\csc[e + f*x])^{(1 + m)}}{(b*f*(2 + m))}\right) + (\text{Sqrt}[2]*a*(a + b)*\text{AppellF1}[1/2, 1/2, -1 - m, 3/2, (1 - \csc[e + f*x])/2, (b*(1 - \csc[e + f*x]))/(a + b)]*\cot[e + f*x]*(a + b*\csc[e + f*x])^m)/(b^2*f*(2 + m)*\text{Sqrt}[1 + \csc[e + f*x]]*((a + b*\csc[e + f*x])/(a + b))^m - (\text{Sqrt}[2]*(a^2 + b^2*(1 + m))*\text{AppellF1}[1/2, 1/2, -m, 3/2, (1 - \csc[e + f*x])/2, (b*(1 - \csc[e + f*x]))/(a + b)]*\cot[e + f*x]*(a + b*\csc[e + f*x])^m)/(b^2*f*(2 + m)*\text{Sqrt}[1 + \csc[e + f*x]]*((a + b*\csc[e + f*x])/(a + b))^m)$

Rule 138

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]/(b*(m + 1)*(b/(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c*f), 0]) && SimplerQ[c + d*x, a + b*x] && !(GtQ[f/(f*a - e*b), 0] && GtQ[f/(f*c - e*d), 0]) && SimplerQ[e + f*x, a + b*x]

Rule 139

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))
^(p_), x_Symbol] := Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 3834

```
Int[csc[(e_) + (f_)*(x_)]*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_), x_
Symbol] := Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]
```

Rule 3840

```
Int[csc[(e_) + (f_)*(x_)]^3*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_),
x_Symbol] := -Simp[(Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(b*f*(m + 2)
), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m +
1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2
, 0] && !LtQ[m, -1]
```

Rule 4007

```
Int[csc[(e_) + (f_)*(x_)]*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_)*(cs
c[(e_) + (f_)*(x_)]*(B_) + (A_)), x_Symbol] := Dist[(A*b - a*B)/b, Int[C
sc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + Dist[B/b, Int[Csc[e + f*x]*(a
+ b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, A, B, e, f, m}, x] && NeQ
[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \csc^3(e + fx)(a + b \csc(e + fx))^m dx &= -\frac{\cot(e + fx)(a + b \csc(e + fx))^{1+m}}{bf(2 + m)} + \frac{\int \csc(e + fx)(b(1 + m) - a \csc(e + fx)) dx}{b(2 + m)} \\
&= -\frac{\cot(e + fx)(a + b \csc(e + fx))^{1+m}}{bf(2 + m)} - \frac{a \int \csc(e + fx)(a + b \csc(e + fx))^m dx}{b^2(2 + m)} \\
&= -\frac{\cot(e + fx)(a + b \csc(e + fx))^{1+m}}{bf(2 + m)} - \frac{(a \cot(e + fx)) \text{Subst}\left(\int \frac{(a+bx)^{1+n}}{\sqrt{1-x}\sqrt{1+bx}} dx\right)}{b^2 f(2 + m)\sqrt{1 - \csc(e + fx)}} \\
&= -\frac{\cot(e + fx)(a + b \csc(e + fx))^{1+m}}{bf(2 + m)} + \frac{\left(a(-a - b) \cot(e + fx)(a + b \csc(e + fx))^m\right)}{b^2 f} \\
&= -\frac{\cot(e + fx)(a + b \csc(e + fx))^{1+m}}{bf(2 + m)} + \frac{\sqrt{2} a(a + b)F_1\left(\frac{1}{2}; \frac{1}{2}, -1 - m; \frac{3}{2}; \frac{1}{2}\right)}{b^2 f}
\end{aligned}$$

Mathematica [F] time = 4.51, size = 0, normalized size = 0.00

$$\int \csc^3(e + fx)(a + b \csc(e + fx))^m dx$$

Verification is Not applicable to the result.

[In] Integrate[Csc[e + f*x]^3*(a + b*Csc[e + f*x])^m,x]

[Out] Integrate[Csc[e + f*x]^3*(a + b*Csc[e + f*x])^m, x]

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b \csc(fx + e) + a\right)^m \csc(fx + e)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)^3*(a+b*csc(f*x+e))^m,x, algorithm="fricas")

[Out] integral((b*csc(f*x + e) + a)^m*csc(f*x + e)^3, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \csc(fx + e) + a)^m \csc(fx + e)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)^3*(a+b*csc(f*x+e))^m,x, algorithm="giac")

[Out] integrate((b*csc(f*x + e) + a)^m*csc(f*x + e)^3, x)

maple [F] time = 2.10, size = 0, normalized size = 0.00

$$\int (\csc^3(fx + e))(a + b \csc(fx + e))^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(f*x+e)^3*(a+b*csc(f*x+e))^m,x)

[Out] int(csc(f*x+e)^3*(a+b*csc(f*x+e))^m,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \csc(fx + e) + a)^m \csc(fx + e)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)^3*(a+b*csc(f*x+e))^m,x, algorithm="maxima")

[Out] integrate((b*csc(f*x + e) + a)^m*csc(f*x + e)^3, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + \frac{b}{\sin(e+fx)}\right)^m}{\sin(e+fx)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/sin(e + f*x))^m/sin(e + f*x)^3,x)

[Out] int((a + b/sin(e + f*x))^m/sin(e + f*x)^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \csc(e + fx))^m \csc^3(e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)**3*(a+b*csc(f*x+e))**m,x)

[Out] Integral((a + b*csc(e + f*x))**m*csc(e + f*x)**3, x)

3.55 $\int \csc^2(e + fx)(a + b \csc(e + fx))^m dx$

Optimal. Leaf size=220

$$\frac{\sqrt{2} a \cot(e + fx)(a + b \csc(e + fx))^m \left(\frac{a+b \csc(e+fx)}{a+b}\right)^{-m} F_1\left(\frac{1}{2}; \frac{1}{2}, -m; \frac{3}{2}; \frac{1}{2}(1 - \csc(e + fx)), \frac{b(1-\csc(e+fx))}{a+b}\right)}{bf\sqrt{\csc(e + fx) + 1}} \sqrt{2} (a + b)$$

[Out] $-(a+b)*\text{AppellF1}(1/2, -1-m, 1/2, 3/2, b*(1-\csc(f*x+e))/(a+b), 1/2-1/2*\csc(f*x+e))$
 $*\cot(f*x+e)*(a+b*\csc(f*x+e))^{m*2^{(1/2)}/b/f/(((a+b*\csc(f*x+e))/(a+b))^{m})/(1+$
 $\csc(f*x+e))^{(1/2)+a*\text{AppellF1}(1/2, -m, 1/2, 3/2, b*(1-\csc(f*x+e))/(a+b), 1/2-1/2*$
 $\csc(f*x+e))*\cot(f*x+e)*(a+b*\csc(f*x+e))^{m*2^{(1/2)}/b/f/(((a+b*\csc(f*x+e))/(a$
 $+b))^{m})/(1+\csc(f*x+e))^{(1/2)}$

Rubi [A] time = 0.23, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3838, 3834, 139, 138}

$$\frac{\sqrt{2} a \cot(e + fx)(a + b \csc(e + fx))^m \left(\frac{a+b \csc(e+fx)}{a+b}\right)^{-m} F_1\left(\frac{1}{2}; \frac{1}{2}, -m; \frac{3}{2}; \frac{1}{2}(1 - \csc(e + fx)), \frac{b(1-\csc(e+fx))}{a+b}\right)}{bf\sqrt{\csc(e + fx) + 1}} \sqrt{2} (a + b)$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Csc}[e + f*x]^2*(a + b*\text{Csc}[e + f*x])^m, x]$

[Out] $-((\text{Sqrt}[2]*(a + b)*\text{AppellF1}[1/2, 1/2, -1 - m, 3/2, (1 - \text{Csc}[e + f*x])/2, (b$
 $*(1 - \text{Csc}[e + f*x]))/(a + b)]*\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^m)/(b*f*\text{Sqrt}$
 $[1 + \text{Csc}[e + f*x]]*((a + b*\text{Csc}[e + f*x])/(a + b))^m) + (\text{Sqrt}[2]*a*\text{AppellF}$
 $1[1/2, 1/2, -m, 3/2, (1 - \text{Csc}[e + f*x])/2, (b*(1 - \text{Csc}[e + f*x]))/(a + b)]*$
 $\text{Cot}[e + f*x]*(a + b*\text{Csc}[e + f*x])^m)/(b*f*\text{Sqrt}[1 + \text{Csc}[e + f*x]]*((a + b*\text{Cs}$
 $c[e + f*x])/(a + b))^m)$

Rule 138

$\text{Int}[(a_ + (b_)*(x_))^{(m_)}*((c_ + (d_)*(x_))^{(n_)}*((e_ + (f_)*(x_))^{(p_)}, x_Symbol] :> \text{Simp}[(a + b*x)^{(m + 1)}*\text{AppellF1}[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]/(b*(m + 1)*(b/(b*c - a*d))^{n*(b/(b*e - a*f))^{p}}, x] /; \text{FreeQ}\{a, b, c, d, e, f, m, n, p\}, x] \&\& !\text{IntegerQ}[m] \&\& !\text{IntegerQ}[n] \&\& !\text{IntegerQ}[p] \&\& \text{GtQ}[b/(b*c - a*d), 0] \&\& \text{GtQ}[b/(b*e - a*f), 0] \&\& !(\text{GtQ}[d/(d*a - c*b), 0] \&\& \text{GtQ}[d/(d*e - c*f), 0]) \&\& \text{SimplerQ}[c + d*x, a + b*x] \&\& !(\text{GtQ}[f/(f*a - e*b), 0] \&\& \text{GtQ}[f/(f*c - e*d), 0]) \&\& \text{SimplerQ}[e + f*x, a + b*x]$

Rule 139

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))
^(p_), x_Symbol] := Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 3834

```
Int[csc[(e_) + (f_)*(x_)]*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_), x_
Symbol] := Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]
```

Rule 3838

```
Int[csc[(e_) + (f_)*(x_)]^2*(csc[(e_) + (f_)*(x_)]*(b_) + (a_))^(m_),
x_Symbol] := -Dist[a/b, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] + D
ist[1/b, Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x] /; FreeQ[{a,
b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned} \int \csc^2(e + fx)(a + b \csc(e + fx))^m dx &= \frac{\int \csc(e + fx)(a + b \csc(e + fx))^{1+m} dx}{b} - \frac{a \int \csc(e + fx)(a + b \csc(e + fx))^m dx}{b} \\ &= \frac{\cot(e + fx) \operatorname{Subst}\left(\int \frac{(a+bx)^{1+m}}{\sqrt{1-x}\sqrt{1+x}} dx, x, \csc(e + fx)\right)}{bf\sqrt{1 - \csc(e + fx)}\sqrt{1 + \csc(e + fx)}} - \frac{(a \cot(e + fx)) \operatorname{Subst}\left(\int \frac{(a+bx)^m}{\sqrt{1-x}\sqrt{1+x}} dx, x, \csc(e + fx)\right)}{bf\sqrt{1 - \csc(e + fx)}\sqrt{1 + \csc(e + fx)}} \\ &= -\frac{\left(a \cot(e + fx)(a + b \csc(e + fx))^m \left(-\frac{a+b \csc(e+fx)}{-a-b}\right)^{-m}\right) \operatorname{Subst}\left(\int \frac{\left(-\frac{a}{-a-b}\right)^m}{\sqrt{1-x}\sqrt{1+x}} dx, x, \csc(e + fx)\right)}{bf\sqrt{1 - \csc(e + fx)}\sqrt{1 + \csc(e + fx)}} \\ &= -\frac{\sqrt{2}(a + b)F_1\left(\frac{1}{2}; \frac{1}{2}, -1 - m; \frac{3}{2}; \frac{1}{2}(1 - \csc(e + fx)), \frac{b(1 - \csc(e+fx))}{a+b}\right) \cot(e + fx)}{bf\sqrt{1 + \csc(e + fx)}} \end{aligned}$$

Mathematica [F] time = 2.77, size = 0, normalized size = 0.00

$$\int \csc^2(e + fx)(a + b \csc(e + fx))^m dx$$

Verification is Not applicable to the result.

[In] Integrate[Csc[e + f*x]^2*(a + b*Csc[e + f*x])^m,x]

[Out] Integrate[Csc[e + f*x]^2*(a + b*Csc[e + f*x])^m, x]

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b \csc(fx + e) + a\right)^m \csc(fx + e)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)^2*(a+b*csc(f*x+e))^m,x, algorithm="fricas")

[Out] integral((b*csc(f*x + e) + a)^m*csc(f*x + e)^2, x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \csc(fx + e) + a)^m \csc(fx + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)^2*(a+b*csc(f*x+e))^m,x, algorithm="giac")

[Out] integrate((b*csc(f*x + e) + a)^m*csc(f*x + e)^2, x)

maple [F] time = 1.32, size = 0, normalized size = 0.00

$$\int (\csc^2(fx + e))(a + b \csc(fx + e))^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(f*x+e)^2*(a+b*csc(f*x+e))^m,x)

[Out] int(csc(f*x+e)^2*(a+b*csc(f*x+e))^m,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \csc(fx + e) + a)^m \csc(fx + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)^2*(a+b*csc(f*x+e))^m,x, algorithm="maxima")

[Out] integrate((b*csc(f*x + e) + a)^m*csc(f*x + e)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + \frac{b}{\sin(e+fx)}\right)^m}{\sin(e+fx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/sin(e + f*x))^m/sin(e + f*x)^2,x)`

[Out] `int((a + b/sin(e + f*x))^m/sin(e + f*x)^2, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \csc(e + fx))^m \csc^2(e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(csc(f*x+e)**2*(a+b*csc(f*x+e))**m,x)`

[Out] `Integral((a + b*csc(e + f*x))**m*csc(e + f*x)**2, x)`

3.56 $\int \csc(e + fx)(a + b \csc(e + fx))^m dx$

Optimal. Leaf size=104

$$\frac{\sqrt{2} \cot(e + fx)(a + b \csc(e + fx))^m \left(\frac{a+b \csc(e+fx)}{a+b} \right)^{-m} F_1 \left(\frac{1}{2}; \frac{1}{2}, -m; \frac{3}{2}; \frac{1}{2}(1 - \csc(e + fx)), \frac{b(1-\csc(e+fx))}{a+b} \right)}{f \sqrt{\csc(e + fx) + 1}}$$

[Out] -AppellF1(1/2, -m, 1/2, 3/2, b*(1-csc(f*x+e))/(a+b), 1/2-1/2*csc(f*x+e))*cot(f*x+e)*(a+b*csc(f*x+e))^m*2^(1/2)/f/(((a+b*csc(f*x+e))/(a+b))^m)/(1+csc(f*x+e))^(1/2)

Rubi [A] time = 0.07, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3834, 139, 138}

$$\frac{\sqrt{2} \cot(e + fx)(a + b \csc(e + fx))^m \left(\frac{a+b \csc(e+fx)}{a+b} \right)^{-m} F_1 \left(\frac{1}{2}; \frac{1}{2}, -m; \frac{3}{2}; \frac{1}{2}(1 - \csc(e + fx)), \frac{b(1-\csc(e+fx))}{a+b} \right)}{f \sqrt{\csc(e + fx) + 1}}$$

Antiderivative was successfully verified.

[In] Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m,x]

[Out] -((Sqrt[2]*AppellF1[1/2, 1/2, -m, 3/2, (1 - Csc[e + f*x])/2, (b*(1 - Csc[e + f*x]))/(a + b)]*Cot[e + f*x]*(a + b*Csc[e + f*x])^m)/(f*Sqrt[1 + Csc[e + f*x]])*((a + b*Csc[e + f*x])/(a + b))^m)

Rule 138

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]/(b*(m + 1)*(b/(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])

Rule 139

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b

*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]

Rule 3834

Int[csc[(e_.) + (f_.)*(x_.)]*(csc[(e_.) + (f_.)*(x_.)]*(b_.) + (a_.))^m], x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m]

Rubi steps

$$\begin{aligned} \int \csc(e + fx)(a + b \csc(e + fx))^m dx &= \frac{\cot(e + fx) \operatorname{Subst}\left(\int \frac{(a+bx)^m}{\sqrt{1-x}\sqrt{1+x}} dx, x, \csc(e + fx)\right)}{f\sqrt{1 - \csc(e + fx)}\sqrt{1 + \csc(e + fx)}} \\ &= \frac{\left(\cot(e + fx)(a + b \csc(e + fx))^m \left(-\frac{a+b \csc(e+fx)}{-a-b}\right)^{-m}\right) \operatorname{Subst}\left(\int \frac{\left(-\frac{a}{-a-b} - \frac{bx}{-a}\right)}{\sqrt{1-x}\sqrt{1+x}} dx, x, \csc(e + fx)\right)}{f\sqrt{1 - \csc(e + fx)}\sqrt{1 + \csc(e + fx)}} \\ &= \frac{\sqrt{2} F_1\left(\frac{1}{2}; \frac{1}{2}, -m; \frac{3}{2}; \frac{1}{2}(1 - \csc(e + fx)), \frac{b(1 - \csc(e+fx))}{a+b}\right) \cot(e + fx)(a + b \csc(e + fx))^m}{f\sqrt{1 + \csc(e + fx)}} \end{aligned}$$

Mathematica [F] time = 1.97, size = 0, normalized size = 0.00

$$\int \csc(e + fx)(a + b \csc(e + fx))^m dx$$

Verification is Not applicable to the result.

[In] Integrate[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x]

[Out] Integrate[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x]

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(b \csc(fx + e) + a\right)^m \csc(fx + e), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)*(a+b*csc(f*x+e))^m,x, algorithm="fricas")

[Out] integral((b*csc(f*x + e) + a)^m*csc(f*x + e), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \csc(fx + e) + a)^m \csc(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)*(a+b*csc(f*x+e))^m,x, algorithm="giac")

[Out] integrate((b*csc(f*x + e) + a)^m*csc(f*x + e), x)

maple [F] time = 1.72, size = 0, normalized size = 0.00

$$\int \csc(fx + e) (a + b \csc(fx + e))^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(csc(f*x+e)*(a+b*csc(f*x+e))^m,x)

[Out] int(csc(f*x+e)*(a+b*csc(f*x+e))^m,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \csc(fx + e) + a)^m \csc(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(f*x+e)*(a+b*csc(f*x+e))^m,x, algorithm="maxima")

[Out] integrate((b*csc(f*x + e) + a)^m*csc(f*x + e), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{b}{\sin(e+fx)}\right)^m}{\sin(e+fx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/sin(e + f*x))^m/sin(e + f*x),x)

[Out] int((a + b/sin(e + f*x))^m/sin(e + f*x), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \csc(e + fx))^m \csc(e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*csc(f*x+e))**m,x)
```

```
[Out] Integral((a + b*csc(e + f*x))**m*csc(e + f*x), x)
```

3.57 $\int (a + b \csc(e + fx))^m dx$

Optimal. Leaf size=15

$$\text{Int}\left((a + b \csc(e + fx))^m, x\right)$$

[Out] Unintegrable((a+b*csc(f*x+e))^m,x)

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a + b \csc(e + fx))^m dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*Csc[e + f*x])^m,x]

[Out] Defer[Int][(a + b*Csc[e + f*x])^m, x]

Rubi steps

$$\int (a + b \csc(e + fx))^m dx = \int (a + b \csc(e + fx))^m dx$$

Mathematica [A] time = 1.50, size = 0, normalized size = 0.00

$$\int (a + b \csc(e + fx))^m dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*Csc[e + f*x])^m,x]

[Out] Integrate[(a + b*Csc[e + f*x])^m, x]

fricas [A] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b \csc(fx + e) + a\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*csc(f*x+e))^m,x, algorithm="fricas")

[Out] integral((b*csc(f*x + e) + a)^m, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \csc(fx + e) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*csc(f*x+e))^m,x, algorithm="giac")

[Out] integrate((b*csc(f*x + e) + a)^m, x)

maple [A] time = 1.24, size = 0, normalized size = 0.00

$$\int (a + b \csc(fx + e))^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*csc(f*x+e))^m,x)

[Out] int((a+b*csc(f*x+e))^m,x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \csc(fx + e) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*csc(f*x+e))^m,x, algorithm="maxima")

[Out] integrate((b*csc(f*x + e) + a)^m, x)

mupad [A] time = 0.00, size = -1, normalized size = -0.07

$$\int \left(a + \frac{b}{\sin(e + fx)} \right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/sin(e + f*x))^m,x)

[Out] int((a + b/sin(e + f*x))^m, x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \csc(e + fx))^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csc(f*x+e))**m,x)
```

```
[Out] Integral((a + b*csc(e + f*x))**m, x)
```

3.58 $\int (a + b \csc(e + fx))^m \sin(e + fx) dx$

Optimal. Leaf size=22

$$\text{Int}(\sin(e + fx)(a + b \csc(e + fx))^m, x)$$

[Out] Unintegrable((a+b*csc(f*x+e))^m*sin(f*x+e), x)

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a + b \csc(e + fx))^m \sin(e + fx) dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*Csc[e + f*x])^m*Sin[e + f*x], x]

[Out] Defer[Int] [(a + b*Csc[e + f*x])^m*Sin[e + f*x], x]

Rubi steps

$$\int (a + b \csc(e + fx))^m \sin(e + fx) dx = \int (a + b \csc(e + fx))^m \sin(e + fx) dx$$

Mathematica [A] time = 6.36, size = 0, normalized size = 0.00

$$\int (a + b \csc(e + fx))^m \sin(e + fx) dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*Csc[e + f*x])^m*Sin[e + f*x], x]

[Out] Integrate[(a + b*Csc[e + f*x])^m*Sin[e + f*x], x]

fricas [A] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b \csc(fx + e) + a\right)^m \sin(fx + e), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*csc(f*x+e))^m*sin(f*x+e), x, algorithm="fricas")

[Out] integral((b*csc(f*x + e) + a)^m*sin(f*x + e), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \csc(fx + e) + a)^m \sin(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*csc(f*x+e))^m*sin(f*x+e),x, algorithm="giac")

[Out] integrate((b*csc(f*x + e) + a)^m*sin(f*x + e), x)

maple [A] time = 1.07, size = 0, normalized size = 0.00

$$\int (a + b \csc(fx + e))^m \sin(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*csc(f*x+e))^m*sin(f*x+e),x)

[Out] int((a+b*csc(f*x+e))^m*sin(f*x+e),x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \csc(fx + e) + a)^m \sin(fx + e) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*csc(f*x+e))^m*sin(f*x+e),x, algorithm="maxima")

[Out] integrate((b*csc(f*x + e) + a)^m*sin(f*x + e), x)

mupad [A] time = 0.00, size = -1, normalized size = -0.05

$$\int \sin(e + fx) \left(a + \frac{b}{\sin(e + fx)} \right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(e + f*x)*(a + b/sin(e + f*x))^m,x)

[Out] int(sin(e + f*x)*(a + b/sin(e + f*x))^m, x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \csc(e + fx))^m \sin(e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csc(f*x+e))**m*sin(f*x+e),x)
```

```
[Out] Integral((a + b*csc(e + f*x))**m*sin(e + f*x), x)
```

$$3.59 \quad \int (a + b \csc(e + fx))^m \sin^2(e + fx) dx$$

Optimal. Leaf size=24

$$\text{Int}(\sin^2(e + fx)(a + b \csc(e + fx))^m, x)$$

[Out] Unintegrable((a+b*csc(f*x+e))^m*sin(f*x+e)^2,x)

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a + b \csc(e + fx))^m \sin^2(e + fx) dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*Csc[e + f*x])^m*Sin[e + f*x]^2,x]

[Out] Defer[Int] [(a + b*Csc[e + f*x])^m*Sin[e + f*x]^2, x]

Rubi steps

$$\int (a + b \csc(e + fx))^m \sin^2(e + fx) dx = \int (a + b \csc(e + fx))^m \sin^2(e + fx) dx$$

Mathematica [A] time = 6.24, size = 0, normalized size = 0.00

$$\int (a + b \csc(e + fx))^m \sin^2(e + fx) dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*Csc[e + f*x])^m*Sin[e + f*x]^2,x]

[Out] Integrate[(a + b*Csc[e + f*x])^m*Sin[e + f*x]^2, x]

fricas [A] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\cos(fx + e)^2 - 1\right)\left(b \csc(fx + e) + a\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*csc(f*x+e))^m*sin(f*x+e)^2,x, algorithm="fricas")

[Out] `integral(-(cos(f*x + e)^2 - 1)*(b*csc(f*x + e) + a)^m, x)`

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \csc(fx + e) + a)^m \sin(fx + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*csc(f*x+e))^m*sin(f*x+e)^2,x, algorithm="giac")`

[Out] `integrate((b*csc(f*x + e) + a)^m*sin(f*x + e)^2, x)`

maple [A] time = 4.45, size = 0, normalized size = 0.00

$$\int (a + b \csc(fx + e))^m (\sin^2(fx + e)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*csc(f*x+e))^m*sin(f*x+e)^2,x)`

[Out] `int((a+b*csc(f*x+e))^m*sin(f*x+e)^2,x)`

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \csc(fx + e) + a)^m \sin(fx + e)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*csc(f*x+e))^m*sin(f*x+e)^2,x, algorithm="maxima")`

[Out] `integrate((b*csc(f*x + e) + a)^m*sin(f*x + e)^2, x)`

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \sin(e + fx)^2 \left(a + \frac{b}{\sin(e + fx)} \right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(sin(e + f*x)^2*(a + b/sin(e + f*x))^m,x)`

[Out] `int(sin(e + f*x)^2*(a + b/sin(e + f*x))^m, x)`

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \csc(e + fx))^m \sin^2(e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*csc(f*x+e))*m*sin(f*x+e)**2,x)
```

```
[Out] Integral((a + b*csc(e + f*x))*m*sin(e + f*x)**2, x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
```

```

If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
  If[LeafCount[result]<=2*LeafCount[optimal],
    "A",
    "B"],
  "C"],
If[FreeQ[result,Integrate] && FreeQ[result,Int],
  "C",
"F"]]

```

```

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)

```

```

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
        Max[ExpnType[expn[[1]],2]],
      Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
    If[Head[expn]===Plus || Head[expn]===Times,
      Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
    If[ElementaryFunctionQ[Head[expn]],
      Max[3,ExpnType[expn[[1]]],
    If[SpecialFunctionQ[Head[expn]],
      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
    If[HypergeometricFunctionQ[Head[expn]],
      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
    If[AppellFunctionQ[Head[expn]],
      Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],

```

```

If[Head[expn]===RootSum,
  Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
If[Head[expn]===Integrate || Head[expn]===Int,
  Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
9]]]]]]]]]]

ElementaryFunctionQ[func_] :=
MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]

SpecialFunctionQ[func_] :=
MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]

HypergeometricFunctionQ[func_] :=
MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]

AppellFunctionQ[func_] :=
MemberQ[{AppellF1},func]

```

4.0.2 Maple grading function

```

# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000

```

```

#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                      see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

    leaf_count_result:=leafcount(result);
    #do NOT call ExpnType() if leaf size is too large. Recursion problem
    if leaf_count_result > 500000 then
        return "B";
    fi;

    leaf_count_optimal:=leafcount(optimal);

    ExpnType_result:=ExpnType(result);
    ExpnType_optimal:=ExpnType(optimal);

    if debug then
        print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
ExpnType_optimal);
    fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
#   is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
#   antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F";
end if;

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;

```

```

if is_contains_complex(result) then
  if is_contains_complex(optimal) then
    if debug then
      print("both result and optimal complex");
    fi;
    #both result and optimal complex
    if leaf_count_result<=2*leaf_count_optimal then
      return "A";
    else
      return "B";
    end if
  else #result contains complex but optimal is not
    if debug then
      print("result contains complex but optimal is not");
    fi;
    return "C";
  end if
else # result do not contain complex
  # this assumes optimal do not as well
  if debug then
    print("result do not contain complex, this assumes optimal do not
as well");
  fi;
  if leaf_count_result<=2*leaf_count_optimal then
    if debug then
      print("leaf_count_result<=2*leaf_count_optimal");
    fi;
    return "A";
  else
    if debug then
      print("leaf_count_result>2*leaf_count_optimal");
    fi;
    return "B";
  end if
end if
else #ExpnType(result) > ExpnType(optimal)
  if debug then
    print("ExpnType(result) > ExpnType(optimal)");
  fi;
  return "C";
end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false

```

```

#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  end if
end proc

```



```

elif HypergeometricFunctionQ(op(0,expn)) then
  max(5,apply(max,map(ExpnType,[op(expn)])))
elif AppellFunctionQ(op(0,expn)) then
  max(6,apply(max,map(ExpnType,[op(expn)])))
elif op(0,expn)='int' then
  max(8,apply(max,map(ExpnType,[op(expn)]))) else
9
end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

```

```
#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
    MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
        asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
        asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
        fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
        gamma,loggamma,digamma,zeta,polylog,LambertW,
        elliptic_f,elliptic_e,elliptic_pi,exp_polar
        ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'`^`')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)
))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type
(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))

```

```

elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
expn))
    m1 = max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif is_appell_function(expn.func):
    m1 = max(map(expnType, list(expn.args)))
    return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
elif isinstance(expn,RootSum):
    m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,
Apply[List,expn]],7]],
    return max(7,m1)
elif str(expn).find("Integral") != -1:
    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```

4.0.4 SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
#       Albert Rich to use with Sagemath. This is used to
#       grade Fracas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#       'arctan2', 'floor', 'abs', 'log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:

```

```

        print ("func ", func , " is elementary_function")
    else:
        print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U
']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in
sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:

```

```

        return expn.operator() is None
    if expn.parent() in (ZZ, QQ, AA, QQbar):
        return expn in expn.parent() # Should always return True
    if hasattr(expn.parent(), "base_ring") and hasattr(expn.parent(), "gens"):
        return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
    return False

except AttributeError as error:
    return False

def expnType(expn):

    if debug:
        print (">>>>Enter expnType, expn=", expn)
        print (">>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #instance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(instance(expn.args[0],
Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
    elif expn.operator() == operator.pow: #instance(expn,Pow)
        if type(expn.operands()[1])==Integer: #instance(expn.args[1],Integer)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #instance(expn.args[1],
Rational)
            if type(expn.operands()[0])==Rational: #instance(expn.args[0],
Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
instance(expn,Add) or instance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))

```

```

    return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()): #is_elementary_function(expn.
func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(4,m1) #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(5,m1) #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(6,m1) #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
#is checked before calling the grading function that is passed.
#but kept it here.
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

```

```
#main function
```

```
def grade_antiderivative(result,optimal):
```

```

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

```

```

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

```

```

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)

```

```

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex

```



```
        if leaf_count_result <= 2*leaf_count_optimal:
            return "A"
        else:
            return "B"
    else: #result contains complex but optimal is not
        return "C"
else: # result do not contain complex, this assumes optimal do not as
well
    if leaf_count_result <= 2*leaf_count_optimal:
        return "A"
    else:
        return "B"
else:
    return "C"
```